A signal selection method for heterogeneous positioning sources

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Abstract. It can receive different signals in multi-system positioning. There might be dozens, even hundreds of signal sources. It is a great computation burden of hardware to process so many signals. On the other hands, the positioning accuracy of different signal systems is different; the worse signal may affect the positioning accuracy if there are too many signal sources. So it is necessary to select the better signal sources for positioning. This paper proposed a signal selection method for heterogeneous positioning sources. It combines the factors which affect the positioning accuracy to a function. Then assess the quality of the signals and select the best source set. The simulation results showed the efficiency of the algorithm.

1. Introduction

There are more than one signal systems in the multi-source heterogeneous positioning system [1]. Each of the signal system has different measurement and positioning accuracy [2]. It should consider the effect of the signal system when select the signal sources in heterogeneous positioning system. Meanwhile, there might be great amount of signal sources which may be up to dozens even hundreds. If there are some bad quality signals in positioning, the measurement error may be very large which lead to a low accuracy positioning [3]. On the other hand, huge amount of the signal sources leads to a huge computation which is a great burden of the hardware [4]. So it is necessary to assess the quality of the signals, and then select the best signal source set for a best positioning accuracy.

GDOP (Geometric Dilution of Precision) is often used for signal source selection of TOA (Time of Arrival) positioning system [5]. But it only considered the geometric relations between the signal sources and the receiver. It doesn't reflect the effect of the signal system and the environment factors which will greatly affect the positioning accuracy [6]. For this reason, we proposed a new signal selection method for heterogeneous sources. It considered the signal system, the positioning algorithm, the receiver parameter, the environment factors and the geometric relation between the signal sources and receiver to assess the quality of each signal sources. And then select the best signal sources set for positioning.

2. The factors that affect the positioning accuracy

The positioning error $\boldsymbol{\varepsilon}_x$ of TOA positioning system is:

$$\boldsymbol{\varepsilon}_{\mathbf{x}} = (\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\boldsymbol{\varepsilon}_{\rho} \tag{1}$$

Where $\boldsymbol{\varepsilon}_{\rho}$ the pseudorange measurement error of each signal source, G is is the geometric matrix, and:

$$\begin{cases} \mathbf{G} = \begin{bmatrix} -\mathbf{l}_{x,1} & -\mathbf{l}_{y,1} & -\mathbf{l}_{z,1} & 1 \\ -\mathbf{l}_{x,2} & -\mathbf{l}_{y,2} & -\mathbf{l}_{z,2} & 1 \\ \cdots & \cdots & \cdots & \cdots \\ -\mathbf{l}_{x,N} & -\mathbf{l}_{y,N} & -\mathbf{l}_{z,N} & 1 \end{bmatrix} \\ \mathbf{l}_{X,i} = \dfrac{x_i - \hat{x}_u}{\hat{r}_i} \end{cases} \tag{2}$$

Where $\mathbf{l}_{X,i}$ is the measurement vector of each observation direction The positioning error is related to the geometric matrix and the pseudo range measurement errors as Equation (1) shows. Assuming that the pseudorange measurement errors of each source have the same statistic features for easy. If the pseudorange measurement errors accord with normal distribution which mean is 0 and variance is σ_{ρ}^2, and each of them is incoherent. We have:

$$\mathrm{Cov}(\boldsymbol{\varepsilon}_{\mathbf{x}}) = (\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\sigma_{\rho}^2 \tag{3}$$

Where Cov(•) is the covariance of a matrix enote that:

$$\mathbf{H} = (\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1} \tag{4}$$

According to Equation (3) and (4), we can see that the positioning accuray is only related to the matrix **H** and the variance of the pseudo rang error σ_ρ^2. Because **H** is related to the relative position of the signal sources and the receiver, we can define the following equation:

$$\sigma_p^2 = \mathbf{H}\sigma_m^2 \tag{5}$$

Where σ_p^2 the variance of the positioning error is, σ_m^2 is the measurement error of each signal. The positioning error is:

$$\varepsilon_p = \sqrt{\mathrm{trace}(\mathbf{H})}\varepsilon_m = GDOP \cdot \varepsilon_m \tag{6}$$

Where ε_p is the positioning error, ε_m is the measurement error, *GDOP* is called the Geometric Dilution of Precision.

Equation (6) can also be used in non-TDOA positioning systems. For example, the matrix **H** is related to the number of APs and their position in the fingerprint positioning system [7]. So we summarized that the positioning error is related to the matrix **H** which is due to the position of the signal sources and the measurement errors $\boldsymbol{\varepsilon}_m$.

3. Signal selection method for heterogeneous sources

We assumed each measurement error is the same according to the analysis above. But when there are more than one positioning systems or the signal condition of each signal sources is different, the hypothesis will not be true. So only GDOP itself can't assess the positioning accuracy perfectly.

When each measurement error is not the same, the Equation (3) becomes:

$$\begin{aligned} \mathrm{Cov}(\boldsymbol{\varepsilon}_x) &= \mathrm{E}[\boldsymbol{\varepsilon}_x\boldsymbol{\varepsilon}_x^{\mathrm{T}}] \\ &= (\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}E[\boldsymbol{\varepsilon}_m\boldsymbol{\varepsilon}_m^{\mathrm{T}}]\mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1} \end{aligned} \tag{7}$$

$\boldsymbol{\varepsilon}_m$ Includes all the factors that affect the measurement errors, such as the CN0 (Carrier to noise ratio), multipath, interference of the people streams, etc. It is the features of the building's environment, the interference of the people streams, etc., that can equivalent to the CN0 of the signal. Namely they affect the CN0s first, and then CN0s affect the positioning accuracy. And they are time-variant. While the signal features, measurement circus parameters, etc. are related to the signal itself and the design of the receiver. And they usually are non-time-variant.

In the real circumstances, the factors that affect the time-variant and non-time-variant errors are usually correlative. For example, the variance of the code loop noise in GPS (Global Positioning System) is:

$$\sigma_{tDLL} \approx \begin{cases} \sqrt{\dfrac{B_n}{2C/N_0}D\left[1+\dfrac{2}{TC/N_0(2-D)}\right]}, & D \geq \dfrac{\pi}{B_{fe}T_c} \\ \sqrt{\dfrac{B_n}{2C/N_0}\left[\dfrac{1}{B_{fe}T_c}+\dfrac{B_{fe}T_c}{\pi-1}\left(D-\dfrac{1}{B_{fe}T_c}\right)^2\right]\cdot \left[1+\dfrac{2}{TC/N_0(2-D)}\right]}, & \dfrac{1}{B_{fe}T_c} \leq D < \dfrac{\pi}{B_{fe}T_c} \\ \sqrt{\dfrac{B_n}{2C/N_0}\dfrac{1}{B_{fe}T_c}\left[1+\dfrac{2}{TC/N_0(2-D)}\right]}, & D \leq \dfrac{1}{B_{fe}T_c} \end{cases} \tag{8}$$

where T is the redetection integration time, T_c is the code length, B_{fe} is the front-to-end bandwidth, B_n is the loop bandwidth, and D is the earl-to-late correlator spacing. They are all related to the signal system or the receiver's parameters. So they are non-time-variant. While the CN0 is related to the environment which is time-variant. So we must decouple them firstly:

$$\sigma(a_1a_2\cdots a_k) \approx \sigma_1(a_1)\sigma_2(a_2)\cdots\sigma_k(a_k) \tag{9}$$

For Equation (8), it becomes:

$\tilde{\sigma}_{tDLL}$ [illegible] (C/N_0) (10)

Where

$$\alpha = \begin{cases} \sqrt{\dfrac{B_n}{2}D}, & D \geq \dfrac{\pi}{B_{fe}T_c} \\ \sqrt{\dfrac{B_n}{2}\left[\dfrac{1}{B_{fe}T_c}+\dfrac{B_{fe}T_c}{\pi-1}\left(D-\dfrac{1}{B_{fe}T_c}\right)^2\right]}, & \dfrac{1}{B_{fe}T_c} \leq D < \dfrac{\pi}{B_{fe}T_c} \\ \sqrt{\dfrac{B_n}{2B_{fe}T_c}}, & D \leq \dfrac{1}{B_{fe}T_c} \end{cases} \tag{11}$$

$$f(C/N_0) = \frac{\sqrt{C/N_0+\eta}}{C/N_0} \tag{12}$$

Where

$$\eta = \frac{2}{T(2-D)} \tag{13}$$

The $1_{\sigma tDLL}$ results when B_{fe}=2.046MHz, T_c=1.023MHz, B_n=2Hz, D=1chip and T=1ms or 10ms respectively using Equation (8) and (10) are as Figure (1) and (2) shows. And we can see that the curves before and after decoupled are very similar.

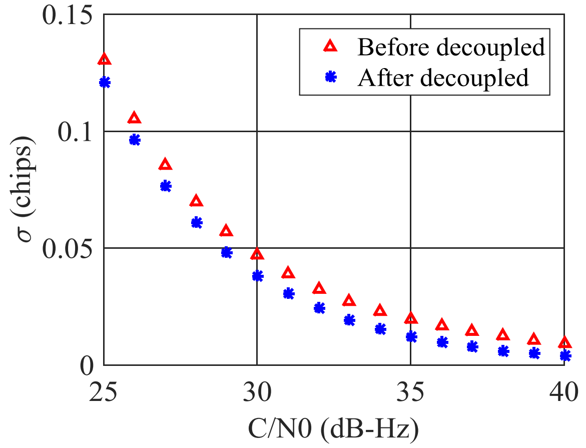

Figure 1. Comparison between the before and after decoupled $1\sigma_{tDLL}$ curves of GPS signal (T=1ms).

$$\mathrm{E}\left[\boldsymbol{\varepsilon}_m\boldsymbol{\varepsilon}_m^{\mathrm{T}}\right] \approx \begin{pmatrix} \left[\sigma_{1,1}(a_1)\sigma_{2,1}(a_2)\cdots\sigma_{k,3}(a_k)\right]^2 & 0 & \cdots & 0 \\ 0 & \left[\sigma_{1,2}(a_1)\sigma_{2,2}(a_2)\cdots\sigma_{k,2}(a_k)\right]^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left[\sigma_{1,N}(a_1)\sigma_{2,N}(a_2)\cdots\sigma_{k,N}(a_k)\right]^2 \end{pmatrix} \tag{14}$$
$$= \boldsymbol{\sigma}_1^2\boldsymbol{\sigma}_2^2\cdots\boldsymbol{\sigma}_k^2$$

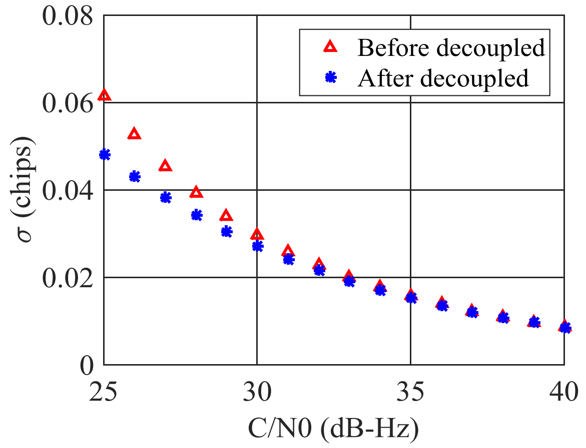

Figure 2. Comparison between the Before and after Decoupled $1\sigma_{tdll}$ Curves of gps Signal (t=10ms).

We assume that the measurement errors of each signal from each source are incoherent. We can get Equation (14). Where

$$\boldsymbol{\sigma}_k = \begin{pmatrix} \sigma_{k,1}(a_k) & 0 & \cdots & \\ 0 & \sigma_{k,2}(a_k) & \cdots & \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{k,N}(a_k) \end{pmatrix} \tag{15}$$

If $\boldsymbol{\sigma}_{1\sim i}$ is non-time-variant noise and $\boldsymbol{\sigma}_{i\sim k}$ is time-variant noise, then define:

$$\mathrm{E}\left[\boldsymbol{\varepsilon}_m\boldsymbol{\varepsilon}_m^{\mathrm{T}}\right] \approx \boldsymbol{\sigma}_1^2\boldsymbol{\sigma}_2^2\cdots\boldsymbol{\sigma}_k^2 = \boldsymbol{\alpha}^2\boldsymbol{\gamma}^2 \tag{16}$$

Where $\boldsymbol{\gamma}$ is related to the CN0 and the external factors. It is time-variant affected by the environment around. While $\boldsymbol{\alpha}$ is only related to the signal characteristics and the receiver parameters which are usually fixed or known in the receiver. So we can easily get $\boldsymbol{\alpha}$ via the signal systems because the same signal system usually has the same parameters in a receiver.

Then we can define the Measurement Error Coefficient Rate (MECR) as

$$\chi_{\mathrm{sig}k} = \frac{\alpha_{\mathrm{sig}k}}{\alpha_{\mathrm{sigRef}}} \tag{17}$$

Where the subscript sig$k$ represents the k^{th} signal system, sigRef represents the reference signal system. MECR is only related to the signal characteristics and the receiver parameters. It can clearly reflect the inherent measurement errors of different signal systems with the same CN0. It depends on the signal characteristics itself, such as the signal system type, signal frequency, etc. Another, it also depends on the receiver's performance, such as the front-to-end bandwidth, the noise bandwidth, etc. MECR is usually fixed after the design of the receiver. On the other hand, the MECR also normalized the $\boldsymbol{\alpha}$. using the MECR, Equation (16) becomes:

$$\mathrm{E}\left[\boldsymbol{\varepsilon}_m\boldsymbol{\varepsilon}_m^{\mathrm{T}}\right] \approx \boldsymbol{\chi}^2\boldsymbol{\gamma}^2\alpha_{\mathrm{sigRef}} \tag{18}$$

Put Equation (18) to Equation (7) we can get:

$$\begin{aligned} \mathrm{Cov}(\boldsymbol{\varepsilon}_{\mathbf{x}}) &= (\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\boldsymbol{\chi}^2\boldsymbol{\gamma}^2\mathbf{G}(\mathbf{G}^{\mathrm{T}}\mathbf{G})^{-1}\alpha_{\mathrm{sigRef}} \\ &= \left(\mathbf{G}^{\mathrm{T}}\boldsymbol{\chi}^{-2}\boldsymbol{\gamma}^{-2}\mathbf{G}\right)^{-1}\alpha_{\mathrm{sigRef}} \end{aligned} \tag{19}$$

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Similar to GDOP, we can define Multivariate Dilution of Precision (MDOP) as:

$$MDOP = \sqrt{\text{trace}\left(\mathbf{G}^{\mathrm{T}}\boldsymbol{\chi}^{-2}\boldsymbol{\gamma}^{-2}\mathbf{G}\right)^{-1}} \qquad (20)$$

MDOP reflects the range of the positioning error. The smallest MDOP means the best accuracy. And the largest MDOP means the worst one. So we can use MDOP to assess the quality of the signal source. And then select the best source set.

4. Simulation analysis

There were 2 kinds of positioning signal sources in simulation. And each of them had 10 notes. Their distribution is as Figure (3) shows. We assumed that the MECR=1:0.6, σ of the time-variant was inversely proportional to CN0. The CN0s of each note were 35dBHz except the one of A2# (the second not of the Source A) was 25dBHz. The standard deviation of measurement error of source A was 5m at 35dBHz.

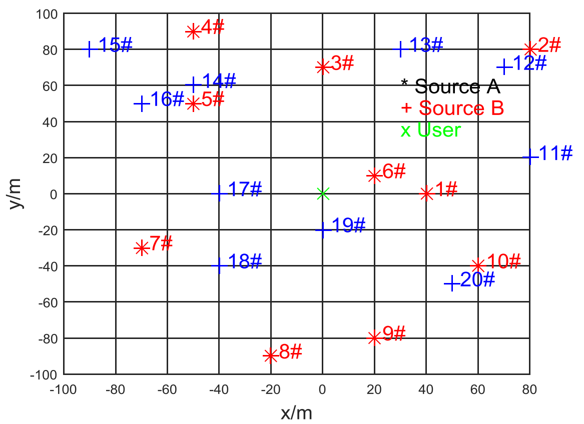

Figure 3. Distribution of All Signal Sources.

We selected 4 sources from 20 sources. And calculated CEP50s by 1000 times solution. Then acquired $C_{20}^{4} = 4845$ GDOPs, MDOPs and CEP50s. Figure (4) shows the relationships between MDOP and CEP50 while Figure (5) shows the relationships between GDOP and CEP50. The MDOPs were multiplied by 100 for clearly comparison. We can see from the figures that the CEP50 increases with the increasing of MDOP, while not that trend between CEP50 and GDOP. This means the GDOP don't acquire the best source set while the MDOP can.

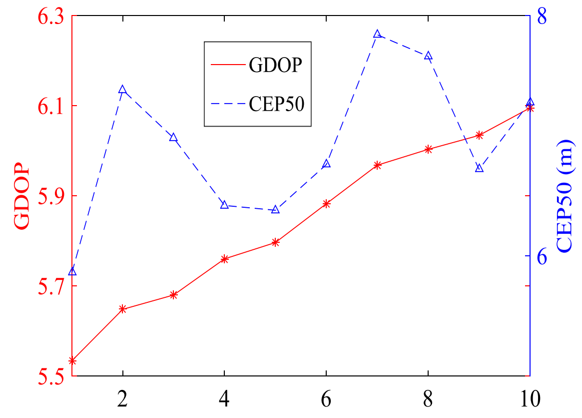

Figure 4. Relationship between CEP50 and GDOP.

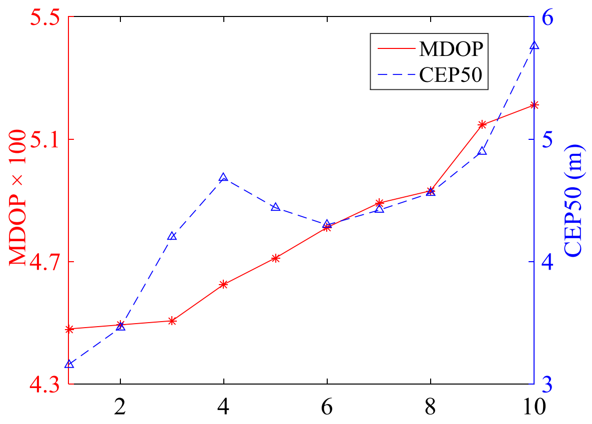

Figure 5. Relationship between CEP50 and MDOP.

Table 1 shows the source sets of the 5 smallest GDOP and MDOP. A2# is appeared in the GDOP column because of its good position. But it is not appeared in the MDOP column because of its bad CN0. So the MDOP eliminates the effect of the bad signal sources.

Table 1. The Source Sets of the 5 Smallest Gdops and Mdops

No.	Source sets of the 5 smallest GDOP	Source sets of the 5 smallest MDOP
1	A2, A6, A8, B5	A1, A5, A9, B2
2	A2, A6, A9, B5	A1, B5, B9, B10
3	A2, A4, A6, A8	A1, A4, A9, B2
4	A2, A6, A8, B6	A1, A4, A8, B1
5	A2, A8, B5, B9	A1, A7, A9, B2

5. Conclusion

With the development of the indoor/outdoor seamless positioning technology, there are more and more different positioning systems appeared. We can't have high positioning accuracy with any one single system. We must use multi positioning system for good accuracy. So the multi-source heterogeneous positioning system becomes more important. But in this kind of positioning, quality of the signal sources takes great effect to the positioning accuracy, how to assess the source quality and then select the best source set becomes very important. In this paper, we proposed a signal selection method for heterogeneous positioning sources. It combines the factors which affect the positioning accuracy to a function. Then assess the quality of the signal sources and select the best source set. The simulation results showed the efficiency of the algorithm.

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References

1. M. Glocker, H. Landau, R Leandro, et.al. 6th ESA Workshop on Satellite Navigation Technologies and European Workshop on GNSS Signals and Signal. 1-8 (2012)
2. H. K. Su, Z. X. Liao, C. H. Lin, et.al. A hybrid indoor-position mechanism based on bluetooth and WiFi communications for smart mobile devices. International Symposium on Bioelectronics and Bioinformatics. 188-191 (2015)
3. B. Bernhard, S. Felix and B. Jan. GPS-equipped wireless sensor network node for high-accuracy positioning applications. 9th European Conference on Wireless Sensor Networks. 179-195 (2012)
4. J. Roth, M. Tummala, and J. McEachen. A computationally efficient approach for hidden-Markov model-augmented fingerprint-based positioning. INTERNATIONAL JOURNAL OF SYSTEMS SCIENCE. 47, 12, 2847-2858 (2016)
5. Y.L. Teng, J.L. Wang, Q. Huang. Mathematical minimum of Geometric Dilution of Precision (GDOP) for dual-GNSS constellations. ADVANCES IN SPACE RESEARCH. 57, 1, 183-188 (2016)
6. J. Liu. Research on Satellite Selection Algorithm in Complex Environment. Radio Engineering of China. 41, 7, 39-41 (2011)
7. J. G. Hwang, K. E. Lee, and J. G. Park. An AP Selection Criteria for Enhanced Indoor Positioning using IEEE 802.11 RSSI Measurements and AP Configuration Information. J Electr Eng Technol. 11, 2, 537-542 (2016)