Prediction of residential property prices using machine learning algorithms

Sing Yee Oh[1], *See Pheng* Hang[1*], and *Jasmine Thye Wei* Wang[1]

[1]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Skudai, Johor, Malaysia

Abstract. Residential property prices prediction is essential for evaluating market value and identifying over-pricing or under-pricing. This study investigates the performance of various machine learning algorithms, including Decision Tree (DT), Random Forest (RF), and Multilayer Perceptron (MLP) in predicting residential property prices. The study performs exploratory data analysis and principal components analysis (PCA) to reduce the dimensionality of the variables and extract the most useful variables affecting terrace house prices in Kuala Lumpur, Malaysia. A publicly available dataset is used for training and testing the algorithms, with a 70:30 proportion after pre-processing procedures. Performance indicators such as Kappa statistics, r-squared, Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Squared Error (RMSE) are used to evaluate the algorithms. The results show that RF outperforms DT and MLP, achieving the highest accuracy score of 85.82%, and highest Kappa statistics of 0.8307. The study also finds that the predicted data by RF algorithm are reliable from the train set. After performing exploratory data analysis and PCA, RF-PCA demonstrated the best performance in residential property price prediction, with an r-squared value of 0.7497, the lowest values of MAE (0.6091), MAPE (19.23%), and RMSE (1.066) compared to DT-PCA and MLP-PCA.

1 Introduction

In the 1940s, the implementation of technology transformed scientific prediction approaches by allowing scientists to perform complex computations [1]. Then, data mining techniques such as regression, clustering, and classification were then developed to enable scientists to analyse vast volumes of exponentially growing data. It is technically a collection of techniques that allow scientists to detect hidden patterns in datasets [2]. To forecast future-related factors, data scientists use a variety of methodologies to find patterns in a dataset. Machine learning and data mining benefit many fields, such as finance, healthcare, web development, robotics, image processing, and retail [3].

In economics and marketing, price prediction is especially important in the real estate market because housing prices have risen dramatically making most people now have

*Corresponding author: sphang@utm.my

difficulties buying a house [4]. Hence, having a brainy housing price predictive model is becoming more important to consumers to avoid buying an overpriced housing unit. The forecasting model of the traditional residential property prices is based on the comparison of cost and sale price that lacks an acknowledged standard and certification process, which can lead to an inefficient and inaccurate forecasting model. Hence, the presence of an accurate residential property price prediction model is especially important to help fill up the information disparity and improve the efficiency and transparency of the housing market [5].

As a consequence of the growing tendency of big data, machine learning has become a crucial prediction approach because it can solve predictive modeling issues more accurately based on their attributes and features [6]. Machine learning algorithms provide an opportunity for large volumes of data to be included in the development of predictive analytics, without the restrictions and limitations of standard modeling methodologies.

This paper adopted several machine learning approaches such as DT, RF, and MLP with the application of PCA. The dataset is obtained from the Kaggle website covering 53,883 records of all residential property types in Kuala Lumpur, Malaysia. The dataset undergoes the data preparation process inclusive of narrowing down the property type to only terrace houses, eliminating instances with missing information, and filtering meaningless data to have consistent datasets. Exploratory data analysis and PCA has been carried out to further reduce the dimensionality of the data thus obtaining the most useful variables to be used in training and testing the models, to reduce the computational cost of the study and improve the performance of the algorithms.

This research aims to (1) model the residential property price using machine learning algorithms, (2) assess the impact of PCA on the selected machine learning algorithms in residential property price prediction and (3) determine the machine learning algorithms with good performance in predicting the residential property prices.

2 Literature review

Prior research has explored the factors that impact customers' decisions when purchasing residential properties. Rachmawati et al. [7] conducted a survey on purchase decision among residents randomly chosen from specific locations in Selangor, Malaysia. The research findings suggest that customers' purchase decisions are positively and significantly impacted by factors including quality, price, location, promotion, and corporate image. Additionally, Thanaraju et al. [8] undertook investigations to identify the significant factors influencing decision-making in the house purchasing process in Kuala Lumpur. The results highlight the crucial role of locational factors, emphasizing the importance of strategic proximity to workplaces, schools, and nurseries, as well as easy accessibility in Kuala Lumpur, significantly influencing buyer preferences when purchasing a house. Analysing the factors that impact housing decisions across different generations is essential [9], as it offers valuable insights into the key elements that have a substantial influence on housing prices. This, in turn, can contribute to the development of effective models for predicting residential property prices.

Several research studies were conducted to explore the efficacy of diverse models in predicting residential property prices across different countries and regions. A comparative study between the DT classifier and multiple linear regression had been conducted to predict the residential property prices in India, with the implementation of the Scikit-Learn machine learning tool [10]. The proposed work involves utilizing various house attributes, such as the number of bedrooms, house age, transportation options, nearby schools, and the availability of shopping malls, to inform the research. Their study found that multiple linear regression model outperforms DT in predicting house prices. On the other hand, Kuvalekar et al. [11] utilized a decision tree regressor to forecast house prices, achieving an accuracy of 89%. The dataset was enriched with supplementary features such as air quality and crime rate to improve the predictive capabilities. In the study of Jha et al. [12], various machine learning algorithms, including Logistic Regression, RF, Voting Classifier, and XGBoost, are utilized to predict property sale prices in Volusia County, Florida. The primary goal of their study was to forecast whether the final sale price surpasses or falls below the initially listed sale price. Their findings demonstrates that XGBoost exhibits superior performance and model robustness in comparison to the other models.

Wang et al. [13] proposed a RF approach to estimate the house price estimation model in North Virginia. Their findings indicate that RF model outperformed the basic linear regression model, capturing hidden nonlinear relationships between house prices and features resulting in a more accurate overall estimation. Furthermore, a predictive analysis was conducted for estimating house prices in South Korea using an RF classifier. The validation was carried out with a 90:10 test-train split ratio, resulting in a MAPE value of 5.5% [14]. Adetunji et al. [15] investigates the application of the Random Forest machine learning method for predicting house prices in Boston. Comparing the predicted and actual prices indicated that the model produced reasonably accurate predictions, with an error margin of ±5 when compared to the actual values. Another research on residential price prediction was conducted in Petaling Jaya, Selangor, Malaysia using an RF regressor, yielding an R-squared of 0.991 and RMSE of 0.044 [16]. Nevertheless, despite the fact that RF display minimal prediction errors and strong generalization abilities, an in-depth review of numerous papers in the literature that apply machine learning algorithms reveals a notable neglect of the real estate mass appraisals problem [17]. This emphasizes the significance of the current work in showcasing the importance of machine learning, particularly the role of RF in mass appraisals.

In recent years, there has been a growing adoption of machine learning models like ANN to address complex and non-linear problems, specifically in the context of house price forecasting. The ANN model has been established to forecast Malaysian housing prices, utilizing a dataset of double-storey terraced houses in Johor Bahru. The model demonstrates a high value of R-squared with low values of RMSE and MAPE [18]. In the study of Rampini et al. [19], six numerical features, comprising geographic coordinates, unit surface area, number of car spots, construction year, and energy performance index, were employed to forecast housing prices in Italy. The results demonstrated superior performance from the ANN, achieving a MAE that was 5% lower compared to XGBoost. However, the models encountered a decline in accuracy when predicting the most expensive house cases due to the insufficient amount of data. This highlights the revealed reality that deep learning, with its complex multi-layer structure, demands a substantially larger amount of data compared to traditional machine learning algorithms [20]. Also, to address the complexities arising from high dimensionality in real estate price datasets, Mostofi et al. [21] utilized principal component analysis (PCA) with the goal of enhancing the accuracy of a machine learning model for predicting prices.

Hence, additional investigation into the machine learning model for predicting residential property prices in Malaysia, considering various attributes, is crucial due to its potential to improve decision-making for real estate investors, mortgage lenders, and financial institutions [8]. This underscores the model's importance in offering valuable insights into the complex dynamics of property markets.

3 Methodology

Figure 1 depicts the research framework of the study. The framework includes five main phases which are: Phase One, data acquisition and data pre-processing; Phase Two, exploratory data analysis; Phase Three, feature selection; Phase Four, model implementation and performance evaluation; and Phase Five, result analysis and conclusion.

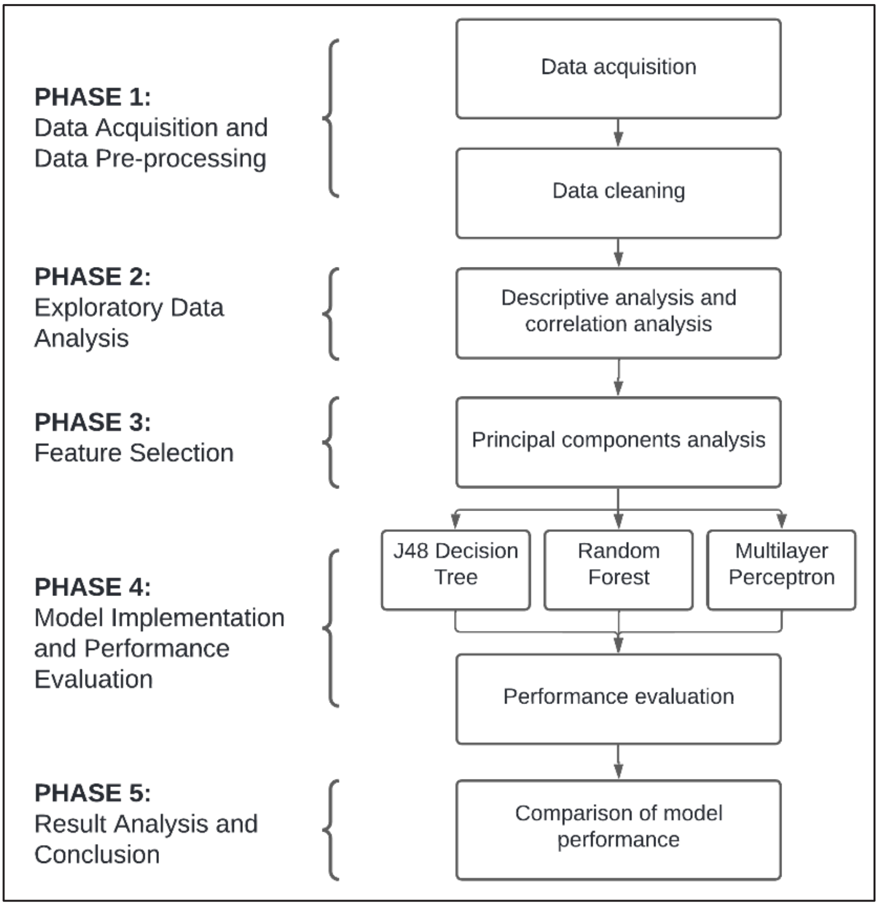

Fig. 1. Research framework of prediction of residential property prices.

3.1 Data preparation

The dataset used in this study is taken from the Kaggle website. The acquired dataset consists of 53,883 residential properties in Kuala Lumpur with eight attributes. There are three categorical attributes which are location, property type, and furnishing while there are another five numerical attributes namely price, rooms, bathrooms, car parks, and size.

In data pre-processing, the dataset undergoes the cleansing process inclusive of narrowing down the property type to only terrace houses, eliminating missing information, and filtering meaningless data to have consistent datasets. Up to 46,072 records are removed due to different property types, 4,412 records are removed due to missing and incomplete details, and 3,025 records are identified as outliers and removed as well. The attributes were then renamed, categorical data types were encoded as numbers, unusual features were removed, and the data type was changed to ensure the dataset was complete for model development. In the process of attribute engineering, 1 attribute has been removed, and 3 new attributes are added. The attribute Property_type is separated into Property_lot and Floor. While Distance_range which is calculated from the unit location to the city center, is added to check their correlation with the housing prices. Figure 2 shows the process of data pre-processing.

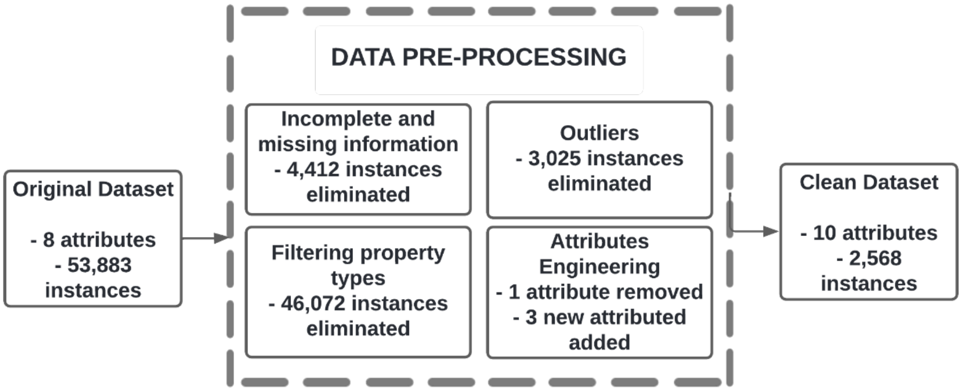

Fig. 2. Process of data pre-processing.

After data pre-processing, the original dataset consisting of 53,883 instances has been reduced to 2,568 instances. Exploratory data analysis and Pearson correlation analysis have been conducted to get a preliminary understanding of the clean dataset before applying machine learning algorithms. Table 1 summarizes the attributes description after the data pre-processing.

Table 1. Attributes description after data pre-processing.

Category	Attributes	Description
Locational attributes	Location	36 residence areas in Kuala Lumpur
	Distance_range	the distance to the city center in km *(1: less than 7.5km, 2: 7.5km-10km, 3: 10km-12.5km, 4: 12.5km-15km, 5: 15km-17.5km, 6: 17.5km-20km, 7: more than 20km)*
Structural attributes	Size_range	unit size in sq. ft. *(1: less than 1000sqft, 2: 1000-1250sqft, 3: 1250-1500sqft, 4: 1500-1750sqft, 5: 1750-2000sqft, 6: 2000-2250sqft, 7: more than 2250sqft)*
	Rooms	number of units
	Bathrooms	number of units
	Furnishing	furnishing condition *(0: unfurnished, 1: partly furnished, 2: fully furnished)*
Neighborhood attributes	Carparks	number of parking lot
	Property_lot	building type *(0: end lot, 1: intermediate, 2: corner lot)*
	Floor	floor level
Cost attributes	Price_range	categorized listed price in MYR *(1: below 500k, 2: 500k-700k, 3: 700k-900k, 4: 900k-1100k, 5: 1100-1300k, 6: 1300-1500k, 7: above 1500k)*

3.2 Exploratory data analysis

Exploratory data analysis is an unavoidable process before building a classifier or regression model to examine a dataset for patterns and find possible relations among the features based on the understanding of the dataset. The distribution of the terrace house listing prices is shown as a histogram in Figure 3.

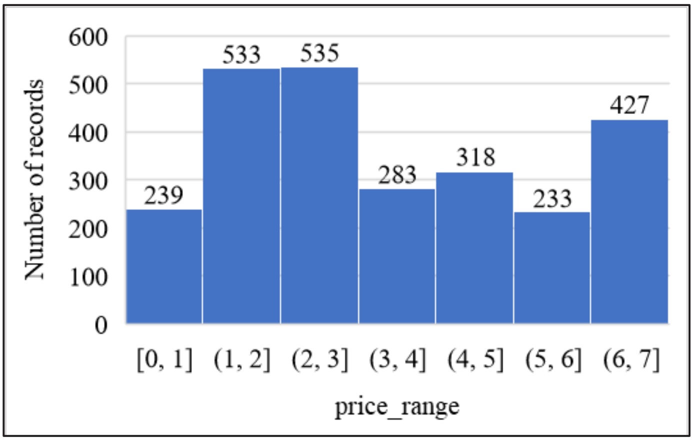

Fig. 3. Distribution of terrace house listing prices.

3.2.1 Correlation analysis

Correlation analysis determines the strength of a relationship between two attributes. In the study, price_range undergoes correlation analysis with all the other attributes to identify the strength of the linear relationship between the other attributes with price_range. Price_range has the strongest correlation with bathrooms and size_range with the value of r at 0.555 and 0.547. While price_range shows a moderate correlation with location, rooms, and floor with the value of r at 0.339, 0.479, and 0.353 respectively. The property_lot, car parks, and furnishing_status show a low correlation with price_range. Lastly, distance_range shows a very low negative correlation with price_range indicated by the value of r at -0.048, although the statistical significance is 0.014, indicating it is statistically significant. Figure 4 below shows the Pearson correlation analysis between the housing speculation and the listing price of the property unit in a heatmap.

https://doi.org/10.1051/itmconf/20246701042

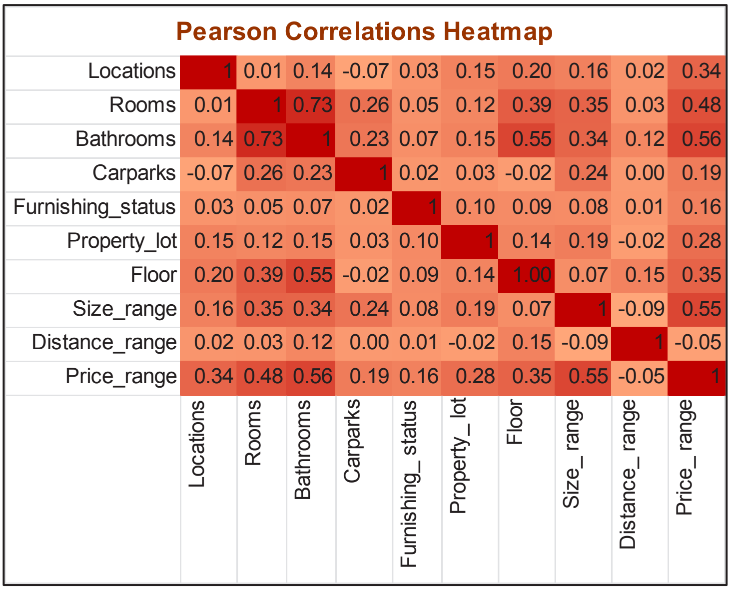

Fig. 4. Correlation analysis heatmap.

3.3 Dimensionality reduction

PCA is an unsupervised learning algorithm that is used in machine learning to reduce dimensionality. It is a statistical methodology that uses an orthogonal transformation to convert the observations of correlated features into a set of linearly uncorrelated features. There are 2 tests to perform to determine whether the data can be further processed using PCA, which are the Kaiser-Meyer-Olkin (KMO) Test and Bartlett's Test. KMO measures the sampling adequacy for each observed variable and the complete model by estimating the proportion of variance that might be a common variance among all observed variables. Bartlett's test indicates statistical significance by using a correlation matrix of the measured variables to the identity matrix which would then be consistent with the assumption that the matrix should be treated as factorable.

3.4 Machine learning approaches

All training and testing procedures are carried out using WEKA software using the default parameter and settings in the software without customization to the model of prediction of residential property prices. The J48 classifier is a Java implementation of the C4.5 algorithm which has been developed by Ross Quinlan to generate DT. RF is a supervised algorithm that is verified by many researchers in predictive studies. MLP is feed-forward ANN which is known as efficient in predictive study.

3.5 Performance indicator

The performances of different models are to be compared using performance indicators such as Kappa statistics, R-squared, MAE, MAPE, and RMSE which are commonly used in predictive models.

4 Results and discussion

4.1 Results of principal components analysis

PCA is a linear feature extractor used for unsupervised feature selection. It is essential to reduce the number of measure variables to only those are deemed to be the most useful variables to the model, to establish an accurate yet effective predictive model. PCA is performed twice in the study, the first attempt is to figure out which variable to exclude from the dataset, while the second attempt is to make sure all the variables fulfill the communalities and loading criterion. The results of both attempts of PCA are shown in the following sub-sections.

4.1.1 First principal component analysis

Before performing PCA, it is important to evaluate whether the observed variables fulfilled the KMO and Bartlett's test. In the first test, all 9 attributes are included. A p-value smaller than 0.001 indicates the test is statistically significant [22], while a KMO value of 0.672 > 0.6 indicates the sampling is adequate [22]. Hence, factor analysis is appropriate to be performed.

Communalities determine the variance proportion of each variable that can be explained by the factors. The acceptable cut-off values for communalities are at least 0.4. Generally, the stricter these cut-off values, the better fit the model has with the variables that remained. In this case, there is only the Furnishing_status attribute falls out of the acceptable region of communalities. The Kaiser criterion also known as the eigenvalue cut-off rule can be used to retain those components that have an eigenvalue that is greater than 1 [22]. The eigenvalue of 1.796 means the first component summarizes as much variation as 1.796 of the original measured variables. The second component and third components account for as much variation as 1.779 and 1.362 of the measured variables respectively. The first component accounts for 19.555% of the variation in the measured variables, the second component accounts for an additional 19.766% of the variation, and the third component accounts for another additional 15.128% of the variation. The component matrix contains correlations between each of the measured variables and the components. A loading criterion valuing an absolute value that is greater than 0.4 is probably comprised and more stable to retain certain indicators on that particular component. Judging from the rotated component matrix, the attribute Furnishing_status fails to fulfill the loading criterion with an absolute value greater than 0.4 [22].

4.1.2 Second principal component analysis

The second PCA will be done without the attribute Furnishing_status since it failed to fulfill all the criteria to identify if there are any other attributes that need to be excluded. A p-value smaller than 0.001, indicates the test is statistically significant, while a KMO value of 0.670 > 0.6, indicates the sampling is adequate. Therefore, factor analysis is appropriate to be performed.

Judging from the extractions of all variables in communalities, all variables scored more than 0.4, considered a good value for communalities. The eigenvalue of 1.804 means the first component summarizes as much variation as 1.804 of the original measured variables. The second component accounts for as much variation as 1.780 of the measured variables, whereas the third component accounts for as much variation as 1.316 of the measured variables. The first component accounts for 22.550% of the variation in the measured variables, the second component accounts for an additional 22.248% of the variation, and the third component accounts for another additional 16.453% of the variation. Judging from the second rotated component matrix, all the attributes scored greater than 0.4 at least in a component. Hence, there are no variables that need to be excluded.

4.1.3 Summary of principal components analysis

In the first attempt, there is only one attribute which is Furnishing_status, that fails to fulfill the communalities and loading criterion, therefore, it is excluded from the PCA second attempt. In the second attempt, all attributes fulfill the communalities and loading criterion. Table 2 summarizes the feature selection of the dataset based on the results of PCA.

Table 2. Different feature selection of the dataset.

Attributes	All attributes	Selected attributes
Location	√	√
Rooms	√	√
Bathrooms	√	√
Carparks	√	√
Furnishing_status	√	Not included
Property_lot	√	√
Floor	√	√
Size_range	√	√
Distance_range	√	√

The results from PCA are then implemented to the machine learning algorithms to check on the impact of PCA on different machine learning algorithms.

4.2 Model implementation

The train-test split procedure is applied to evaluate the performance of the machine learning algorithms in predictive models. The models are created by letting the machine learn through a train set, thus the performance of the prediction results is then evaluated in the test set. In the study, the datasets are split into 70:30 ratios for training and testing purposes. 70% are allocated for training, while the remaining 30% are allocated for testing. The total number of instances is 2568, hence 1798 instances are for training, while 770 instances are for testing. The training and testing procedures have to be carried out twice to check the impact of PCA on the selected machine learning algorithms. The train-test procedure has been carried out twice as the first attempt will involve all 10 attributes, while the second attempt will consider the results of PCA by removing the attribute furnishing_status since it is identified to be less appropriate to be included in the research.

Comparing the training results before and after removing the least correlated attribute furnishing_status, the accuracy of MLP increases slightly, while RF drops significantly from 89.4327% to 85.8176%. The accuracy of DT drops slightly. The kappa statistics of DT and RF increase a bit, while MLP increases by 0.002. The value of MAE of MLP decreases, while the value of RMSE of MLP increases. MAE and RMSE values of both DT and RF increase. The results show that MLP tends to capture the information better after removing the less appropriate attribute, while DT and RF perform better when there are more attributes in the dataset. MLP is one of the most fundamental approaches in ANN, while ANN is proven to have the ability to incorporate PCA well to achieve the desired superiority and more stable predictions [23]. Table 3 compares the training results before and after feature selection.

Table 3. Comparison of training results before and after feature selection.

Classifier	Before Feature Selection			After Feature Selection		
	DT	RF	MLP	DT-PCA	RF-PCA	MLP-PCA
Accuracy	71.3014	89.4327	60.8454	70.4116	85.8176	61.0122
Kappa	0.6556	0.8738	0.5271	0.6450	0.8307	0.5287
MAE	0.1141	0.0751	0.1385	0.1185	0.0817	0.1348
RMSE	0.2388	0.1604	0.2770	0.2434	0.1755	0.2792

However, based on the prediction results before and after removing the attribute furnishing_status, only MLP shows a setback with smaller r-squared, greater MAE, MAPE, and RMSE values. Meanwhile, DT and RF show improvements by obtaining greater r-squared, and smaller MAE, MAPE, and RMSE values, indicating DT and RF are more likely to fit the model. The results indicate that DT and RF tend to make better predictions with highly correlated attributes, while MLP is only able to predict better when there are more attributes in the dataset. Table 4 compares the prediction results with and without the attribute furnishing_status.

Table 4. Comparison of prediction results before and after feature selection.

Classifier	Before Feature Selection			After Feature Selection		
	DT	RF	MLP	DT-PCA	RF-PCA	MLP-PCA
R-squared	0.6376	0.7444	0.6316	0.6714	0.7497	0.6217
MAE	0.7649	0.6351	0.7987	0.7182	0.6091	0.8169
MAPE	22.3772	20.1212	22.6327	21.7096	19.2307	24.2180
RMSE	1.2817	1.0757	1.3421	1.2184	1.0660	1.3775

Figure 5 shows the RMSE of different machine learning models before and after the feature selection. DT-PCA and RF-PCA tend to have slightly higher RMSE values while MLP-PCA tends to produce a better result of RMSE value compared to the models before applying feature selection. Therefore, RF is still the best model among DT and MLP before and after combined with PCA.

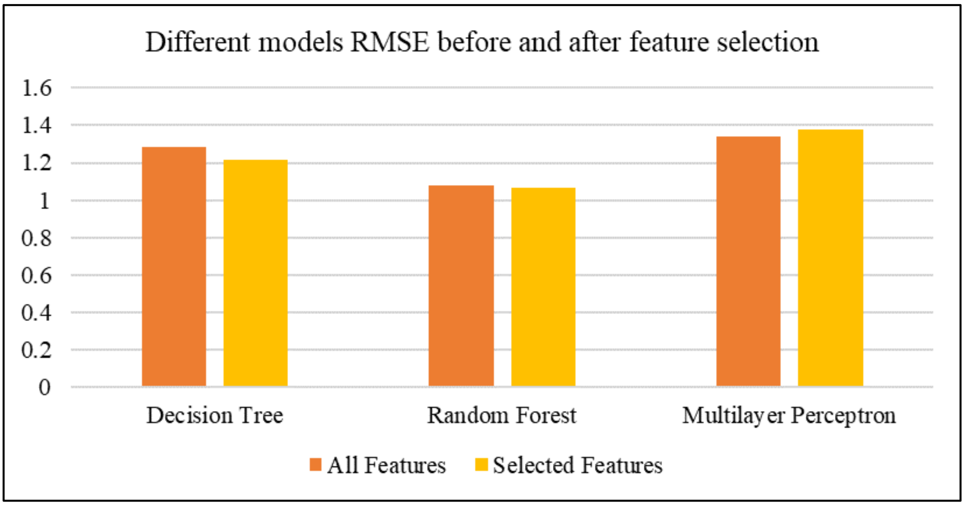

Fig. 5. RMSE of different models before and after feature selection.

Comparing the performance of the machine learning algorithms, RF outperforms J48 DT and MLP. This is because tree-based algorithms provide great accuracy and stability to predictive models. With the property of combining trees into one ensemble model, RF is able to reduce the high variance from a flexible model. In addition, MLP and DT are both weak learners which can easily be over-fitting, they can only produce a classifier that is only slightly more accurate than random classification. While RF is an ensemble method of DT, thus, the problem of instability and high variance of DT will not exist. The main process of RF is to develop lots of DT based on random sampling, and each DT is independent of each other. The results of the research fulfilled that the main advantage of RF is the chance of overfitting the model is low and high dimensionality can be avoided. As each tree does not consider all the features, the feature space reduces, making it immune to dimensionality. Thus, in the results, RF scores the best results from every perspective.

5 Conclusion

This study investigated the ability of machine learning algorithms to predict residential property prices based on different attributes. Machine learning algorithms were incorporated with the PCA method, which was utilized for predicting residential property prices in Kuala Lumpur, Malaysia. The residential property price prediction will be beneficial to all members of society especially the consumers, real estate developers, policymakers, and the government. Machine learning algorithms can optimize the forecasting processes better, as they can detect odd patterns in data, classify them into several time series classes, and match a time series to a method.

There are some limitations in this study; one of them was that the methodologies chosen are applied with the default parameters and settings in the WEKA without customization to be more adaptable to the model of prediction of residential property prices. Moreover, this study is only applicable to terrace houses in Kuala Lumpur, Malaysia. Besides, the data used were of listing price but not sale price. Therefore, for future work, it is suggested that all the housing types are considered, which are terrace houses, bungalows, apartments, condominiums, and others in the chosen regions. In addition, the data can be incorporated with factor analysis to enhance the performance of the machine learning algorithms. Furthermore, further studies can adopt different kinds of approaches such as XGBoost and k-Nearest Neighbors.

This work was supported by the Malaysian Ministry of Higher Education (MOHE) and Universiti Teknologi Malaysia (UTM) under UTMER Grant (Cost centre no: Q.J130000.3854.31J57).

References

1. F. Azimlu, S. Rahnamayan, M. Makrehchi, *House price prediction using clustering and genetic programming along with conducting a comparative study,* In Proceedings of the Genetic and Evolutionary Computation Conference Companion, 1809-1816 (2021)
2. M. Kantardzic, DataMining Concepts,1 – 18 (2003)
3. J. McCarthy, E. A. Feigenbaum, AI Magazine, **11**, 10 (1990)
4. G. P. Yeap, H. H. Lean, Malaysian Journal of Economic Studies, **54**(2), 301-314 (2017)
5. C. A. Calhoun, Housing Finance International, **17**, 31-41 (2003)
6. Q. Truong, M. Nguyen, H. Dang, B. Mei, Procedia Comput. Sci., **174**, 433-442 (2019)
7. D. Rachmawati, S. Shukri, S. Azam, A. Khatibi, Manag. Sci. Lett., **9**, 1341-1348 (2019)
8. P. Thanaraju, P. A. M. Khan, N. H. Juhari, S. Sivanathan, N. M. Khair, Planning Malaysia, **17** (2019)
9. H. Ismail, S. M. Shaari, EDP Sciences, **266**, 10-16 (2019)
10. M. Thamarai, S. Malarvizhi, IJIEEB, **12**(2), 15-20 (2020)
11. A. Kuvalekar, S. Manchewar, S. Mahadik, S. Jawale, ICAST (2020)
12. S. B. Jha, V. Pandey, R. K. Jha, R. F. Babiceanu, ArXiv (2020)
13. C. Wang, H. Wu, New Trends in Mathematical Sciences, **6**, 165-171 (2018)
14. J. Hong, H. Choi, W.S. Kim, Int. J. Strateg. Prop. Manag., **24**, 140-152 (2020)
15. A. B. Adetunji, O. N. Akande, F. A. Ajala, O. Oyewo, Y. F. Akande, G. Oluwadara, Procedia Computer Science, **199**, 806-813 (2022)
16. T. Mohd, S. Masrom, N. Johari, IJRTE, **8**, 542-546 (2019)
17. T. Dimopoulos, H. Tyralis, N. P. Bakas, D. Hadjimitsis, Advances in Geosciences, **45**, 377-382 (2018)
18. S. N. A. Rahma, N. H. A. Maimun, M. N. M. Razali, S. Ismail, Planning Malaysia, **17** (2019)
19. L. Rampini, F. Re Cecconi, Journal of Property Investment & Finance, **40**, 588-611 (2022)
20. M. M. Taye, Computers, 12, 91 (2023)
21. F. Mostofi, V. Toğan, H. B. Başağa, Organization, Technology and Management in Construction: an International Journal, **14**, 2741-2759 (2022)
22. N. Shrestha, Am. J. Appl. Math., **9**, 4-11 (2021)
23. M. M. Hameed, M. K. AlOmar, W. J. Baniya, M. A. AlSaadi, Asian J. Civ. Eng., **22**, 1019-1031 (2021)