

Hollow system with fin. Transient Green function method combination for two hollow cylinders

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Abstract. In this paper we develop mathematical model for three dimensional heat equation for the system with hollow wall and fin and construct its analytical solution for two hollow cylindrical sample. The method of solution is based on Green function method for one hollow cylinder. On the conjugation conditions between both hollow cylinders we construct solution for system wall with fin. As result we come to integral equation on the surface between both hollow cylinders. Solution is obtained in the form of second kind Fredholm integral equation. The generalizing of Green function method allows us to use Green function method for regular non-canonical domains.

1 INTRODUCTION

Systems with extended surfaces (fins, spines) are related to very different branches of technique. Usually their mathematical modeling is realized by one dimensional steady-state assumptions [1]-[3]. In our previous papers we have constructed various two and three dimensional analytical approximate and exact solutions [4 -25].

In [2] the so-called Murray – Gardner assumptions are formulated. Some of them are:

- 1) The heat flow in the fin and the temperature at any point on the fin remains constant with time;
- 4) The temperature of the medium surrounding the fin is uniform;
- 5) The fin width is small compared to its height, so the temperature gradients across the fin width may be neglected;
- 6) The temperature at the base of the fin is uniform;
- 7) The heat transferred through the outmost edge of the fin (the fin tip) is negligible compared to that through the lateral surfaces (faces) of the fin.

In this paper we obtain exact transient solution by the Green function method [11 -13]. We give up all of these Murray – Gardner assumptions.

We consider three-dimensional statement for non-homogeneous equation with partly non-homogeneous boundary conditions.

In recent years, we have been able to generalize the Green's function method to areas, which consist of several canonical connected sub-areas, and thus we have obtained the exact solutions for the L-, T- and II-type areas [11 -13], [15,], 21, 23]. We have constructed two cylinders [17], and two-layer sphere [15]. Cylinder was investigated in the paper [18].

2 FORMULATION OF 3-D PROBLEM

For this purpose instead of the time depended ordinary differential equation [1] – [3], we consider the following partial differential equation for the first cylinder:

$$\frac{1}{a^2} \frac{\partial U}{\partial t} = \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2} + \frac{\partial^2 U}{\partial z^2} \right] + F(r, \varphi, z, t), r \in [R_0, R], \varphi \in [0, 2\pi], \quad (1)$$

$$z \in [0, l], a^2 = \frac{k}{c\rho}.$$

Here c is specific heat capacity, k - heat conductivity coefficient, ρ - density. For the second cylinder which can be of different material:

$$\frac{1}{a_0^2} \frac{\partial U_0}{\partial t} = \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial U_0}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 U_0}{\partial \varphi^2} + \frac{\partial^2 U_0}{\partial z^2} \right], \quad (2)$$

$$r \in [R_0, R_1], \varphi \in [0, 2\pi], z \in [l, l_0], a_0^2 = \frac{k_0}{c_0\rho_0}.$$

We formulate the conjugations conditions as ideal thermal contact for $r \in [R_0, R_1]$:

$$U|_{z=l-0} = U_0|_{z=l+0}, \quad (3)$$

$$k \frac{\partial U}{\partial z} \Big|_{z=l-0} = k_0 \frac{\partial U_0}{\partial z} \Big|_{z=l+0}. \quad (4)$$

We assume that the outer diameter of second cylinder is smaller as for primary cylinder: $R_1 < R$. The boundary conditions we assume partly homogeneous:

$$\left(\frac{\partial U}{\partial r} - k_0 U \right) \Big|_{r=R_0} = 0, k_0 = \frac{h}{k}, \quad (5)$$

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$$\left(\frac{\partial U}{\partial r} + k_0 U\right)\Big|_{r=R} = g_0(\varphi, z, t), \quad (6)$$

$$\left(\frac{\partial U}{\partial z} - k_0 U\right)\Big|_{z=0} = g_1(r, \varphi, t), \quad (7)$$

$$\left(\frac{\partial U}{\partial z} + k_1 U\right)\Big|_{z=l, R_1 < r < R} = g_2(r, \varphi, t), \quad (8)$$

$$\left(\frac{\partial U_0}{\partial r} - k_2 U_0\right)\Big|_{r=R_0} = 0, k_2 = \frac{h_0}{k_0}, \quad (9)$$

$$\left(\frac{\partial U_0}{\partial r} + k_2 U_0\right)\Big|_{r=R_1} = 0, \quad (10)$$

$$\left(\frac{\partial U_0}{\partial z} + k_3 U_0\right)\Big|_{z=l_1} = g_3(r, \varphi, t). \quad (11)$$

Here h, h_0 is heat exchange coefficient. The initial conditions for the both cylinders are assumed in following form:

$$U|_{t=0} = U_0(r, \varphi, z), \quad (12)$$

$$U_0|_{t=0} = U_{00}(r, \varphi, z), \quad (13)$$

For 3-D mathematical model it is important that solution in φ -direction is continuous and smooth. These 2 conditions are important for the reduction of 3-D model to 2-D model by conservative averaging method [9, 23] and [24]:

$$U|_{\varphi=0} = U|_{\varphi=2\pi}, \quad (14)$$

$$\frac{\partial U}{\partial \varphi}\Big|_{\varphi=0} = \frac{\partial U}{\partial \varphi}\Big|_{\varphi=2\pi}. \quad (15)$$

3 SOLUTION OF 3-D PROBLEM

The Green function for two cylinders is not known. The new idea is to combine the two conjugations conditions in the form of third type boundary condition with unknown non-homogeneity in the right hand side.

3.1. Solution for the wall

As for the fin we combine the conjugations conditions (3), (4) for $R_0 < r < R_1$:

$$\left(\frac{\partial U}{\partial z} + k_1 U\right)\Big|_{z=l-0} = F_0(r, \varphi, t),$$

$$F_0(r, \varphi, t) = \left(\frac{k_0}{k} \frac{\partial U_0}{\partial z} + k_1 U_0\right)\Big|_{z=l+0}. \quad (16)$$

The Green function (1) is known; see [28]:

$$G(r, \varphi, z, \xi, \eta, \zeta, t) = G_1(r, \varphi, \xi, \eta, t) G_2(z, \zeta, t),$$

$$G_1(r, \varphi, \xi, \eta, t) = \frac{1}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_n \mu_{nm}^2 \times \frac{Z_n(\mu_{nm} r) Z_n(\mu_{nm} \xi)}{B_{nm}} \times \frac{\cos[n(\varphi - \eta)] \sin(\lambda_{nms} t)}{\lambda_{nms}} \exp(-a_\tau^2 \mu_{nm}^2 t), \quad (17)$$

$$B_{nm} = (k_2^2 R + \mu_{nm}^2 R^2 - n^2) Z_n^2(\mu_{nm} R) - (k_2^2 R_0 + \mu_{nm}^2 R_0^2 - n^2) Z_n^2(\mu_{nm} R_0),$$

$$G_2(z, \zeta, t) = \sum_{s=1}^{\infty} \frac{h_s(z) h_s(\zeta)}{\|h_s\|^2} \exp(-a^2 \lambda_s^2 t).$$

The solution for three-dimensional problem for the wall (1) is in following form:

$$U(r, \varphi, z, t) = H(r, \varphi, z, t) + a^2 \int_0^t d\tau \times \int_0^{2\pi} d\zeta \int_{R_0}^{R_1} F_0(\xi, \zeta, \tau) G(r, \varphi, z, \xi, \zeta, l, t - \tau) d\xi + \int_{R_0}^{R_1} \xi d\xi \int_0^{2\pi} d\zeta \times \int_l^{l_1} U_0(\xi, \eta, \zeta) G(r, \varphi, z, \xi, \eta, \zeta, t) d\eta. \quad (18)$$

Here

$$H(r, \varphi, z, t) = a^2 R \times \int_0^t d\tau \int_0^{2\pi} d\eta \int_0^l g_0(\xi, \zeta, \tau) G(r, \varphi, z, R, \eta, \zeta, t - \tau) d\zeta + -a^2 \times \int_0^t d\tau \int_0^{2\pi} d\zeta \int_{R_0}^{R_1} g_1(\xi, \zeta, \tau) \xi G(r, \varphi, z, \xi, \zeta, 0, t - \tau) d\xi + a^2 \int_0^t d\tau \int_0^{2\pi} d\zeta \int_{R_1}^R g_2(\xi, \zeta, \tau) G(r, \varphi, z, \xi, \zeta, l, t - \tau) d\xi.$$

Here $J_n(r), Y_n(r)$ are Bessel's functions

$$Z_n(\mu_{nm}r) = \left[\mu_{nm}J'_n(\mu_{nm}R_0) - k_2J_n(\mu_{nm}R_0) \right] Y_n(\mu_{nm}r) - \left[\mu_{nm}Y'_n(\mu_{nm}R_0) - k_2Y_n(\mu_{nm}R_0) \right] J_n(\mu_{nm}r) \quad (19)$$

and μ_{nm} are the positive roots of the transcendental equation:

$$\frac{\left[\mu J'_n(\mu R_0) - k_0J_n(\mu R_0) \right]}{\left[\mu J'_n(\mu R_1) + k_0J_n(\mu R_1) \right]} = \frac{\left[\mu Y'_n(\mu R_0) - k_0Y_n(\mu_{nm}R_0) \right]}{\left[\mu Y'_n(\mu R_1) + k_0Y_n(\mu R_1) \right]} \quad (20)$$

For the second part of the Green function $G_2(z, \zeta, t)$ we have:

$$h_s(z) = \cos(\lambda_s z) + \frac{k_0}{\lambda_s} \sin(\lambda_s z),$$

$$\|h_s\|^2 = \frac{k_1}{2\lambda_s^2} \frac{\lambda_s^2 + k_0^2}{\lambda_s^2 + k_1^2} + \frac{k_0}{2\lambda_s^2} + \frac{l}{2} \left(1 + \frac{k_0^2}{\lambda_s^2} \right), \quad (21)$$

$$A_n = \begin{cases} 1, & \text{if } n = 0, \\ 2, & \text{if } n > 0. \end{cases}$$

The eigenvalues β_s are positive roots of the transcendental equations:

$$\frac{tg(\lambda l)}{\lambda} = \frac{k_0 + k_1}{\lambda^2 - k_0k_1} \quad (22)$$

The wall solution (18) contains the fin temperature and its derivative $F_0(\xi, \varsigma, \tau)$ in the formula (16). It means that we must solve the solution for the fin.

3.2. Solution for the fin

The combination of conjugations conditions (3), (4) gives such third type boundary condition :

$$\left(\frac{\partial U_0}{\partial z} - U_0 \right) \Big|_{z=l} = F(r, \varphi, t),$$

$$F(r, \varphi, t) = \left(\frac{k}{k_0} \frac{\partial U}{\partial z} - U \right) \Big|_{z=l=0} \quad (23)$$

The solution in three-dimensional problem for the fin (2) is in following form:

$$U_0(r, \varphi, z, t) = H_0(r, \varphi, z, t) + \int_{R_0}^{R_1} \xi d\xi \int_0^{2\pi} d\eta \times \int_l^{l_1} U_{00}(\xi, \eta, \varsigma) G_0(r, \varphi, z, \xi, \eta, \varsigma, t) d\varsigma - a^2 \int_l^t d\tau \times \int_0^{2\pi} d\eta \int_{R_0}^{R_1} F(\xi, \eta, \tau) \xi G_0(r, \varphi, z, \xi, \eta, l, t - \tau) d\xi \quad (24)$$

Here in the function we have collected all known boundary right hand side functions:

$$H_0(r, \varphi, z, t) = a^2 \times \int_0^t d\tau \int_0^{2\pi} d\eta \int_{R_0}^{R_1} g_3(\xi, \eta, \tau) \xi G_0(r, \varphi, z, \xi, \eta, l_1, t - \tau) d\xi \quad (25)$$

The Green function is similar as (17) for initial-boundary problem for Klein-Gordon equation (2). It is known; see [27]:

$$G_0(r, \varphi, z, \xi, \eta, \zeta, t) = G_{01}(r, \varphi, \xi, \eta, t) G_{02}(z, \zeta, t),$$

$$G_{01}(r, \varphi, \xi, \eta, t) = \frac{1}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_n \mu_{0nm}^2 \times \frac{Z_n(\mu_{0nm}r) Z_n(\mu_{0nm}\xi)}{B_{0nm}} \times \frac{\cos[n(\varphi - \eta)] \sin(\lambda_s t)}{\lambda_s} \exp(-a_\tau^2 \mu_{0nm}^2 t), \quad (26)$$

$$B_{0nm} = (k_2^2 R_1 + \mu_{0nm}^2 R_1^2 - n^2) Z_n^2(\mu_{0nm} R_1) - (k_2^2 R_0 + \mu_{0nm}^2 R_0^2 - n^2) Z_n^2(\mu_{0nm} R_0),$$

$$G_2(z, \zeta, t) = \sum_{s=1}^{\infty} \frac{h_{0s}(z) h_{0s}(\zeta)}{\|h_{0s}\|^2} \exp(-a_0^2 \lambda_s^2 t).$$

Here μ_{0nm} are the positive roots of the transcendental equation:

$$\left[\mu J'_n(\mu R_0) - k_2J_n(\mu R_0) \right] \times \left[\mu Y'_n(\mu R_1) + k_2Y_n(\mu R_1) \right] = \left[\mu Y'_n(\mu R_0) - k_2Y_n(\mu_{nm}R_0) \right] \times \left[\mu J'_n(\mu R_1) + k_2J_n(\mu R_1) \right].$$

For the second part of the Green function $G_{02}(z, \zeta, t)$ we have:

$$h_{0s}(z) = \cos(\lambda_{0s}z) + \frac{\sin(\lambda_{0s}z)}{\lambda_{0s}},$$

$$\|h_{0s}\|^2 = \frac{k_3}{2\lambda_{0s}^2} \frac{\lambda_{0s}^2 + 1}{\lambda_{0s}^2 + k_3^2} + \frac{1}{2\lambda_{0s}^2} + \frac{l_1 - l}{2} \left(1 + \frac{1}{\lambda_{0s}^2}\right),$$

$$A_n = \begin{cases} 1, & \text{if } n = 0, \\ 2, & \text{if } n > 0. \end{cases}$$

The eigenvalues β_s are positive roots of the transcendental equations:

$$\frac{tg[\lambda(l_1 - l)]}{\lambda} = \frac{1 + k_3}{\lambda^2 - k_3}.$$

4 SOLUTION AS COMBINATION OF WALL AND FIN SOLUTIONS

The solution in three-dimensional problem for the fin (24) is in following form:

$$U_0(r, \varphi, z, t) = -\bar{H}_0(r, \varphi, z, t) - a^2 \int_0^t d\tau \int_0^{2\pi} d\eta \times$$

$$\int_{R_0}^{R_1} F(r, \varphi, t) G_0(r, \varphi, z, \xi, \eta, l, t - \tau) d\xi, \quad (27)$$

$$\bar{H}_0(r, \varphi, z, t) = H_0(r, \varphi, z, t) + \int_{R_0}^{R_1} \xi d\xi \int_0^{2\pi} d\eta \times$$

$$\int_l^{l_1} U_{00}(\xi, \eta, \varsigma) G_0(r, \varphi, z, \xi, \eta, \varsigma, t) d\varsigma.$$

Unfortunately, the representation (27) is unusable as solution for the fin because of the unknown function $F(r, \varphi, t)$, i.e. temperature in the wall $U(r, \varphi, z = l, t)$ and derivative of temperature. From equation (16) we can write the combination $F_0(r, \varphi, t)$:

$$F_0(r, \varphi, t) = -\hat{H}_0(r, \varphi, t) - a^2 \int_0^t d\tau \int_0^{2\pi} d\eta \times$$

$$\int_{R_0}^{R_1} F(\xi, \eta, \tau) \hat{G}_0(r, \varphi, \xi, \eta, \varsigma, t - \tau) d\xi. \quad (28)$$

Here

$$\hat{H}_0(r, \varphi, t) = \left(\frac{k_0}{k} \frac{\partial}{\partial z} \bar{H}_0(r, \varphi, z, t) - \bar{H}_0(r, \varphi, z, t) \right) \Bigg|_{z=l},$$

$$\hat{G}_0(r, \varphi, \xi, \eta, \varsigma, t) = \left[\frac{k_0}{k} \frac{\partial}{\partial z} G_0(r, \varphi, z, \xi, \eta, \varsigma, t) - G_0(r, \varphi, z, \xi, \eta, \varsigma, t) \right] \Bigg|_{z=l}.$$

Similarly we do with equations (23) and (18):

$$U(r, \varphi, z, t) = \bar{H}(r, \varphi, z, t) + a^2 \int_0^t d\tau \times$$

$$\int_0^{2\pi} d\varsigma \int_{R_0}^{R_1} F_0(\xi, \varsigma, \tau) G(r, \varphi, z, \xi, \varsigma, l, t - \tau) d\xi,$$

$$\bar{H}(r, \varphi, z, t) = H(r, \varphi, z, t) + \int_{R_0}^{R_1} \xi d\xi \int_0^{2\pi} d\varsigma \times$$

$$\int_l^{l_1} U_0(\xi, \eta, \varsigma) G(r, \varphi, z, \xi, \eta, \varsigma, t) d\eta.$$

From equation (23) we can write the combination $F(r, \varphi, t)$:

$$F(r, \varphi, t) = \hat{H}(r, \varphi, t) + a^2 \int_0^t d\tau \times$$

$$\int_0^{2\pi} d\eta \int_{R_0}^{R_1} F_0(\xi, \eta, \tau) \hat{G}(r, \varphi, l, \xi, \eta, l, t - \tau) d\xi. \quad (29)$$

Here

$$\hat{H}(r, \varphi, t) = \left[\frac{k}{k_0} \frac{\partial}{\partial z} \bar{H}(r, \varphi, t) - \bar{H}(r, \varphi, t) \right] \Bigg|_{z=l},$$

$$\hat{G}(r, \varphi, l, \xi, \eta, \varsigma, l, t) = \left[\frac{k}{k_0} \frac{\partial}{\partial z} G(r, \varphi, z, \xi, \eta, \varsigma, l, t) - G(r, \varphi, z, \xi, \eta, \varsigma, l, t) \right] \Bigg|_{z=l}.$$

As the last step we substitute the combination $F_0(\xi, \eta, \tau)$ from (28)

$$F_0(\xi, \eta, \tau) = -\hat{H}_0(\xi, \eta, \tau) - a^2 \int_0^\tau d\tau_1 \int_0^{2\pi} d\eta_1 \times$$

$$\int_{R_0}^{R_1} F(\xi_1, \eta, \tau_1) \hat{G}_0(\xi, \eta, \xi_1, \eta_1, \varsigma, \tau - \tau_1) d\xi_1,$$

in the equation (29):

$$F(r, \varphi, t) = \hat{H}(r, \varphi, t) + a^2 \int_0^t d\tau \times \int_0^{2\pi} d\eta \int_{R_0}^{R_1} F_0(\xi, \eta, \tau) \hat{G}(r, \varphi, l, \xi, \eta, l, t - \tau) d\xi.$$

Finally we obtain following non-homogeneous Fredholm integral equation of 2nd kind:

$$F(r, \varphi, t) = \Phi(r, \varphi, t) + a^2 \int_0^t d\tau \int_0^\tau d\tau_1 \int_0^{2\pi} d\eta_1 \times \int_0^{2\pi} d\eta \int_{R_0}^{R_1} F(\xi_1, \eta, \tau_1) \Gamma(r, \varphi, l, \xi, \eta, l, t - \tau) d\xi. \quad (30)$$

Here

$$\Gamma(r, \varphi, l, \xi, \eta, l, t) = \int_{R_0}^{R_1} \hat{G}(r, \varphi, l, \xi, \eta, l, t) d\xi \times \int_{R_0}^{R_1} \hat{G}_0(\xi, \eta, \xi_1, \eta_1, \zeta, \tau - \tau_1) d\xi_1,$$

$$\Phi(r, \varphi, t) = \hat{H}(r, \varphi, t) - a^2 \int_0^t d\tau \times \int_0^{2\pi} d\eta \int_{R_0}^{R_1} \hat{H}_0(\xi, \eta, \tau) \hat{G}(r, \varphi, l, \xi, \eta, l, t - \tau) d\xi.$$

We finish our paper with the following remark. The problem with non-homogeneous environment temperatures (6), (7), (8) and (11) and its solution allow conjugating temperature field with hydrodynamic field.

5 CONCLUSIONS

We have constructed exact three-dimensional transient analytical solution for one element with hollow cylinder fin. The solution is obtained in the form of Fredholm integral equation of 2nd kind and has continuous kernel. As the result we come to integral equation on the surface between both hollow cylinders. The generalizing of Green function method allows us to use Green function method for regular non-canonical domains.

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