

# Research on Joint Torque Optimization Method of Redundant Space Manipulators with Vibration Suppression

Xin Gao <sup>1</sup>, Ming-Tao Du <sup>2</sup>, Lin Zhai <sup>3</sup>, Yan-heng Zhang <sup>4</sup>, Han-Xu Sun <sup>5</sup>, Xin-Ping Diao <sup>6</sup>

<sup>1</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>2</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>3</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>4</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>5</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>6</sup>School of Automation Beijing University of Posts and Telecommunications, Beijing, China

<sup>1</sup>xlhhh74@bupt.edu.cn, <sup>2</sup>dmt2013@bupt.edu.cn, <sup>3</sup>zhailin0522@sina.com, <sup>4</sup>zyh620@bupt.edu.cn, <sup>5</sup>hxsun@bupt.edu.cn, <sup>6</sup>dxp0724@163.com

**Abstract:** The space manipulator can exert its advantages of high efficiency and low cost, which can assist or even replace astronauts to complete a variety of space operations. However, the research has less consideration on joint torque optimization method with considering the influence of flexible factors on the manipulator. A joint torque optimization method of redundant space flexible manipulator with vibration suppression is proposed in this paper. The bending deformation of flexible connecting rod is described by the assumed mode method and the dynamic model of space flexible manipulator system is derived by the Lagrange method. On the basis of decomposing the dynamic equation of the flexible redundant manipulator, the joint torque is optimized as well as restraining the terminal vibration through introducing the Lagrange multiplier. Finally, simulation result shows that the increase rate of joint torque is not very obvious based on the Particle Swarm Optimization algorithm compared with the experiment without vibration suppression. The forward torque of first joint increases 20.2% and that of second joint only increases 4.1% while the backward torque of both two joints decrease obviously. Meanwhile, the running trajectory has excellent effect on vibration suppression of end effector of flexible manipulators. The terminal deformation in two directions is reduced by 21.3% and 78.6%. The decline of vibration deformation of end effector is obviously larger than the rise of joint torque, which verifies the feasibility and effectiveness of the algorithm.

## 1. Introduction

Redundant space manipulator, an advanced manipulator system, has been invented as the development of space technology and higher demands of advanced manufacturing technology [1-2]. Redundant manipulator refers to the kind of mechanical arm with more than the minimum number of degree of freedom (DOF) required to complete the task. The particular self-motion ability of redundant manipulator can overcome the weaknesses of general ones, such as poor flexibility, poor dynamic performance, limited joints, and poor obstacle avoidance, and it can also ensure the completion of actuator tasks and optimize every system performance index [3-4].

It is one of important performance indexes to optimize dynamics of redundant manipulator for the purpose of rational allocation of joint torque, and its premise is to satisfy the limit of the manipulator joint torque. The research at the present stage mainly focuses on two aspects: on the premise of meeting mission requirements, scientists often take action to minimize joint torque and reduce system energy consumption [5-7]. The method

proposed by Hollerbach et al [8] and by Osumi et al[9] is typical method of joint torque local optimization and global optimization for redundant manipulator. The former may lead to local optimal solution, while when the dynamic performance is optimized in global method; the control equation will be so complex that the optimization process is very complicated. However, the research has less consideration on joint torque optimization method with considering the influence of flexible factors on the manipulator [10].

The joint optimization of space flexible manipulators in the condition of vibration suppression is mainly based on the following two aspects: 1) from the perspective of safety of working process, the end-effector will cause joints severe jitter if vibrating violently in the process of task execution, which may degrade dynamic performance, fail algorithm or even damage the manipulator itself. Reducing the terminal vibration can improve the stability and tracking accuracy of manipulators movement; reduce the wear and resonance of manipulators; delay the attenuation rate of mechanical structure caused by severe changes in the joint stress; reduce the use cost of the manipulator and prolong its service life. 2) We hope the

torque of each manipulator joint becomes small when it finishes the prescribed procedures and we also hope to reduce energy consumption improving the situation of limitation of fuel supply. Thus, it has important theoretical value and practical significance to carry out the research on joint torque optimization method of space flexible manipulator in the condition of vibration suppression, and it has a very wide application prospect.

A joint torque optimization method of redundant manipulator with vibration suppression is proposed in this paper. The bending deformation of flexible connecting rod is described by the assumed mode method and the dynamic model of space flexible manipulator system is derived by the Lagrangian formulation. On the basis of decomposing the dynamic equation of the flexible redundant manipulator, the joint torque is optimized as well as restraining the terminal vibration through introducing the Lagrange multiplier. Simulation shows its feasibility and effectiveness.

## 2. Research on the Optimization Method of Joint Torque of Space Manipulator Considering Flexibility

The deformation description of flexible manipulator during working process is the fundamental for model construction and operation. To reduce the model complexity, combined with the structure character of the target itself, this paper only considers the flexibility of the connecting rod. In this part, assumed modes method was used to approximately describe the deformation of connecting rod, regardless the vibration mode at higher-order. Then we use Lagrange method to analyze the kinetic characteristics of the flexible manipulator. Accompanied with system momentum conservation, kinetic model of the flexible manipulator system can be inferred. Further, inhibiting the vibration of terminal effector and optimizing the joint torque can be implemented, which is based on splitting the kinetic equation of the flexible manipulator, through introducing the Lagrange multiplier.

### 2.1 Establishment of Dynamic Model of Flexible Manipulator

The assumed mode method was used to describe the deformation caused by the flexibility of the manipulator. For flexible manipulator with multiple connection rods, to describe the deformation accurately, following assumptions were made [11-12]:

- The length-to-diameter ratio for each rod is higher enough to neglect the transverse force and rotary inertia
- Deformation can only occurred within the motor level of each rod.
- Each rod had higher rigid level so that the deflection of each rod can be ignored.
- The vibration and joint torque of each rod do not affect others' vibration frequency and modal shape.
- The orthogonality of the vibration function is independent on external force and other factors.

The description of the deformed flexible manipulator in inertial frame is shown in Fig.1.

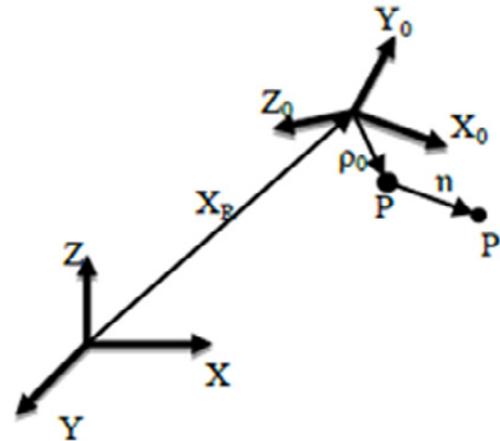


Figure 1. Description of Flexible Body in Inertial Coordinate System

In Fig.1, point P move to P' after the deformation. The pose, after shape changing, of P in inertial frame can be expressed as:

$$r_{p'} = X_R + \rho_0 + \eta \quad (1)$$

where,  $X_R$  is the pose of FM according to the original point in inertial frame;  $\rho_0$  is the pose of P in its own coordinate system;  $\eta$  is the elastic deformation vector of P.

Taking the derivative of type (1) with respect to time, the velocity of any point on flexible manipulator can be obtained in the inertial frame.

$$V = \frac{dr_{p'}}{dt} = V_R + \omega \times (\rho_0 \times \eta) + \dot{\rho}_0 + \dot{\eta} \quad (2)$$

The kinetic energy of a typical flexible manipulator is:

$$T = \frac{1}{2} \int V^T \cdot V dm \quad (3)$$

Bring type (2) into type (3)

$$T = \frac{1}{2} \int \{ V_R \cdot V_R + [\omega \times (\rho_0 + \eta)] \cdot [\omega \times (\rho_0 + \eta)] + \dot{\eta} \cdot \dot{\eta} + 2V_R \cdot [\omega \times (\rho_0 + \eta)] + 2[\omega \times (\rho_0 + \eta)] \cdot \dot{\eta} \} dm \quad (4)$$

Then, we solve the kinetic energy of the flexible manipulator in Space rectangular coordinate system.

Before this, we definite a form of expression, that is, we use  $\Phi_k \dot{\zeta}_k$  to express  $\sum_{k=1}^N \Phi_k \dot{\zeta}_k$ .

The kinetic energy of the flexible manipulator can be expressed in quadratic form of generalized velocity:

$$T = \frac{1}{2} U^T M U \quad (5)$$

According to the previous conclusions, the kinetic energy can be expressed with generalized coordinates as:

$$T = \frac{1}{2} \dot{q}^T (H^T M H) \dot{q} = \frac{1}{2} \dot{q}^T \tilde{M} \dot{q} \quad (6)$$

As the space manipulator works in the microgravity environment, the influence of its own gravity can be neglected. The elastic potential energy produced by the flexible deformation of the manipulator is the only potential energy on the system. According to the theory of mechanics of materials, the elastic potential energy can be expressed as:

$$V = \frac{1}{2} \xi^T K \xi \quad (7)$$

Where,  $K$  is modal stiffness matrix of the manipulator;  $\omega_i$  is the corresponding frequency of  $i$ th-order natural vibration  $\xi$  is modal displacement matrix,  $\xi = [\xi_1 \ \xi_2 \ \xi_3]^T$

The kinetic energy and potential energy required for solving the dynamic equations of the manipulator system have been obtained in former two sections. At the same time, the expression forms of the generalized coordinates of the system kinetic energy and potential energy have been obtained through the transformation of the corresponding matrix. Then, the dynamic equations of a single flexible body can be obtained and its matrix form is shown as follows:

$$\tilde{M} \ddot{q} + \tilde{C} \dot{q} + \tilde{K} q = Q + \lambda \alpha - \tilde{M} \dot{q} + \frac{1}{2} \dot{q}^T \frac{\partial \tilde{M}}{\partial q} \dot{q} \quad (8)$$

So far, dynamic differential equation has been established. We can draw a conclusion from the derivation of the equations that on account of considering the elastic deformation, the dynamic equations of flexible body become a nonlinear strong coupling differential equation set.

If  $n$ -,  $nr$ -,  $nf$ ( $n=nr+nf$ ) are respectively total degree of freedom, rigid degree of freedom and flexible degree of freedom of the manipulator, The dynamic equation of the flexible manipulator is shown as follows:

$$M(\theta, q) \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + D(\theta, q) \begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} + K(\theta, q) \begin{bmatrix} \theta \\ q \end{bmatrix} + Q = I \tau \quad (9)$$

where  $M$ -,  $D$ -,  $K \in R^{n \times n}$  is respectively the generalized mass matrix, the damping matrix and the stiffness matrix;  $Q$  is  $n$  order column vector which is the sum of gravity, centrifugal force and Coriolis force etc.;  $\tau \in R^{nr}$  is control torque on joint;  $I$  is  $nr$  dimension of the unit matrix;  $\theta \in R^{nr}$  is column vector of joint angle;  $q \in R^{nf}$  is generalized coordinates matrix of flexible deformation.

$$M = \begin{bmatrix} M_{rr} & M_{rf} \\ M_{rf} & M_{ff} \end{bmatrix}$$

Where  $M_{rr} \in R^{nr \times nr}$ ,  $M_{fr} \in R^{nf \times nr}$  and  $M_{ff} \in R^{nf \times nf}$  are respectively sub block matrix of  $M$ . Similarly, we divide the matrix  $D$  and  $K$  into blocks.  $Q = [Q_r \ Q_f]^T$ ,  $Q_r \in R^{nr}$ ,  $Q_f \in R^{nf}$  are respectively sub block matrix of  $Q$ . Thus, the above formula can be decomposed as follows:

$$M_{rr} \ddot{\theta} + M_{rf} \ddot{q} + Q_r = \tau \quad (10)$$

$$M_{ff} \ddot{q} + D_{ff} \dot{q} + K_{ff} q = -M_{fr} \ddot{\theta} - Q_f \quad (11)$$

The above two types are dynamic equations of the flexible redundant manipulator, where type (10) is torque control equation of space manipulator and type (11) is vibration equation of manipulator system.

## 2.2 The Algorithm of Joint Torque Optimization for Flexible Space Manipulator with Vibration Suppression

We assumed  $m$  is dimension of working space of the manipulator;  $n$ -,  $nr$ -,  $nf$ ( $n=nr+nf$ ) are respectively total degree of freedom, rigid degree of freedom and flexible degree of freedom of the manipulator;  $X \in R^m$  is the vector of position and pose of effector. Thus, the following relationship is established for the flexible manipulator:

$$X = f(\theta, q) \quad (12)$$

Taking the derivative of type (12), the equation of velocity and acceleration between Rectangular Coordinates and generalized coordinates (including joint displacement coordinates and flexible deformation coordinates) can be obtained as follows:

$$\begin{aligned}\dot{X} &= J_r \dot{\theta} + J_f \dot{q} \\ \ddot{X} &= J_r \ddot{\theta} + J_f \ddot{q} + \dot{J}_r \dot{\theta} + \dot{J}_f \dot{q}\end{aligned}\quad (13)$$

Where,  $J_r = \partial f / \partial \theta \in R^{m \times nr}$ ,  $J_f = \partial f / \partial q \in R^{m \times nf}$  is respectively Jacobian matrix of rigid and flexible motion of the manipulator. The rigid motion and flexible motion are coupled, but the amplitude of the flexible motion is relatively small, which can be regarded as a disturbance to the rigid motion. The influence of the flexible motion is usually neglected in the trajectory planning and the inverse kinematics solution and the error is eliminated with vibration suppression. Thus, the above formula can be simplified as:

$$\ddot{X} = J_r \ddot{\theta} + \dot{J}_r \dot{\theta} \quad (14)$$

As for redundant manipulator,  $\theta$  has infinite solution on non-singular position ( $rank(J_r) = m < n$ ) which can be optimized. Then, the complex modal excitation force of the vibration of flexible manipulator can be obtained through complex modal transformation of the dynamic equations.

$$Q_e = V_{RHL}^T M_{ff}^{-1} f_e = V_{RHL}^T M_{ff}^{-1} [-M_{ff} \ddot{\theta} - Q_f] \quad (15)$$

Where,  $V_{RHL} \in R^{nf \times nf}$  is real part of left modal matrix.

We can make the exciting force become small or even zero via reasonably selecting the self-motion variables of the manipulator, which can restrain the vibration of the terminal manipulator. Now, we want to optimize the joint torque on the basis of vibration suppression. Thus, the optimization problem becomes as follows:

$$\begin{aligned}\min(Z) &= \min(\tau^T \tau + Q_e^T Q_e) \\ \text{s.t. } & J_r \ddot{\theta} + \dot{J}_r \dot{\theta} - \ddot{X} = 0\end{aligned}\quad (16)$$

By introducing Lagrange multiplier, we can obtain:

$$Z = \tau^T \tau + Q_e^T Q_e + \lambda^T (J_r \ddot{\theta} + \dot{J}_r \dot{\theta} - \ddot{X}) \quad (17)$$

$$\begin{aligned}A &= -2M_r^T M_r - 2 \left[ (V_{RHL}^T M_{ff}^{-1} M_{fr})^T (V_{RHL}^T M_{ff}^{-1} M_{fr}) \right] \\ B &= 2M_r^T\end{aligned}$$

$(M_{fr}^T \dots + 2(V_{RHL}^T M_{ff}^{-1} M_{fr})^T (V_{RHL}^T M_{ff}^{-1} Q_f))$  and solve the following conditions:

$$\begin{aligned}\frac{\partial Z}{\partial \theta} &= 0 \\ \frac{\partial Z}{\partial \lambda} &= 0\end{aligned}\quad (18)$$

We can obtain:

$$\begin{aligned}\ddot{\theta} &= A^{-1} B + A^{-1} J_r^T \lambda \\ J_r \ddot{\theta} + \dot{J}_r \dot{\theta} - \ddot{X} &= 0\end{aligned}\quad (19)$$

Then, the simultaneous equation can be obtained from type (24)

$$\ddot{\theta} = D(\ddot{X} - \dot{J}_r \dot{\theta}) + (I - D J_r) A^{-1} B \quad (20)$$

Where  $D = A^{-1} J_r^T (J_r A^{-1} J_r^T)^{-1}$ . It's easy to verify  $J_r D = I$ , so D is right inverse of  $J_r$ .

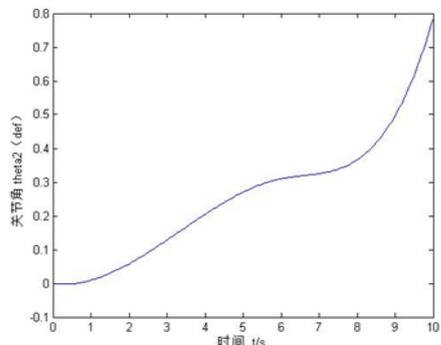
Until now, we have accomplished the optimization of the dynamic equation. Then, we can realize the corresponding control algorithm. Comparing with the pseudo-inverse method, the above algorithm can optimize joint torque of redundant space manipulator on the basis of vibration suppression with complex modal method.

### 2.3 Numerical Simulation

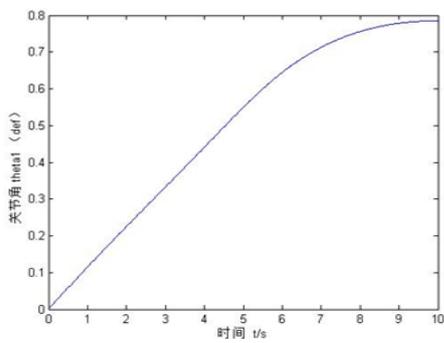
To verify the feasibility and efficiency of the upper algorithm, we use a two-link flexible manipulator to do numerical simulation research. We assumed that the mass of two connecting rods are  $m_1 = m_2 = 10\text{kg}$ , the concentrated mass at the end of two connection rod are  $m_1 = 50\text{kg}$ ,  $m_2 = 1000\text{kg}$ , the length of two rods are  $l_1 = l_2 = 3\text{m}$ , the elastic parameter of flexible rod is  $EI = 1200$ , the density of the rods is  $\rho = 2.7 \times 10^3 \text{kg/m}^3$ , the cross-sectional area is  $A = 1.2 \times 10^{-4} \text{m}^2$ . The initial angles of given configuration are  $\theta_{in} = [0^\circ, 0^\circ]$  and the termination angles are  $\theta_{des} = [45^\circ, 45^\circ]$ . The planning time is  $t = [0, 10\text{s}]$  and the limitation of two joint angles is  $\theta \in [-180^\circ, 180^\circ]$ . Specific value of each parameter in given particle swarm optimization algorithm is as follows: the velocity range of particles is limited to  $v \in [-5, 5]$ ; the maximum update iteration is  $max\_length = 80$ ; the number of particles is  $Popnum = 30$ ; the inertial factor is gradually fading from 0.8 to 0.2.

Cubic spline curve was used to trace the moving trajectory of the manipulator and the middle node of the spline curve can be regarded as the control parameter of the optimal computing. Finally, we got the optimal solution by using PSO algorithm. The cubic spline curve of two joint angles of the manipulator is shown in Fig 2; Fig 3-4 reflects its torque changing and terminal deformation caused by vibration. To evaluate the test

result, another manipulator, whose joint torque was optimized only, was designed under the same circumstance, regardless of the inhibition of its terminal vibration. Its changing curve of the joint angle is shown in Fig 5. Fig 6-7 reflects its torque changing and terminal deformation caused by vibration.

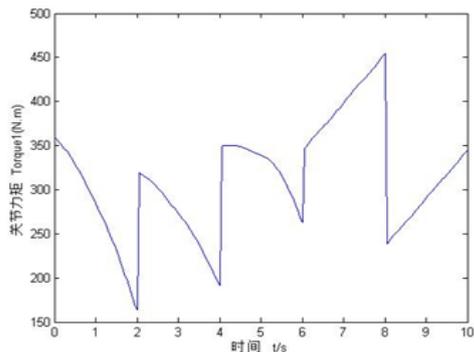


(a)

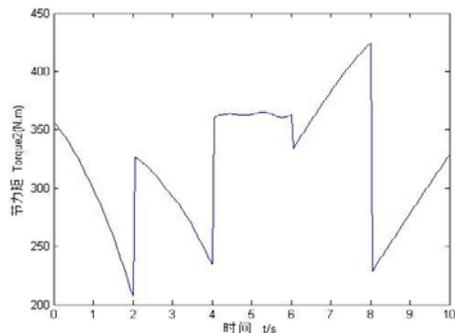


(b)

Figure 2. Cubic Spline Curve of Two Joint Angles of the Manipulator



(a)



(b)

Figure 3. Variation Curve of Two Joint Torque of the Manipulator

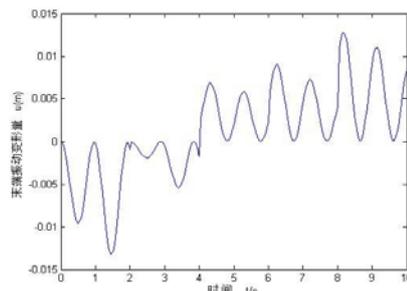
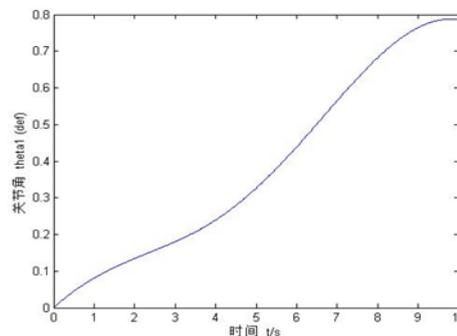
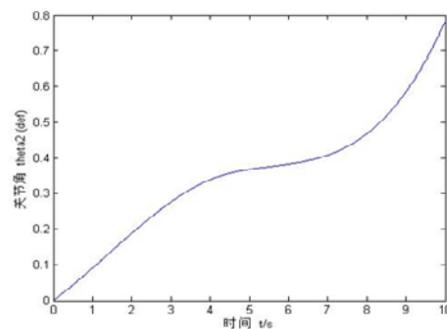


Figure 4. The variation curve of Terminal Vibration Deformation of the Manipulator

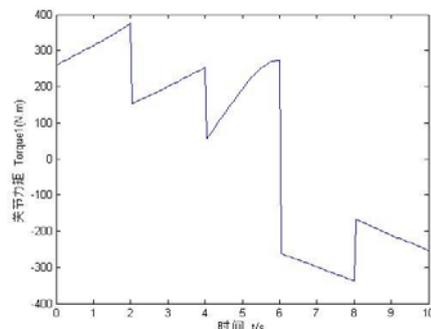


(a)



(b)

Figure 5. The Variation Curve of Two Joint Angles of the Manipulator in Contrast Experiment



(a)

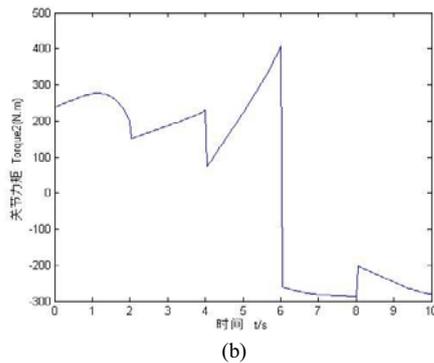


Figure 6. The variation Curve of Two Joint Torque of the Manipulator in Contrast Experiment

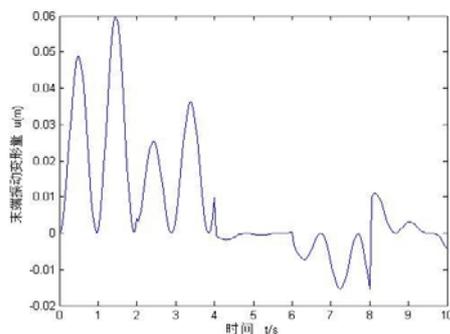


Figure 7. The Variation Curve of Terminal Vibration Deformation of the Manipulator in Contrast Experiment

From the simulation result, after doing the interpolation calculation towards the joint angle via cubic spline, and doing the practical swarm optimization, the trajectory of the manipulator can be obtained. The amplitude of variation of its two joint torques are

$$Torque1 \in [163.5\text{Nm}, 451.1\text{Nm}]$$

,  $Torque2 \in [207.7\text{Nm}, 424.3\text{Nm}]$  respectively, while in the same lab environment, when optimizing the joint torque of the connection rod itself without considering the inhibition of the endpoint vibration, the amplitude of variation of its two joint torque is

$$\tilde{Torque1} \in [-337.8\text{Nm}, 375.2\text{Nm}]$$

,  $\tilde{Torque2} \in [-288.8\text{Nm}, 407.7\text{Nm}]$  respectively. Further, the terminal vibration amplitude of the manipulator whose trajectory was optimized by PSO is  $[-0.0122\text{m}, 0.0128\text{m}]$ , while that of the control group is  $[-0.0155\text{m}, 0.0599\text{m}]$ .

From the numerical simulation result, we can infer that, comparing with experiment that only optimizing the joint torque while neglect the inhibiting of terminal vibration, the increase of joint torque when applying the PSO algorithm is quite negligible. The first forward joint torque increased 20.2%, and the second forward joint torque increased only 4.1%, however, the backward of two joint torque dropped significantly. But, when it comes to the operation trajectory and its inhibiting towards the terminal vibration, PSO is quite obvious when applying to manipulator. Its terminal deformation decrease 78.6% and 21.3% in two directions respectively. The decrease of terminal deformation is significantly

larger than the increase of its joint torque, which verifies the feasibility and efficiency of this algorithm.

### 3. Conclusion

We have carried out the research on joint torque optimization method of space flexible manipulator and we have proposed an algorithm with vibration suppression. First of all, the bending deformation of the flexible rod is approximated by the assumed mode method. Then, using Lagrange method to derive the dynamic equations of flexible manipulator and separate the rigid and flexible terms from the dynamic equation, the vibration excitation of the manipulator was obtained. The weighted sum of joint torque and terminal deformation are objective functions of the particle swarm optimization algorithm. At last, we use cubic spline curve to describe the trajectory of space manipulator in joint coordinate system and the middle control node of spline curve is chosen as control parameters of optimization. The research realizes restraining the terminal vibration as well as optimizing the joint torque based on Particle Swarm Optimization algorithm and the simulation has verified its feasibility and effectiveness.

### Acknowledgment

The authors would like to thank their colleagues from the Robotics Research Group for helpful discussions and comments on this paper. This work is supported in part by Key Project of Chinese National Programs for Fundamental Research and Development (973 program) (No. 2013CB733000), National Natural Science Foundation of China (No. 61573066) and National Natural Science Foundation of China (No. 61403038).

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