ANALYTICAL AND NUMERICAL SOLUTION OF CONJUGATE HEAT AND MASS TRANSFER IN FALLING FILM ABSORPTION PROCESS

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Abstract. Investigation of heat and mass transfer at nonisothermal absorption on the falling liquid film surrounded by pure vapour is presented. The analytical method for entrance region and numerical method for solving the problem are proposed. The analytical solution is obtained for the entry region, where temperature and concentration perturbations are near the interface. Then, the analytical solution is used as an inlet condition for further numerical solution by the finite-difference method. The key dimensionless criteria characterizing heat and mass transfer at absorption on the falling liquid film are defined.

1 Introduction

Thin liquid films are used in various technological processes more often to intensify the heat and mass transfer processes. The numerous theoretical and experimental studies of heat and mass transfer in liquid films are explained by the applied significance of film flows. The classical problem of heat transfer in a smooth laminar film flowing down the wall with a fixed temperature was firstly described in the work of Nusselt [1]. Graetz was the first, who considered a similar problem of liquid flow heating in a cylindrical pipe with isothermal wall [2]. One of the important applications of non-isothermal film flow is the absorption thermotransformers. So, the absorption heat pump is a unique device consisted of the desorber, condenser, evaporator, and absorber. In such devices, vapor absorption in liquid goes along with heat release, so the heat and mass transfer processes are conjugated. The first theoretical description of the processes of simultaneous heat and mass transfer at vapor absorption in a laminar falling liquid film was made in [3-6] with some simplifications of the problem formulation. The authors of [7] investigated the process of vapor absorption by the liquid film for two limiting cases: constant wall temperature and thermally insulated wall. Equations of heat transfer and diffusion were solved jointly throughout the entire flow region by numerical method and analytical method (for the thermal entry region). The profiles of temperature and concentration were presented in the form of power series of the transverse coordinate with coefficients dependent on the longitudinal coordinate. In subsequent theoretical studies [8-19] the process of simultaneous heat and mass transfer at gas absorption by the liquid film was also examined under various simplifying assumptions. In [8] the problem of the vapour absorption into a falling film,

where the concentration levels of absorbate and absorbent are comparable was studied. The conjugated heat and mass transfer processes were analysed. The adiabatic and isothermal wall conditions were considered. In [11] the falling film absorption in aqueous LiBr solution was investigated numerically using package Fluent. The convective boundary condition at the cooling water side was considered. In [14] the comparison of absorption and desorption has been conducted using a detailed model describing heat and mass transfer. In [15] the conjugate heat and mass transfer process taking place during absorption of a refrigerant vapor into a falling liquid film has been analyzed using the Laplace transform method. In [16] the numerical and experimental investigations of a vertical LiBr falling film absorber considering wave regimes have been performed. In [17] a model was developed for analysis of the combined heat and mass transfer processes for laminar liquid film evaporation. The model was developed in cylindrical coordinate. The energy and diffusion equations are solved using the Laplace transform. The comparison between theoretical calculation and experimental data has been carried out.

This work offers the conjugation of the analytical method for solving the problem of heat and mass transfer in nonisothermal absorption on the falling liquid film surrounded by pure vapour at the entrance region, with numerical method, using the analytical solution as an inlet condition for further numerical calculations.

2 Problem statement

It is assumed that the laminar liquid film with constant thickness h flows under the action of gravity on the wall, inclined at angle θ to the horizon, with uniform velocity u. In the case of falling film absorption, when the vapour

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or gas phase is stationary, coupled heat and mass transfer is considered under the assumptions [18, 19], which are used in absorption heat pumps and refrigerators. Introducing the Cartesian coordinate system with axis Ox, directed to the film flow, and axis Oy, directed from the interface to the film depth. At $x = 0$ we set inlet conditions T_0, C_0. The wall temperature is also equal T_0. Liquid density ρ, dynamic viscosity μ, heat conductivity λ, thermal diffusivity a and diffusivity D are considered constant. The uniform velocity profile in the liquid film takes the form:

$$ u = \frac{gh^2 \sin\theta}{3\nu}. $$

The processes of heat and mass transfer in the solution film are described by the system of heat conduction and diffusion equations:

$$ u\frac{\partial T}{\partial x} = a\frac{\partial^2 T}{\partial y^2}, \tag{1} $$

$$ u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial y^2}, \tag{2} $$

with corresponding conditions at the inlet

$$ x=0 \quad T=T_0,\ C=C_0, \tag{3} $$

with conjugate conditions at the interphase boundary

$$ y=0 \quad C_i=k_1-k_2\cdot T_i,\ \lambda\frac{\partial T}{\partial y} = \frac{r_a \rho D}{1-C_0}\frac{\partial C}{\partial y}, \tag{4} $$

and with the conditions on the wall

$$ y=h \quad T=T_0,\ \frac{\partial C}{\partial y} = 0. \tag{5} $$

We introduce dimensionless variables $\bar{x} = \dfrac{x}{\mathrm{Pe}h}$, $\mathrm{Pe} = \dfrac{uh}{a}$, $\bar{y} = \dfrac{y}{h}$, $\theta = \dfrac{(T-T_0)}{(T_e-T_0)}$, $\gamma = \dfrac{(C-C_0)}{(C_e-C_0)}$. Here $C_e = k_1 - k_2 T_0$ is equilibrium concentration corresponding to the given initial temperature of solution T_0; T_e is the equilibrium temperature corresponding to the given initial concentration of solution C_0 ($C_0 = k_1 - k_2 T_e$). The the coefficients k_1 and k_2 are determined by the pressure. In dimensionless variables, Eq.(1), Eq.(2) and the boundary conditions (3-5) take the form:

$$ \frac{\partial\theta}{\partial\bar{x}} = \frac{\partial^2\theta}{\partial\bar{y}^2}, \tag{1'} $$

$$ \frac{\partial\gamma}{\partial\bar{x}} = \mathrm{Le}\frac{\partial^2\gamma}{\partial\bar{y}^2}, \tag{2'} $$

$$ \bar{x}=0 \quad \theta=0,\ \gamma=0, \tag{3'} $$

$$ \bar{y}=0 \quad \theta_i + \gamma_i = 1,\ \frac{\partial\theta}{\partial\bar{y}} = \mathrm{KaLe}\frac{\partial\gamma}{\partial\bar{y}} \tag{4'} $$

$$ \bar{y} = 1 \quad \theta = 0,\ \frac{\partial\gamma}{\partial\bar{y}} = 0. \tag{5'} $$

where $\mathrm{Ka} = \dfrac{r_a}{C_p\left(T_e - T_0\right)}\dfrac{C_e - C_0}{1-C_0}$, $\mathrm{Le} = \dfrac{D}{a}$.

The parameters for calculations were chosen close to the characteristic values of the two-phase system: water solution of lithium bromide – water vapor, under the assumptions corresponding to the conditions of working absorber of a heat pump.

3 Analytical solution for entrance region

By using self-similar variable $\xi = \dfrac{\bar{y}}{\bar{x}^{1/2}}$ the ordinary differential equations can be obtained

$$ \frac{d^2\theta}{d\xi^2} + \frac{\xi}{2}\frac{d\theta}{d\xi} = 0, \tag{6} $$

$$ \frac{d^2\gamma}{d\xi^2} + \mathrm{Le}^{-1}\frac{\xi}{2}\frac{d\gamma}{d\xi} = 0, \tag{7} $$

with boundary conditions

$$ \xi=0 \quad \theta_i + \gamma_i = 1,\ \frac{\partial\theta}{\partial\xi} = \mathrm{KaLe}\frac{\partial\gamma}{\partial\xi}, \tag{8} $$

$$ \xi \to \infty \quad \theta = 0,\ \gamma = 0. \tag{9} $$

The self-similar solutions of Eq. (6), (7) with boundary conditions (8) and (9) take the form:

$$ \theta = \theta_i\left(1 - erf\left(\xi/2\right)\right), \tag{10} $$

$$ \gamma = \gamma_i\left(1 - erf\left(\mathrm{Le}^{-1/2}\xi/2\right)\right), \tag{11} $$

where $\theta_i = \dfrac{\mathrm{KaLe}^{1/2}}{\mathrm{KaLe}^{1/2}+1}, \gamma_i = \dfrac{1}{\mathrm{KaLe}^{1/2}+1}$. Note that the formulas for temperature and concentration in the entrance region of the free surface of the film θ_i, γ_i agree with the formulas obtained for film absorption at other velocity profiles [18,19].

4 Numerical solution

The numerical calculations by finite-difference method are performed using the self-similar temperature and concentration profiles (10), (11) at the inlet (see Fig.1) at $x = 0.01$. Further y^1 is measured from the wall and varied from 0 to h ($y^1 = 1 - \bar{y}$). Equations (1') and (2')

in the difference form with approximation of order $O\left(\Delta \overline{x}, \Delta y^{1^2}\right)$ are as follows:

$$\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta \overline{x}} = \frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{\Delta (y^1)^2}, \qquad (12)$$

$$\frac{\gamma_{i,j} - \gamma_{i-1,j}}{\Delta \overline{x}} = \frac{\gamma_{i,j+1} - 2\gamma_{i,j} + \gamma_{i,j-1}}{\Delta (y^1)^2}. \qquad (13)$$

Here $\theta_{i,j}$ is the temperature value in node $(\overline{x}_i, y^1{}_j)$. The values of the grid step $\Delta \overline{x} = 10^{-3}$ and $\Delta y^1 = 2 \cdot 10^{-3}$ are rather small, so the results of calculations practically do not change with a further decrease in the grid step. For everyone x_i Eq. (12), (13) have been solved by a sweep method on coordinate y^1 using boundary conditions (3'-5'). The interfacial values of the temperature and concentration have been found from the conjugated calculations of the sweep coefficients.
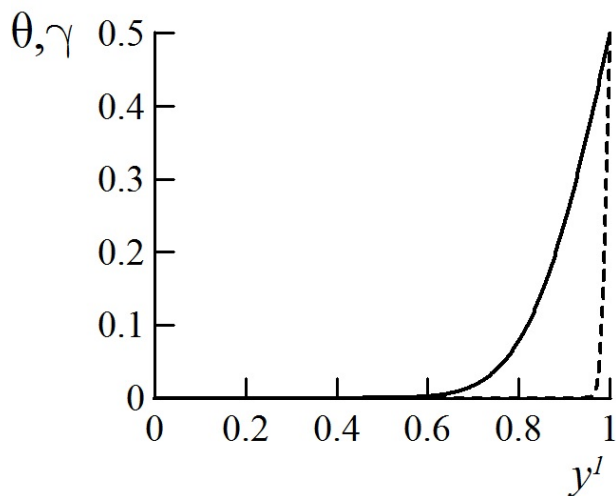
Fig. 1. The self-similar temperature and concentration profiles (10), (11) at the inlet at $x = 0.01$, Ka $= 10$, Le $= 0.01$. Solid line – initial temperature profile, dashed line – initial concentration profile.
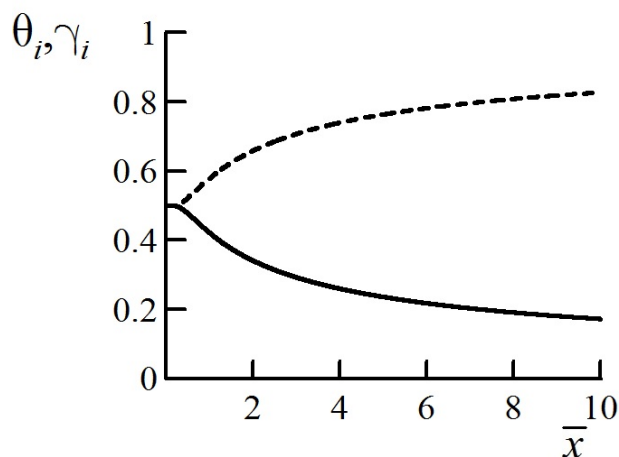
Fig. 2. Film surface temperature and concentration depending on the distance in the case Ka $= 10$, Le $= 0.01$. Solid line – temperature profile, dashed line –concentration profile.
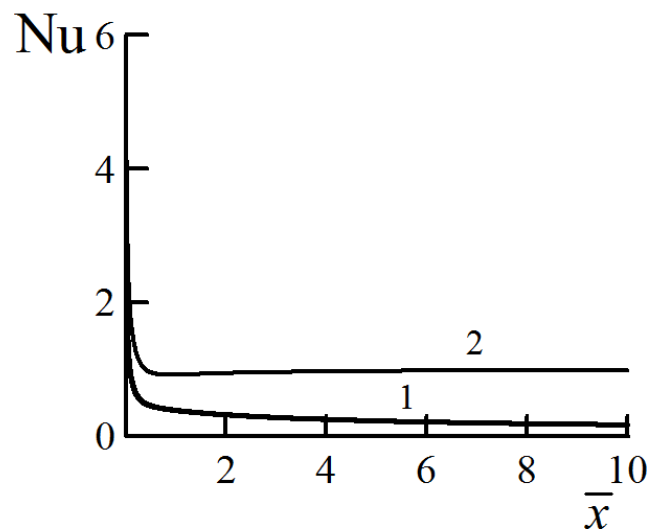
Fig. 3. Nusselt numbers Nu$_1$ and Nu$_2$ along the film-flow direction. 1 – Nu$_1$, 2 – Nu$_2$.

Fig. 2 shows the interfacial temperature and concentration profiles depending on the distance. The Nusselt numbers $\mathrm{Nu}_1 = \dfrac{h}{\Delta T_1} \dfrac{\partial T}{\partial y}\bigg|_{y=0}$, $\mathrm{Nu}_2 = \dfrac{h}{\Delta T_2} \dfrac{\partial T}{\partial y}\bigg|_{y=0}$ are shown in Fig.3 depending on the longitudinal coordinate. Here $\Delta T_1 = T_e - T_0$, $\Delta T_2 = T_i - T_0$.

5 Conclusion

The following conclusions can be drawn from the above study. The self-similar solutions for temperature and concentration appropriated only for the entrance region have been obtained. Although, the self-similar solutions have a limited applicability, these solutions are simple and give a clear interrelation between physical quantities. Another important aspect of the present study is using the self-similar solutions as the initial conditions for solving the problem beyond the entrance region. So, in the following numerical calculations we can vary the thermal regime conditions. Using these self-similar solutions allow us to avoid the difficulties associated with the inconsistency of the boundary conditions. Also, the key criteria characterizing heat and mass transfer in the film absorption with uniform velocity profile and with a constant thickness (Ka – the absorption analog of phase transition criterion, Lewis, Peclet, and Nusselt complexes) have been determined in the present study. The problem was solved for lithium bromide solutions with Lewis number smaller than unity, Le = D/a. It was shown that with increasing distance, the value of Nusselt number significantly decreases to the equilibrium value for the isothermal wall.

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