Abstract. The problem of route optimization of large volume and long distance material distribution is carried out from the rear strategic logistics center to the front line distribution center, researched by turning into multimodal transport problems, and a single-path multimodal transport path optimization model is established. Decision variables about stage safety probability are introduced; the nonlinear model is simplified into a second-order programming model, by changing the probability of task safety and simplified the model to a linear programming model by using the monotonicity of logarithmic functions. An example is presented to solve the simulation with LINGO software, made a reference plan under different constraints, the correctness of the model is verified.

1. Introduction

Logistics distribution is the important content of military traffic transport, is shouldering the important combat goods delivery to combat troop’s task, and it is the link that transforms logistics support into operational capability. Under large-scale combat conditions, military transport are the characteristics of great material consumption, the transport mission is large, the transport scope is wide, and the tasks have a variety of forms; The timeliness of transport is high and the persistent requirements also high, the transport environment is poor and the hidden defense is difficult. So it is difficult to improve the coordination of military transport organization, the requirement of the loading and unloading and the plan of transportation guarantee.

The paper studied the problem of route optimization of railway, highway, waterway and aviation in the context of large-scale operation, a scientific and rational path optimization model is established, by reasonable selecting the running line and transport tools, accurate and efficient delivery plan can be developed.

2. Research Status of Multimodal Transport and Shortest Path Problems

2.1 The Research Status of Multimodal Transport Problem

There is little research on multimodal military transport at home and abroad at present, Xi-rui Yang based on Lagrangian relaxation method, decomposed the problem of multimodal military transport into two sub-problems: cargo flow and vehicle flow, the two subproblems are solved separately; Jiaying Zhang has set up a multimodal military transport model, which take time and cost as the target, and time and capacity as constraints, and solved the problem by Genetic algorithm and Matlab software.

2.2 Research Status of the Shortest Path Problem

The shortest path problem researched the single objective optimization problem of the single-attribute network, for example, the fastest path problem is to use time as the network property, the proposed model is generally unconstrained 0-1 programming model, Dijkstra, Ford, Floyd and other precise algorithms can be used to obtain the optimal solution directly. The constraint path optimization problem is the extension of the shortest path problem, mainly studies the multi-attribute network path optimization problem, consider a target, multiple attribute constraints, the proposed model is a constrained 0-1 nonlinear programming model, it can be solved by heuristic algorithm or improved algorithm.

2.3 Research Status of Path Optimization Algorithm

The research of path optimization is mainly focused on the precise algorithm at first, the accurate algorithm can solve the global optimal solution theoretically, but the disadvantage is that it is computationally intensive, it’s easy to be constrained by problem size. LINGO software
is able to obtain the global optimal solution for large-scale linear programming problem, thus, this NP problem can be solved which considered the tolerability of calculating time and optimality of solution.

3. Single Path Multi-Modal Transport Path Optimization Model

Depending on the mode of transport, large-scale combat logistics distribution problems can be divided into multi-modal transport problems and vehicle path problems, as shown in figure 1. According to the transport task style, the problem of large-scale combat logistics distribution can be divided into multi-modal transport problems and vehicle routing problems, as shown in figure 1. The distribution to front distribution center or force user from rear logistics center or strategic warehouse is usually a long-distance transport, which requires a variety of modes of transport, its multi-modal transport. The distribution from forward distribution center to force users is generally a vehicle path problem. Multimodal transport is a form of transport organization with multiple modes of transport; it can significantly improve transport efficiency, optimize transport structure, and enhance the rationality and economy of transportation. This paper mainly studies the optimization model of single-path multimodal transport path and its solution.

3.1 Determination of the Value of the Intermodal Property

In this paper, we choose to construct a two-dimensional uncoupled transport network with a directed graph \(G(V, E, T, C, P)\). \(V\) is the point set, the total number is \(n\), \(v_i \in V\) is the point in the graph, \(v_i = [1, 2, 3, 4, 5, 6]\) represent logistics centers, railway stations, bus stations, airports, ports and military users respectively; \(E\) is the edge set, \(e_{ij} \in E\) indicates the path \(v_i\) connected to \(v_j\), \(T\) is the time attribute matrix of \(E\), \(t_{ij} \in T\), \(C\) is the cost attribute matrix of \(E\), \(c_{ij} \in C\) indicates the cost required for transport \(v_i\) to \(v_j\), \(P\) is the safety attribute matrix of \(E\), \(p_{ij} \in P\) is the safety probability of road \(e_{ij}\).

If \(e_{ij} \notin E\), make \(t_{ij} = +\infty\), \(c_{ij} = +\infty\), \(p_{ij} = 1\). Introduce the homogeneous judgment matrix \(B = \{b_{ij}\}_{m \times n}\):

\[
b_{ij} = \text{sgn}[v_i - v_j]^2 \quad \forall i, j \in [1, 2, \ldots, n]
\]

\(\text{sgn}[\cdot]\) is symbolic function. When transported homogeneity, \(v_i = v_j\), \(b_{ij} = 0\). When transported non-homogeneous, \(v_i \neq v_j\), \(b_{ij} = 1\).

3.1.1 Time Property

\(t_{ij}\) is the time property of \(e_{ij}\), which indicates the time interval between the arrival of the first vehicle \(v_i\) and the departure of the first vehicle \(v_j\), when transported homogeneity:

\[
t_{ij} = s_{ij} / \mu_{ij}
\]

\(s_{ij}\) is the mileage of \(e_{ij}\), if \(e_{ij} \notin E\) or \(e_{ij} \in E\), \(s_{ij} = +\infty\), \(\mu_{ij}\) is the Average speed of \(e_{ij}\).

When transported non-homogeneous:

\[
t_{ij} = \alpha_i g_i + s_{ij} / \mu_{ij} + \beta_j \cdot \min\{d_j, M\}
\]

\(\alpha_i\) and \(\beta_j\) represented the uninstall time factor \(v_i\) and loading time factor of \(v_j\), \(g_i\) is the standard load for transit vehicles of \(v_i\), \(M\) is the total amount of

![Fig.1 Large-scale combat Military Transportation logistics problems](https://example.com/fig1.png)
delivery (the unit is ton). The volume is calculated according to the relevant standards when transporting the light cargo, \( d_j \) is the standard load for a single vehicle of \( v_j \).

When loading is faster than uninstalling, \( t_{ij} = \alpha_i \cdot \min\{d_j, M\} + s_{ij}/\mu_y + \beta_j \cdot g_i \)  
(4)

\[ t_{ij} = s_{ij}/\mu_y + b_j \cdot \min\{\alpha_i, \beta_j\} \cdot g_i \]

\[ + \max\{\alpha_i, \beta_j\} \cdot \min\{d_j, M\} \] \( \forall i, j \in [1, 2, \cdots, n] \) (5)

### 3.1.2 Cost Attributes

Cost attributes \( c_{ij} \) showed the direct economic cost of the shipping and loading process \( e_{ij} \), the cost included of, loading and unloading cost, transportation cost consisted of the start-up cost and the running cost. If homogeneous transport \( c_{ij} \) is made up of running expenses only, it has to do with transport distance and volume.

\[ c_{ij} = \gamma_{ij} \cdot s_{ij} \cdot M \]  
(6)

\( \gamma_{ij} \) is run base price, unit is Yuan/ton per kilometer.

If it’s Non-homogeneous transport, \( c_{ij} \) including transportation charges in transit, loading and unloading fees, start-up cost of transportation tools in \( v_j \).

\[ c_{ij} = \left( \eta_i + \gamma_{ij} \cdot s_{ij} + \lambda_j + \theta_j \right) M \]  
(7)

\( \eta_i \) is the unloading cost coefficient of \( v_i \) (Yuan/ton), \( \lambda_j \) is the loading cost coefficient of \( v_j \) (Yuan/ton), \( \theta_j \) is the delivery price (Yuan/ton). The computational formula for the cost is:

\[ c_{ij} = \left[ \gamma_{ij} \cdot s_{ij} + b_j (\eta_i + \lambda_j + \theta_j) \right] M \] \( \forall i, j \in [1, 2, \cdots, n] \)  
(8)

### 3.1.3 Transport Properties

If \( e_{ij} \) is Non-homogeneous transport, the capacity constraint consists of the following three components:

\( d_j \) is the standard load of means of transportation in \( v_j \), \( d_j \) not less than the minimum unit of distribution \( m \), \( d_j \geq m \);

The length/width/height \( (l_1^j/l_2^j/l_3^j) \) of the conveyance in \( v_j \) must be bigger than the length/width/height \( (l_1^j/l_2^j/l_3^j) \) of the unit of the minimum unit of distribution, so \( l_1^j > l_1, l_2^j > l_2, l_3^j > l_3 \);

\( D_j \) is the total load weight of all means of transportation in \( v_j \), which not less than the total weight of the distribution \( M \), so \( D_j \geq M \).

### 3.2 Established the Single Path Multi-Modal Transport Path Optimization Model

If the war zone distribute the total amount of material \( M \) from strategic logistics center \( v_1 \) to forward distribution center or military user \( v_n \), satisfying three conditions, that are the total delivery time \( TIME \) no more than \( W_v \), total cost \( COST \) no more than \( W_c \), the probability of safety \( SAFETY \) no less than \( W_p \).

When the path \( \Gamma \) is chosen for transport only, it is a single path multi-modal transport path optimization problem. Set up target function with the total time \( TIME \), total cost \( COST \), Total safety probability \( SAFETY \) individually, established a multi-objective multimodal transport path optimization model.

\[ \text{Min } \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij} \]  
(9)

\[ \text{Min } \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \]  
(10)

\[ \text{Max } \prod_{i=1}^{n} \prod_{j=1}^{n} q_{ij} \]  
(11)

\[ s.t \ x_{ij} = \begin{cases} 0 & \text{if } e_{ij} \notin \Gamma \\ 1 & \text{if } e_{ij} \in \Gamma \end{cases} \] \( \forall i, j \in [1, 2, \cdots, n] \)  
(12)

\[ \sum_{j=1}^{n} x_{1j} = 1 \ \forall j \in [2, 3, \cdots, n] \]  
(13)

\[ \sum_{i=1}^{n} x_{im} = 1 \ \forall i \in [1, 2, \cdots, n-1] \]  
(14)

\[ \sum_{i=1}^{n} x_{ik} = \sum_{j=1}^{n} x_{kj} \ \forall k \in [2, 3, \cdots, n-1] \]  
(15)
\[ u_i - u_j + n \cdot x_{ij} \leq n-1 \quad \forall i, j \in [1, 2, \cdots, n] \quad (16) \]

\[ v_i \in [1, 2, 3, 4, 5, 6] \quad \forall i \in [1, 2, \cdots, n] \quad (17) \]

\[ b_{ij} = \text{sgn}\left[(v_i - v_j)^2\right] \quad \forall i, j \in [1, 2, \cdots, n] \quad (18) \]

\[ t_{ij} = s_y / \mu_y + b_{ij} \min\{\alpha_i, \beta_j\} \cdot g_i \]
\[ + \max\{\alpha_i, \beta_j\} \cdot \min\{d_j, M\} \quad \forall i, j \in [1, 2, \cdots, n] \quad (19) \]

\[ c_{ij} = [y_{ij} \cdot s_y + b_{ij} (\eta_i + \lambda_j + \theta_j)] \cdot M \]
\[ \forall i, j \in [1, 2, \cdots, n] \quad (20) \]

\[ (m - d_j) \sum_{i=1}^{n} (b_{ij} x_{ij}) \leq 0 \quad \forall j \in [1, 2, \cdots, n-1] \quad (21) \]

\[ (M - D) \sum_{i=1}^{n} (b_{ij} x_{ij}) \leq 0 \quad \forall j \in [1, 2, \cdots, n-1] \quad (22) \]

\[ (t^k - t^l) \sum_{i=1}^{n} (b_{ij} x_{ij}) \leq 0 \quad \forall j \in [1, 2, \cdots, n-1]; \quad \forall k \in [1, 2, 3] \quad (23) \]

\[ q_{ij} = \left(\left(p_{ij} - 1\right)x_{ij} + 1\right) \quad \forall i, j \in [1, 2, \cdots, n] \quad (24) \]

\[ \text{TIME} \leq W_t \quad (25) \]

\[ \text{COST} \leq W_c \quad (26) \]

\[ \text{SAFETY} \geq W_p \quad (27) \]

Introduction the decision variable \( p_{ij} \), which is a stage safety probability, stand for the safety probability on the distribution path \( \Gamma \) from the starting point \( v_i \) to \( v_j \).

\[ p_1 = 1, \quad p_j = \sum_{i=1}^{n} p_i \cdot p_{ij} \cdot x_{ij} \quad \forall j \in [2, 3, \cdots, n] \quad (28) \]

So the SAFETY can be expressed in another form.

\[ \text{SAFETY} = \sum_{j=1}^{n} p_j \cdot p_{mn} \cdot x_{mn} \quad (29) \]

This simplified the 0-1 nonlinear programming model to the 0-1 linear programming model.

\[ \ln(\text{SAFETY}) = \ln\left(\prod_{i=1}^{n} \prod_{j=1}^{n} q_{ij}\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_{ij} \cdot \ln p_{ij}) \quad (30) \]

\[ q_{ij} = \exp\left(x_{ij} \cdot \ln p_{ij}\right) \quad \forall i, j \in [1, 2, \cdots, n] \quad (31) \]

### 3.3 Model Solution Method

The problem of single path multi-modal transport path optimization is linear programming; these planning issues involve more variables, the constraint expression is more complex. In this paper, LINGO software for nonlinear programming is selected to programming and solving, it can be greatly improved the speed of the solution, the global optimal solution can be obtained conveniently.

### 4. Simulation Analysis and Solution Of The Model

#### 4.1 Transportation Task

The superior ordered the X Military Region to have a reasonable distribution plan, and organize transportation delivering 40 tons goods to the front distribution center of an island in the Taiwan Strait from a strategic logistics center near the base of Wuhan within 40 hours.

#### 4.2 Implementation Plan

Selected different optimization targets and setting different constraints, Using LINGO software, the model of single path multi-modal transport path optimization is analyzed, made a reference plan as shown in table 1 (assuming that 120 km/h of railway transportation, unloading at the rate of 200 tons per hour; 90 km/h of road transportation, transit transportation 60 km/h; 25 km/h of carrier, loading and unloading speed per hour 150 tons; 800 km/h of air transport, loading and unloading rate of 100 tons per hour. Basic price of railway transport is 10.4 Yuan/ton, basic price of road transport is 0.45 Yuan/ton per kilometers, loading of the goods is 2 Yuan/ton, unloading of the goods is 1.5 Yuan/ton, shipping prices is 5 Yuan/ton, loading and unloading price of 20 Yuan/ton; air transport price is 1097 Yuan/ton, running basic price is 1.628 Yuan/ton per kilometers, loading and unloading price is 500 Yuan/ton). From table1:
<table>
<thead>
<tr>
<th>Reference plan</th>
<th>Constraint conditions</th>
<th>Distance/k m</th>
<th>Time/ h</th>
<th>Cost/ THY</th>
<th>Safety probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>The most economical solution</td>
<td>$\Gamma_1$: Without constraints</td>
<td>1839</td>
<td>79.77</td>
<td>17.95</td>
<td>0.808</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_2$: (TIME &lt; 40\text{h})</td>
<td>1548</td>
<td>36.11</td>
<td>18.18</td>
<td>0.727</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_3$: (TIME &lt; 30\text{h})</td>
<td>1234</td>
<td>28.45</td>
<td>47.82</td>
<td>0.583</td>
</tr>
<tr>
<td>The fastest solution</td>
<td>$\Gamma_4$: Without constraints</td>
<td>1086</td>
<td>23.58</td>
<td>342.74</td>
<td>0.613</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_5$: (COST &lt; 50\text{THY})</td>
<td>1234</td>
<td>28.45</td>
<td>47.82</td>
<td>0.583</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_6$: (COST &lt; 50\text{THY})</td>
<td>1548</td>
<td>36.11</td>
<td>18.18</td>
<td>0.727</td>
</tr>
<tr>
<td>The safest solution</td>
<td>$\Gamma_7$: Without constraints</td>
<td>2196</td>
<td>44.35</td>
<td>446.05</td>
<td>0.808</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_8$: (TIME &lt; 30\text{h}), (COST &lt; 350\text{THY})</td>
<td>1192</td>
<td>28.98</td>
<td>338.95</td>
<td>0.767</td>
</tr>
<tr>
<td></td>
<td>$\Gamma_9$: (TIME &lt; 40\text{h}), (COST &lt; 100\text{THY})</td>
<td>1548</td>
<td>36.11</td>
<td>18.18</td>
<td>0.727</td>
</tr>
</tbody>
</table>

*THY stand for Ten thousand Yuan

If cost is the optimal target, $\Gamma_1$ is the most economic plan, which is water transport after the goods delivery, transport along the Yangtze river from Wuhan to Shanghai, then delivery to the specified location on the island; $\Gamma_2$ is the most economic plan meet the conditions of $TIME < 40\text{h}$ and $SAFETY > 0.7$, which transport from Wuhan to Wenzhou by railway, then turn to the destination by water transportation; $\Gamma_3$ is the most economic plan meet the condition of $TIME < 30\text{h}$, which transport directly to Fuzhou via the highway and then transport to the destination by water.

If time is the optimal goal, $\Gamma_4$ is the quickest plan, which transport directly to Fuzhou by air and transferred to the destination by water. $\Gamma_5$ is the quickest plan if $COST < 50\text{THY}$, which the same as $\Gamma_3$; $\Gamma_6$ is the quickest plan meet the conditions of $COST < 50\text{THY}$ and $SAFETY > 0.7$, which the same as $\Gamma_3$.

If the objective is to optimize the safety probability, $\Gamma_7$ is the safest plan, obtained the maximum safety probability is 0.808, which is to determine the maximum safe probability of unconstrained conditions; $\Gamma_8$ is the safest plan $TIME < 30\text{h}$ and $COST < 350$ THY, which transport to Wenzhou by air then transported to the destination by water. $\Gamma_9$ is the safest plan meet the condition of $TIME < 40\text{h}$ and $COST < 100$ THY, which the same as $\Gamma_2$.

5. Conclusion

In this paper, a single-path multi-objective multimodal transport path optimization model is established; the model of the $H^2$ order characteristic is simplified to a 0-1 linear programming model by simplifying. The solution time is greatly reduced, the global optimal solution is also easy to obtain, and the model is verified by an example. The optimal distribution plan can be developed according to different constraints, the simulation results show that the model is reasonable; it can provide theoretical support for fast accurate and efficient transportation, it can realize the optimal allocation of multi-modal transport resources and capacity of military transport. The next step is to optimize the multi-path multimodal transport path optimization model, and improve the logistics distribution capacity of our military.

Reference