

# A Novel Fast Method for Point-sampled Model Simplification

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**Abstract**—A novel fast simplification method for point-sampled statue model is proposed. Simplifying method for 3d model reconstruction is a hot topic in the field of 3D surface construction. But it is difficult as point cloud of many 3d models is very large, so its running time becomes very long. In this paper, a two-stage simplifying method is proposed. Firstly, a feature-preserved non-uniform simplification method for cloud points is presented, which simplifies the data set to remove the redundancy while keeping down the features of the model. Secondly, an affinity clustering simplifying method is used to classify the point cloud into a sharp point or a simple point. The advantage of Affinity Propagation clustering is passing messages among data points and fast speed of processing. Together with the re-sampling, it can dramatically reduce the duration of the process while keep a lower memory cost. Both theoretical analysis and experimental results show that after the simplification, the performance of the proposed method is efficient as well as the details of the surface are preserved well.

## 1 Introduction

With the development of 3D scanning technologies, obtain complex point cloud is not difficult, and modeling and surface reconstructing complex objects from these samples become a significant recent trend in geometric modeling [1-2]. To address this problem, many surface reconstruction methods have been proposed[3-7]. But the production of such a large point data is due to the fact that scanning devices are not able to determine the required density of the point cloud in order to represent faithfully the real object. The material limitation causes the generation of data with big redundancy. Because modeling such large point cloud is cost so many time, the surface reconstruction is often followed by a step of simplification. Simplification will serve to eliminate the data redundancy, accelerate significantly the surface reconstruction process.

Recently, many point cloud simplifying techniques have been adopted to implement the simplification problem discussed above. Representative simplifying methods for point cloud can be summarized as clustering methods, coarse-to-fine methods and iterative methods. Among this simplifying methods, the clustering methods [8] is paid more and more attention. Clustering means to partition the point cloud into subsets and choose an exemplar in each

subset. Most of clustering algorithms need to randomly select some cluster centers in original point sets but the initial choices will affect the resultant exemplars. Nowadays, a new clustering approach named Affinity Propagation clustering (AP) is devised to overcome the disadvantage. Its main idea is consider all data points as exemplars and passing messages among data points. However, it is not suitable for a large dense similarity matrix as most of other clustering algorithms, so can not handle large-scale point cloud as its high time complexity,.

In this paper, we propose a fast two-stage point cloud simplification method for statue data. Firstly, non-uniform simplification method is used to simplify the point data to a more coarse point cloud. Second, the affinity clustering methods is used to simplify the point cloud. Finally the surface is been reconstructed. Experiments show the surface is smooth and the method works effectively.

## 2 Non-uniformly Down Sampling

The number of samples for surface reconstruction usually is huge. So here we use a technology called non-uniformly down sampling to decrease the number of the points.

If  $S$  is an  $r$ -sample of  $F$  and  $p$  is a point on  $F$ , then the distance between  $p$  and its nearest sample point  $s$  is within

$r \cdot \text{LFS}(p)$ . Since every sample is also a point on  $F$ , the distance between  $s$  and  $s_1$  is no more than  $r \cdot \text{LFS}(s)$ , where  $s_1$  is the nearest point of  $s$  in  $S$ .

As show in figure 1,  $s$  is a point in  $S$ ,  $v$  is the negative pole of  $s$ ,  $s_1$  is another point in  $S$  that  $d(s, s_1) = r_s \cdot \text{LFS}(s)$ . Let  $s$  be the center and  $r_s \cdot \text{LFS}(s)$  be the radius, we have the ball  $B_1$ . Let  $v$  be the center,  $\text{LFS}(s)$  be the radius, we have another ball  $B_2$ . In accordance with the definition of local feature size,  $s_1$  is outside ball  $B_2$ . Passing through  $s$  we make a plane  $L$  tangent to  $F$ . Because of the assumption that the surface is smooth,  $s_1$  and  $B_2$  must be located the same side of  $L$ . From the above discussion, we can see that  $F$  must be in the shaded region of figure 1 if it is in  $B_1$ .

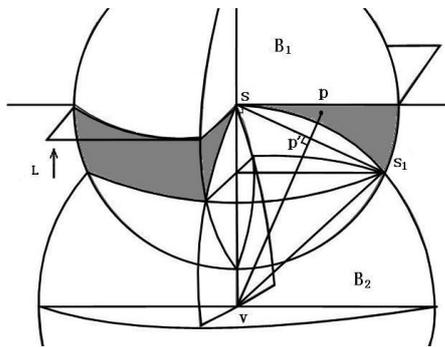


Figure 1. Down sampling

There are two factors influencing local feature size – the curvature and proximity of the other parts of the surface. However, the second factor can't affect the local feature size in a small region, so we need not take into account the factor in a local area. Let  $p$  be a point on the surface in the shaded region, and  $p'$  is the intersection of the line  $pv$  and  $B_2$ . As we all known, the more flat the surface is, the lower the curvature is. It is apparent that the curvature of point  $p$  is smaller than that of point  $p'$ . Since point  $p'$  and  $s$  are both on the ball  $B_2$ , their curvatures are the same. Thus, we have  $\text{LFS}(p) \geq \text{LFS}(s)$ . In addition, on account of that  $p$  is in the shaded region, we have  $d(s, p) \leq d(s, s_1)$ . As a result, we get  $d(s, p) \leq r_s \cdot \text{LFS}(s)$ . As  $S$  satisfies the requirement for  $r$ -sample,  $r_s$  is less than  $r$ . So, we have  $d(s, p) \leq r \cdot \text{LFS}(p)$ .

Then, we can make the following conclusion: if we can find another point  $s' \in S$  that satisfied equation  $d(s, s') \leq r \cdot \text{LFS}(p)$ ,  $S$  is an  $r$ -sample of a surface  $F$ . Therefore, if we delete all the points in the shaded area excepting the farthest one and  $s$  itself, the down sampled point set  $S'$  is still an  $r$ -sample of  $F$ , and an  $r$ -sample point set is sufficiently dense for correctly reconstruction if  $r$  is no more than 0.5[4]. Thus,  $r$  should be less than 0.5 here. In fact, we obtain the adoptive value of  $r$  through affinity propagation clustering algorithm, which will be discussed in the next section.

Down sampling:

- 1 Initial every point in  $S$  as unmarked
- 2 for( $i=0; i < n; i++$ ) {
- 3 if  $s_i$  is unmarked {
- 4  $d_{max}=0; m=0$

- 5 for( $j=0; j < n; j++$ )
- 6 if  $s_j$  is unmarked {
- $S$
- 7 if  $d(s_i, s_j) \leq r \cdot \text{LFS}(s_i)$
- 8 marked  $s_j$ ;
- 9 if  $(d(s_i, s_j) < d_{max})$  update  $d_{max}$  and  $m$
- 10 } }
- 11 unmarked  $s_m$ ;
- 12 select all the unmarked points as the down sampled point set

We first extract global symmetries from point set. We limit the symmetry transformation to reflection symmetry since it is the most dominant symmetry category in ancient architectures. We follow the basic idea in [4] and adapt it to point sets obtained from multi-view reconstruction method. We evenly sample 1000 to 2000 points in the original point set and each pair of sampling points is viewed as a pair of potential symmetric points. In 3D space, the plane of symmetry determined by this pair of points can be expressed as  $Ax+By+Cz+D=0$ . We represent the plane using a 4D vector  $(A, B, C, D)$  which defines the reflection transformation space, and each pair is associated with such a 4D vector. By looking at the original point set, we can deduce that all of the 4D vectors should be clustered around the 4D vector corresponding to the reflection transformation actually present in the object, since there are more pairs agree on the true reflection plane. In practice, the sampling point sets exhibits approximate symmetries instead of precise symmetries. Thus, we utilize affinity propagation clustering algorithm to explore the distribution of the 4D vectors and find clusters in the transformation space.

### 3 Affinity Propagation Clustering algorithm

AP has been proved to be useful for many domains such as in face images, gene expressions and text summarization [8-9]. The main idea of this algorithm is regarding each data point as a node in network and recursively transmits messages among nodes until a good subset appear. AP takes a collection of real-valued similarities among data points as input, denotes  $S(i, k)$  as similarity between point  $i$  and point  $k$ . Similarity  $S(i, k)$  indicates that how well the data point  $k$  is suited to be the exemplar for data point  $i$ . Generally speaking, each similarity is set to a negative squared error. In point cloud simplification, set

$$S(i, k) = -(\|x_i - x_k\|^2 + \|y_i - y_k\|^2 + \|z_i - z_k\|^2) \quad (1)$$

Where  $(x_i, y_i, z_i)$  is the coordinate of point  $i$ . Another parameter of input is  $S(k, k)$ , it locates the diagonal of  $S$ , and shows probability of point  $k$  to be chosen as exemplar. Larger values of  $S(k, k)$  are more likely to be chosen as exemplars. Therefore  $S(k, k)$  are called 'preferences'. In this

sense the numbers of exemplars are influenced by the values of the  $S(k, k)$ .

AP has two kinds of messages to exchange between data points, and each takes a different competition into account, i.e., ‘responsibility’ and ‘availability’. The ‘responsibility’  $R(i, k)$  sends from data  $i$  to candidate exemplar point  $k$ , and reflects the accumulated evidence for point  $k$  as the exemplar for point  $i$ . The ‘availability’  $A(i, k)$  reflects the accumulated evidence for appropriate of point  $i$  to choose point  $k$  as its exemplar.  $R$  and  $A$  update and exchange messages between pairs of points with known similarities. After several iterations, some points are assigned to other exemplars, and availabilities of them will drop below zero by updated rules.

**Table 1.** Algorithm Ap

Algorithm AP:	
Input	$S, P$ (similarities and preferences of point sets).
Output	$D$ (exemplars for point sets)
Step1	initialize $A(i, k)=0$ .
Step2	update $R, R(i, k) = S(i, k) - \{A(i, j) + S(i, j)\},$ $R(k, k) = S(i, k) - \{A(i, j) + S(i, j)\}$
Step3	update $A,$ $A(i, k) = \min\{0, R(k, k) + \sum_{j \in \{i, k\}} \max(0, R(j, k))\},$ $A(k, k) = \sum_{j \in \{i, k\}} \max(0, R(j, k))$
Step4	iteratively update terminated after a fixed number of iterations or messages stay constantly.

From the setp2 of algorithm AP, equal mark both add  $A(i, k)$  to the left and right of  $R(i, k) = S(i, k) - \{A(i, j) + S(i, j)\}$ , we get

$$R(i, k) + A(i, k) = S(i, k) + A(i, k) - \{A(i, j) + S(i, j)\} \quad (2)$$

The value of  $R+A$  shows the effective of affinity points and decides the probability of point  $i$  as exemplar of point  $k$ . In iterations responsibilities and availabilities are both related to the input similarities, and they are combined to confirm exemplars.  $R(k, k)$  and  $A(k, k)$  are increasing along with the accretion of  $S(k, k)$ , and then the probability of point  $k$  as exemplar is increasing. In order to void oscillations, the algorithm introduces damp factor, and generally set  $\lambda=0.5$ . In each iteratively update, the formulas are changed as follows.

$$R^{(t)}(i, k) = (1 - \lambda) * \{S(i, k) - \max\{A(i, j) + R(i, j)\}\} + \lambda * R^{(t-1)}(i, k) \quad (3)$$

$$A^{(t)}(i, k) = (1 - \lambda) * \{0, S(k, k) + \sum_{j \in \{i, k\}} \max\{0, R(j, k)\}\} + \lambda * A^{(t-1)}(i, k) \quad (4)$$

$$A^{(t)}(k, k) = (1 - \lambda) * \{\sum_{j \neq k} \max\{0, R(j, k)\}\} + \lambda * A^{(t-1)}(k, k) \quad (5)$$

Points send messages like Fig 2.

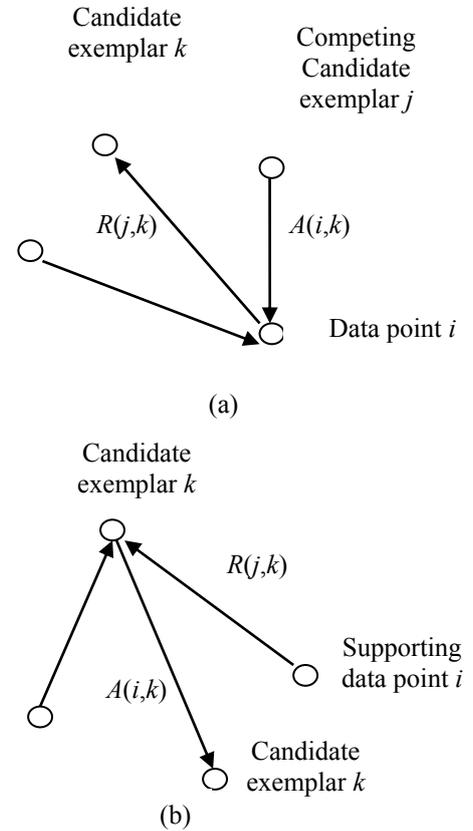


Figure 2. (a)Sending Responsibilities (b)Sending Availabilities

## 4 Simplification Algorithm

Our simplification algorithm is presented as follow:

**Table 2.** Simplification Algorithm

Simplification algorithm	
Input	Original statue point cloud $D$
Output	Final simplified statue point cloud $FSD$
Step1	Get a more simple statue point cloud $SD$ after non-uniformly down sampling
Step2	Compute similarity matrix $T$ of $SD$
Step3	Set the threshold value, run AP clustering with $T$
Step4	If the number of exemplar points output is less than threshold value, back to step1, reset the parameter $r$ of the non-uniformly down sampling method
Step5	Output the point set exemplar points $ED$ as $FSD$

## 5 Experimental Results and Analysis

We experiment with the point-sampled model Twirl and Squirrel. The models both download from the webset of Stanford University computer graphics laboratory. Fig.3 illustrated the results of our proposed method on Twirl and Fig.4 illustrated the results of our proposed method on Squirrel.

From Fig.3, we can see, in the original data, large part of data is missing in the right side, our method still captures the symmetry accurately and completes the missing data successfully. Fig. 4 illustrates that the missing data in the ear, leg and feet of the Squirrel are completed.

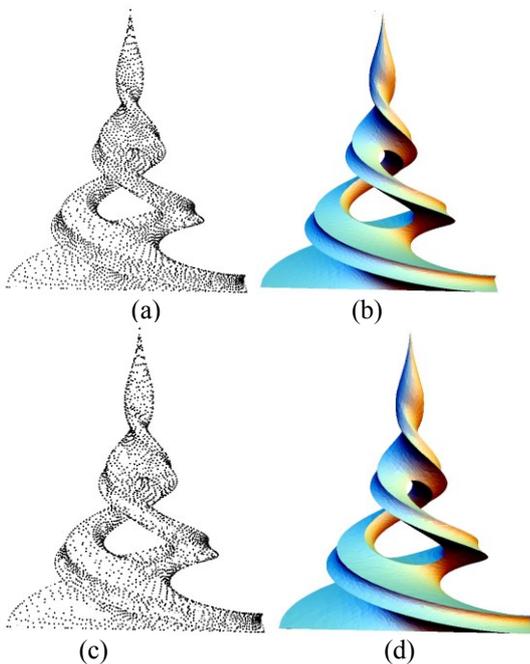


Figure 3. Simplification off Twirl and Squirrel model. (a)Original point clouds Twirl, the number of the point is 7000. (b)Original Twirl model surface. (c) Simplified by our methd, the number of the Twirl preserved is 3600. (d)The output of our algorithm running on the Twist model.

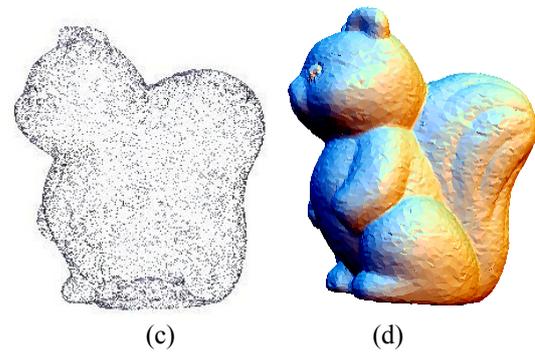
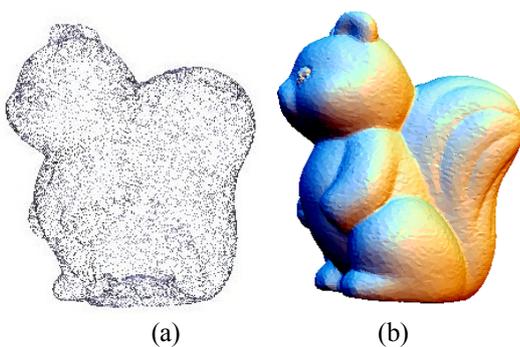


Figure 4. Simplification of Squirrel model. (a)Original point clouds Squirrel, the number of the point is 9000. (b)Original Squirrel model surface. (c) Simplified by our methd, the number of the Squirrel preserved is 4560. (d)The output of our algorithm running on the Squirrel model.

Just as our expectation, almost all the missing parts of point-sampled models are reconstructed and the density of final point cloud is varied according to the surface's detail. For example, the samples are still very dense in the region like the top of Twirl and the eyes of Squirrel.

## 6 Conclusion

We have presented a novel simplification method for point-sampled models using non-uniform down sampling and affinity propagation clustering. Guaranteeing the topological shape, we use non-uniform down sampling to generate a more simple point cloud. With the change of parameter  $r$ , the detail of the point cloud is change. So we use affinity propagation clustering to get an optimistic value of  $r$ . Experiments prove the efficiency of this algorithm.

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