

# Study on Interval Optimal Control Method for Ramp's Merging Areas in Urban Expressway

Jiang-Hong JIN<sup>a</sup>, Wei LI<sup>b</sup>, Jin-Yan HU<sup>c</sup>, and Dan-Dan LI<sup>d</sup>

*Institute of Operational Research and Cybernetic, Hangzhou Dianzi University, Hangzhou, China*

<sup>a</sup>jin8023hong@126.com, <sup>b</sup>weilihz@126.com, <sup>c</sup>hjybbloveyou@163.com, <sup>d</sup>lidandan9203@163.com

**Abstract:** How to solve the problem of urban expressway traffic congestion is an urgent problem for the government and the transportation department. Ramp's merging areas, as the interaction link between internal and external traffic of expressway system, is the main research object of this paper. Taking Hangzhou Qiushi expressway on the road section of the entrance ramp as the target section, the interval optimal control model considering vehicle speed, time occupancy and vehicle spacing as interval variables is proposed, and the single dynamic adjustment algorithm of on-ramp is given.

## 1. Introduction

In recent years, urban traffic congestion has become one of the troubled problems plaguing the government and traffic department. Urban expressway system, as the main road of urban traffic, plays a very important role in urban residents travel. However, due to the high volume of traffic, urban expressway traffic congestion has become increasingly serious.

Taking the traffic system of Hangzhou, Zhejiang as the research object, we find that it is impossible to solve the increasing traffic demand by building more roads and enlarging the scale of the traffic network. Hangzhou has long planned the “four vertical and five horizontal” traffic trunk and gradually completed. However, the problem of traffic congestion has not been improved. In fact, it is the lag of management technology research. Therefore, the management and optimization of urban traffic should arouse people’s attention.

Urban transportation system is a very complicated system. At present, it is impossible to establish a unified interval optimization model for the whole system. However, according to the study we found that traffic jam phenomenon in the large, heavily intertwined ramp is significant. On account of the characteristics of traffic flow, Chen put forward the ramp control method based on time occupancy and velocity as control variables [1]. Actually, the traffic volume, speed and time occupancy of the same road section are interval numbers. Therefore, we propose the interval optimal control model of ramp based on reference [1], and use a single dynamic adjustment algorithm of on-ramp to give the control scheme.

Throughout the paper, we follow the notations given in [2-5].

An interval number is defined as

$$a^I = [\underline{a}, \bar{a}] = \{a \in \mathbb{R} | \underline{a} \leq a \leq \bar{a}\}.$$

The set of all intervals is denoted by  $\text{IR}$ . If  $\underline{a} = \bar{a} = a$ , then  $a^I$  is the real number  $a$ .

Operations over closed real intervals are defined by the general rule

$$a^I * b^I = [\underline{a}, \bar{a}] * [\underline{b}, \bar{b}] = \{a * b | \underline{a} \leq a \leq \bar{a}, \underline{b} \leq b \leq \bar{b}\},$$

where  $* \in \{+, -, \times, \div\}$  and division is defined only if

$$0 \notin b^I = [\underline{b}, \bar{b}].$$

A unit of time, the number of vehicles through a certain point, a section or a lane of the road is known as traffic flow. Generally traffic flow is notated by  $q^I$ .

$$q^I = \{q | \underline{q} \leq q \leq \bar{q}\}.$$

Calculation formula of  $q$  is defined as

$$q = \frac{N}{T},$$

where  $T$  is length of time,  $N$  is the number of vehicles in period  $T$ .

Generally, the degree of compactness between vehicles on the road is called intensity, and it is represented by time occupancy or density. Generally time occupancy is notated by  $o^I$ , formula of  $o$  is defined as

$$o = \frac{\sum_{i=1}^N \Delta t_i}{T},$$

where  $\Delta t_i$  is time for vehicle  $i$  crossing the section.

## 2. Model Establishment

The basic goal of expressway ramp control is that main traffic flow of the expressway is in a steady state and safety of the vehicle is ensured. Based on the analysis for vehicle interaction characteristics and traffic flow stability mechanism of ramp's merging areas in urban expressway[6,7], we propose a guided real-time dynamic coordinated control strategy to limit the amount of traffic entering the expressway, so that the actual traffic volume of the expressway does not exceed its maximum capacity.

Change of time occupancy on the main line can achieve the control of the on-ramp adjustment rate. As much as possible to play the main line capacity, while we guarantee that time occupancy of the main line swing in the vicinity of the desired time share.

Time occupancy control model is given below,

$$f_k^I = q_{k-1}^I + \varphi_1 \cdot [o_c - o_{k-1}^I],$$

where  $q_{k-1}^I$  is range of flow for the on-ramp during the-1 period,  $\varphi$  is regulating coefficient (obtained by a large number of tests),  $k$  is critical value of occupancy on the main road upstream, it is obtained by parameter calibration,  $o_{k-1}^I$  is time occupancy on the main road upstream during the  $k$ -1 period.

Vehicle speed model is given below,

$$g_k^I = q_{k-1}^I + \varphi_2 \cdot [v_{k-1}^I / v_c - 1],$$

where  $v_{k-1}^I$  is average speed on the main road upstream in the  $k$ -1 period,  $v_c$  is critical value of speed on the main road upstream.

In order to reduce the jamming degree of on-ramp traffic to the main traffic flow, we establish the control model considering lane-change safe distance.

$$m_k^I = q_{k-1}^I + \varphi_3 \cdot [S_{k-1}^I / S_c - 1],$$

where  $S_{k-1}^I$  is vehicle spacing of kerb lane in the  $k$ -1 period,  $S_c$  is lane-change safe distance.

For traffic flow under different conditions, different weights are given to the above models to determine the actual on-ramp regulation rate. Finally, the interval optimal control model of ramp is obtained.

$$r_k^I = \mu_1 f_k^I + \mu_2 g_k^I + \mu_3 m_k^I,$$

where  $\mu_1 + \mu_2 + \mu_3 = 1$ .

Remark. If  $r_k^I$  is too small, the vehicle is prohibited from entering the ramp. This is likely to cause the ramp to close in a long time; while  $r_k^I$  is too large to lead traffic congestion on the expressway downstream. Therefore, the minimum ramp rate  $r_{min}^I$  and the maximum regulation rate  $r_{max}^I$  are provided for the expressway ramp. The actual value is determined by comprehensive data analysis.

## 3. Algorithm

In fact, the interval optimization problem is usually transformed into an optimization problem based on real variables. Through  $r_k^I$  we easily obtain that

$$\begin{aligned}\bar{r}_k &= \mu_1 \bar{f}_k + \mu_2 \bar{g}_k + \mu_3 \bar{m}_k, \\ \underline{r}_k &= \mu_1 \underline{f}_k + \mu_2 \underline{g}_k + \mu_3 \underline{m}_k.\end{aligned}$$

In view of the complexity of the Hangzhou traffic problem, we take the lower bound  $\underline{r}_k$  of the on-ramp rate of the target road as an example to realize dynamic adjustment algorithm of on-ramp control.

The control strategy of the on-ramp traffic flow is mainly guided real-time dynamic coordinated control method. First of all, we set the detectors in the main lane and ramp to test traffic flow, speed, time share of each lane and vehicle spacing of the outer lane in a certain period of time; then transmit to the control terminal simultaneously and calculate the on-ramp adjustment rate; finally, we output the control scheme.

We use the signal machine to export control program. The amount of traffic allowed to import the main line is defined as

$$n = \frac{C \cdot \underline{r}_k}{3600},$$

where  $\underline{r}_k$  is the lower bound of the on-ramp rate,  $C$  is sampling period of export in the ramp's merging areas.

Based on the signal timing method in the unified way, we can determine the corresponding display time of light color.

The lane-change safe distance of the lane is determined by the combination of the speed and acceleration of the target lane. Assume  $\underline{r}_k$  is relatively small, that is, traffic flow of the main line is relatively large, at this point we consider whether to generate safe lane spacing.

To sum up, dynamic adjustment algorithm of ramp control has the following steps.

1) The relevant data of the target road section is detected by detectors in the sampling period. Actually, period is generally 20 seconds.

2) Compute

$$\underline{r}_k = \mu_1 \underline{f}_k + \mu_2 \underline{g}_k + \mu_3 \underline{m}_k.$$

3) If  $\underline{r}_k < \underline{r}_{min}$ , then set  $\underline{r}_k := \underline{r}_{min}$ . Consider whether to generate lane-change safe distance. If not, then the on-ramp is normally open; otherwise, go to step 5.

4) If  $\underline{r}_k > \underline{r}_{max}$ , then the on-ramp is open all the time.

5) Output corresponding display time of light color, then go to step 1.

#### 4. Model Evaluation and Analysis

We collected the vehicle speed and vehicle spacing of the target road in the period, vehicle spacing and other data. Using *Vissim* simulation software, we analyzed the above model by traffic simulation, and verified the accuracy and practicability of the interval optimal control model. The model has the following advantages.

a) Although the single-point ramp regulation can not achieve the optimal target of the system, the single-point ramp control is much lower than the multi-ramp coordination control in terms of technical complexity and investment cost.

b) In most cases, local problems are often more serious than the system, only in the local problem is resolved under the premise, in order to better alleviate the system problems.

But our model's shortcomings are given below.

a) The urban expressway ramp control for accidental factors caused by traffic congestion problems difficult to be adjusted in time, flexibility and adaptive performance is relatively poor, the lack of special circumstances for traffic management and control methods of research.

b)  $r_k$  should be adjusted with the other ramp to get an optimal solution on the entrance rate at the same time, perhaps we can maximize the protection of the main road traffic efficiency.

#### 5. Conclusion

Ramp's merging areas are the key nodes of the cross correlation between expressway mainline and urban road network, the traffic operating conditions of which play a direct impact on the effective expressway system functions. We propose interval optimal control model of ramp based on three interval variables (vehicle speed, time occupancy and vehicle spacing), and use the single dynamic adjustment algorithm of on-ramp to give control scheme. Traffic system is a complicated system

composed of people, vehicles, roads and environment. To establish a unified interval optimization model which can describe and explain the complex behavior of various traffic flow is far-reaching.

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