An Evolutionary Game Model of Multi-Topics Diffusion in Social Network

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Abstract: One major function of social networks is the dissemination of information such as news, comments, and rumors. The information passing from a sender to a receiver intrinsically involves both of them by considering their memory, reputation, and preference, which further determine their decisions of whether or not to diffuse the topic. To understand such human aspects of the topics dissemination, we propose a game theoretical model of the multi-topics diffusion mechanisms in a social network. Each individual in the network is considered as both sender and receiver, who transmits different topics taking into account their payoffs and personalities (including memories, reputation and preferences). Several cases were analyzed, and the results suggest that multi-topics dissemination is strongly affected by self-perceived, gregarious and information gain.

1 Introduction

A social network is an ensemble of communicating personalities based on the concept of social proximity [1]. The participants in a social network can form communities [2], influence other participants [3]. One major function of social networks (in particular, massive online social networks) is the dissemination of information such as news, comments, and rumors [4-7]. As an important form of social organization, Information can shape public opinion, inform public behavior [5], further, rumors can spread astoundingly fast through social networks [8].

Due to its significance, information diffusion has been one of the focuses in social network research. In precious work, epidemic models [9] have been widely adopted by researchers for information diffusion due to the analogy between epidemics and the spread of information. Reference [10] investigate the adoption of the classic Susceptible-Infected-Removed (SIR) model for information dissemination. Yang and Leskovec [11] developed a linear influence model to focus on influence of individual node on the rate of dissemination through the implicit network. These studies have a macroscopically eye on the description of information diffusion through social networks.

In recent years, researchers gradually observe that game behaviors between individuals in social network, game-theoretic models, as a new perspective of interpreting social diffusion, are increasingly adopted by computer scientists for analyzing network behaviors. For example, Kostka et al. [8] carried out examinations on the dissemination of competing rumors in social network, using concepts of game theory and location theory, modelling the selection of starting nodes for the rumors as a strategy game. Zinoviev et al. [12, 13] adopted game theoretic models to understand human aspects of information dissemination in which personalities of individuals are considered. Qiu et al. [14] come to a result that information dissemination can be divided into several stages, and the speed of spreading is influenced by characteristic of individuals in the network. Wu et al. [15] focused on the influence of trust in the spreading of information.

However, these researches focus on one topic in the network, several topics will be diffused at the same time in the real network. Sun and Yao [16] discuss the multi-topics, but they aimed at studying the process of competitive information diffusion. In this paper, we focus on more general topics, not only the competitive ones. We introduce a framework taking into consideration that people may care about several aspects. In particular, a utility function is defined to capture what factors shape individuals' choice in social networks.

Our model of multi-topics diffusion and influence as processes taking place on social networks, node is either sender or receiver. We focus on factors that characterize human behaviors, analyze how these factors affect information propagation based on the assumption that individuals aim at maximizing their utilities by picking optimal strategy, and show several laws.

The remainder of this paper is organized as follows. Section II presents the overview of the model and Section III analyses simulation results.

2 Evolutionary Game Model

2.1 Model Setting

Social network is a set of nodes transmit the information from one to another [17]. Nodes represent individuals or organizations, edges stand for social relationships. The information spreading in social network, affects the network in several aspects, such as node's properties, appearance and disappearance of edges. Getting down to fundamentals, it's individuals' various behaviors that lead to the diffusion of information.

As some topics are acknowledged by one node, it will choose someone to disseminate to its neighbor or not, which is decided by several factors, such as its preference, and the information's popularity.

Let us introduce a topic diffusion model in social network. Specifically, every two nodes' interaction is described as a two-player game, in simple terms, different players choose the optimal strategy to achieve best payoffs according to the opponent's strategy.

It is assumed that there is a network consisting of M nodes which represents M participants, $\mathbf{P}=\{p_1, p_2, \ldots, p_M\}$, and there are N topics, $\mathbf{T}=\{t_1, t_2, \ldots, t_N\}$, transmitted in this network. As a real person, we can only recognize and remember the finite topics. Thus, we assume that every participant can remember K ($K \leqslant N$) topics, which is denoted as $\boldsymbol{x_i}=\{x_{i1}, x_{i2}, \ldots, x_{iK}\}$ and these K topics will be updated after each game. And, for p_i, it has $K+1$ pure strategies, $\boldsymbol{s_i}=\{s_{i0}, s_{i1}, s_{i2}, \ldots, s_{iK}\}$, where s_{i0} represents that p_i does not transmit any topic and s_{ik} represents p_i transmits topic x_{ik}, $k=1,2,\ldots,K$.

In order to construct a precise model to explain the real situation, two factors are introduced into this model. One is reputation, each participant is influenced by its neighbor according to the neighbor's reputation. We define $\mathbf{R}$ to describe the influence, where

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1M} \\ r_{21} & r_{22} & \cdots & r_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M1} & r_{M2} & \cdots & r_{MM} \end{bmatrix} \tag{1}$$

$$\sum_{\substack{1 \le i' \le m, \\ i' \ne i}} r_{ii'} = 1,\ 0 \le r_{ii'} \le 1,\ r_{ii} = \ self_i,\ i, i' = 1, 2, \ldots, M.$$

$r_{ii'}$ represents the degree that $p_{i'}$ influence p_i. Specially, when $i'=i$, $r_{ii'}$ represent its self-perceived, which is defined by parameter $self_i$, $0 \leqslant self_i \leqslant 1$. Another is preference, every participant has their unique preference on these topics, which is denoted by $\mathbf{U^0}$, where

$$\mathbf{U^0} = \begin{bmatrix} u^0_{11} & u^0_{12} & \cdots & u^0_{1N} \\ u^0_{21} & u^0_{21} & \cdots & u^0_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ u^0_{M1} & u^0_{M1} & \cdots & u^0_{MN} \end{bmatrix} \tag{2}$$

$$u^0_{ij} \in R,\ i = 1, 2, \ldots M,\ j = 1, 2, \ldots, N.$$

If $u^0_{ij}>0$, it represents how p_i likes t_j, if $u^0_{ij}<0$, it represents how p_i hates t_j; and if $u^0_{ij}=0$, it represents p_i does not care about t_j.

2.2 Utility Function

Now, we take a simple game as example to discuss the problem. Assuming that the two players are p_i and $p_{i'}$, $i \neq i'$, there is a directed edge from p_i to $p_{i'}$, and they both have $K+1$ respective strategies. We assumed that p_i and $p_{i'}$ take σ_i and $\sigma_{i'}$ as their strategy. And during this game, three conditions (as Fig. 1) will happen: (a) such as p_1 and p_4 in Fig. 1, $\sigma_i=t_j$, and $\sigma_{i'}=t_j$, p_i will gain a gregarious profit and pay the cost of diffusion, denoted as (3); (b) such as p_1 and p_3, $\sigma_i=t_j$, $\sigma_{i'}=t_{j'}$, $t_j \neq t_{j'}$ and $t_{j'} \notin \boldsymbol{x_i}$, p_i will get an information gain, denoted as (4); (c) such as p_1 and p_2, $\sigma_i=t_j$, $\sigma_{i'}=t_{j'}$, $t_j \neq t_{j'}$, but $t_j \in \boldsymbol{x_i}$, p_i will have no profit, and its payoff is denoted as (5).

$$u_{ii'} = \alpha * u^0_{ij} - c_j \tag{3}$$

$$u_{ii'} = 1/n * \beta * \ u^0_{ij'} - c_j \tag{4}$$

$$u_{ii'} = -c_j. \tag{5}$$

Where parameters α and c_j represent the gregarious profit and the cost of diffusing t_j respectively. And, β and n represent the information gain and the number of neighbors of p_i. Specially, when $\sigma_i=s_{i0}$, there will be no gregarious profit and no cost of diffusion.

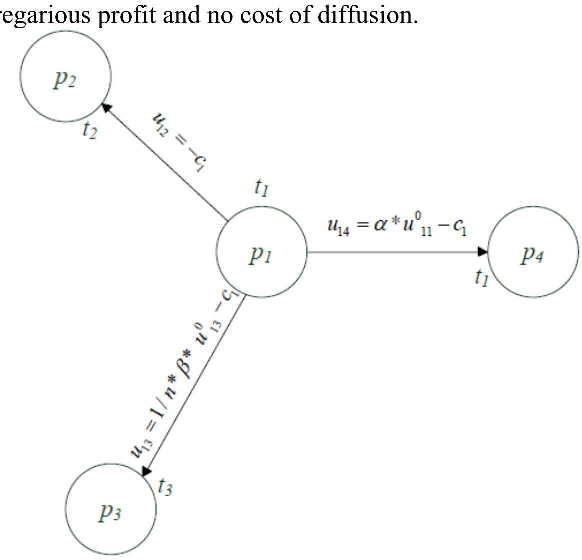

Figure 1. The example of utility function on three conditions, where $t_1 \neq t_2 \neq t_3$, and $t_3 \notin \boldsymbol{x_1}$, $t_2 \in \boldsymbol{x_1}$.

For each game on the network, the whole payoff of p_i is denoted as u_i, where,

$$u_i = \sum\nolimits_{i'=1}^{M} e_{ii'} * u_{ii'},\quad i' \neq i \tag{6}$$

$$e_{ii'} = \begin{cases} 1 & r_{ij} > 0 \\ 0 & r_{ij} = 0 \end{cases}. \tag{7}$$

$e_{ii'}$ denotes whether the edge from i to i' exists or not in the network.

2.3 Situation Update

During the process of information propagation, p_i's memory, strategy, and the neighbors' influence will be updated.

2.3.1 Memory update:

As is known that personal memory is limited and it will be changed by surroundings. Based on the illustration of limit, the update will be demonstrated here. After each game, p_i will send one topic and receive several topics from neighbors. Obviously, the more spreaders transmit the topic and the deeper the influence of spreader is, the higher probability of the topic will be remembered by p_i. So, the influence degree of each topic can be calculated according to the influence matrix **R**. That is to say,

$$w_{ij} = r_{ij} * \sigma_{i'j} \tag{8}$$

$$\sigma_{i'j} = \begin{cases} 1 & \sigma_{i'} = t_j \\ 0 & \sigma_{i'} \neq t_j \end{cases} . \tag{9}$$

Where w_{ij} means the influence of t_j on p_i, and $\sigma_{i'j}$ means whether strategy of $p_{i'}$ is choosing t_j to diffuse or not. Then, the topics are ranked by its influence, and the top K will be taken into $\boldsymbol{x_i}$. Meanwhile, if the number of $w_{ij}>0$ is less than K, the topics in last memory will be selected into $\boldsymbol{x_i}$ according to its order.

2.3.2 Influence update:

Game Theory assumes that the players are rational person and want to gain the most profit. Therefore, each player will seek the players who could bring positive profit and abandon the players who might bring negative profit. In order to imitate such process, the influence of p_i's neighbor need to be updated as

$$r_{ii'} = \begin{cases} e_{ii'} * (r_{ii'} + \sum_{j=1}^{M} \sigma_{i'j} * u^0_{ij}) & i' \neq i \\ self_i & i' = i \end{cases} \tag{10}$$

That is to say, if one of p_i's neighbor transmits a topic which p_i likes, its influence will raise; but if it transmits a topic p_i hates, its influence will descend. Noting that $r_{ii'}$ may become a minus with the game going on. To deal with such situation, a trick is made here. Once $r_{ii'} \leqslant 0$, we find a new participant $p_{i''}$ and let $r_{ii'} =0$ and $r_{ii''} >0$, which imitates the process mentioned above. The neighbors who need to be abandoned is simple to determine, but the new neighbors who needed to be selected is not obvious. In this paper, the p_i's new neighbor is recommended by p_i's present neighbors as follows.

a) Neighbor Selection: The present neighbor of p_i is selected according to $\pi_{ii'}$, where

$$\pi_{ii'} = \frac{r_{ii'}}{\sum_{\substack{1 \le i' \le M \\ i' \neq i}} r_{ii'}} \tag{11}$$

b) Neighbor Recommendation: The present neighbor will select its neighbour $p_{i''}$ randomly and recommend $p_{i''}$ to p_i. And if $r_{ii''}>0$, step *a)* and *b)* should be repeated until $r_{ii''}=0$.

c) Influence Establishment: We give $r_{ii''}$ a rand positive number which is less than 1 as $p_{i''}$'s initial influence on p_i.

Finally, **R** needs to be normalization as (12), so that it can satisfied the assumption in *A*.

$$r_{ii'} = \frac{r_{ii'}}{\sum_{\substack{1 \le i' \le M \\ i' \neq i}} r_{ii'}} \tag{12}$$

2.3.3 Strategy update:

With the memory update of p_i, the new strategy will emerge. We choose the topic x_{i1} which p_i pays most attention on, and calculate its payoff denoted by $u_{i'}$ based on the present situation. Then the probability of changing strategy is $1/\{1+exp[(u_i-u_{i'})/q]\}$, where q is a noise coefficient which represents the bounded rationality of p_i.

3 Case Study

In order to derive the topic dissemination process, we conducted several experiments in a simulation directed network, consisting of M individuals. And whether the edge from p_i to $p_{i'}$ exists obeys the binomial distribution. Where $M=50$ and the probability of link is 0.5.

After the network constructed, the game is initialed as Section □. Here, we initial the variable as follows: $r_{ii}\sim U(0,1)$, $u_{ij}\sim N(0.05)$ and $r_{ii'}$ is normalized as (12). In addition, we set $N=100$ and $K=10$ in this case.

We start our analysis with how topics dissemination differs according to the parameters α, β, and $self_i$ that affect utility function. Before carrying out the contrast experiment, entropy is introduced in to assess the topics distribution.

3.1 Evaluator

During each game, every participant will have a strategy of transmitting some topic or not. Therefore, the proportion of each topic t_j in the step τ will be calculated easily, which is denoted as

$$\delta_j^\tau = \frac{\sum_{i=1}^{M} \sigma_{ij}}{M} \tag{13}$$

Where σ^τ_{ij} means whether p_i disseminate t_j in the step τ. Then the topics' entropy in the step τ is denoted as

$$E^\tau = -\sum_{j=1}^{N} \delta_j^\tau * \log_2 \delta_j^\tau \tag{14}$$

Analogy with the information theory, the smaller the entropy is, the less disorder the information is. In this paper, the smaller the entropy is, the more concentrated the topics are.

3.2 Constract Experiment

3.2.1 Profit parameter

In this paper, we fix $self_i$=0.2, and compare the parameters α, β.

Fig. 2 and Fig. 3 shows the influence of gregarious and information gain on the multi-topics diffusion respectively.

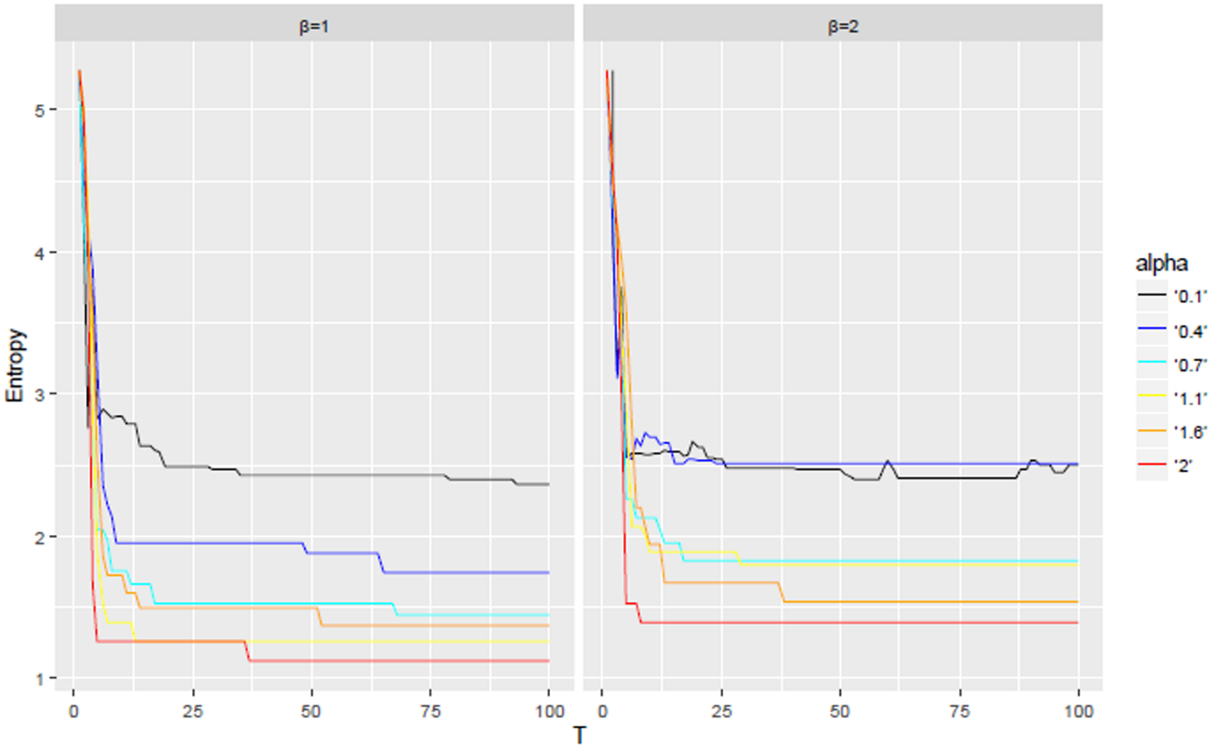

Figure 2. The relation between the changing of α and that of entropy during the game process when $self_i$=0.2, β=*1 or 2*.

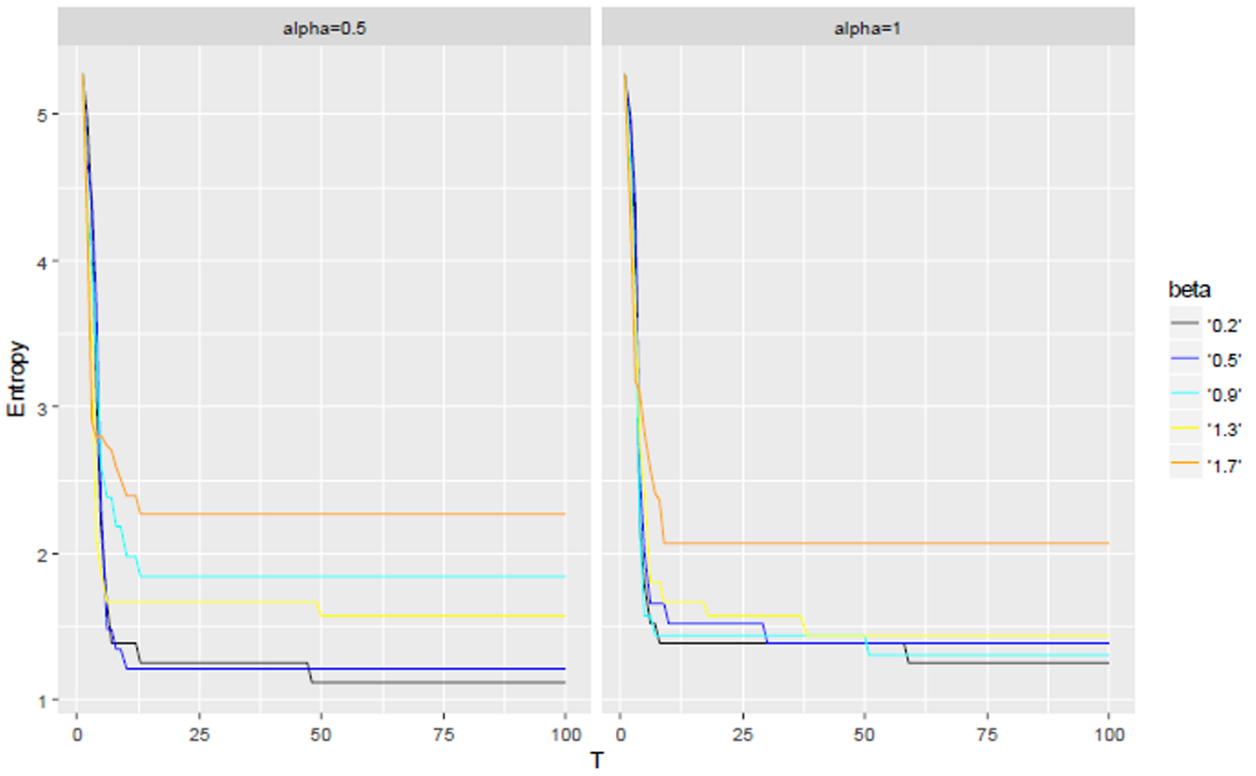

Figure 3. The relation between the changing of β and that of entropy during the game process when selfi=0.2, α=0.5 or 1.

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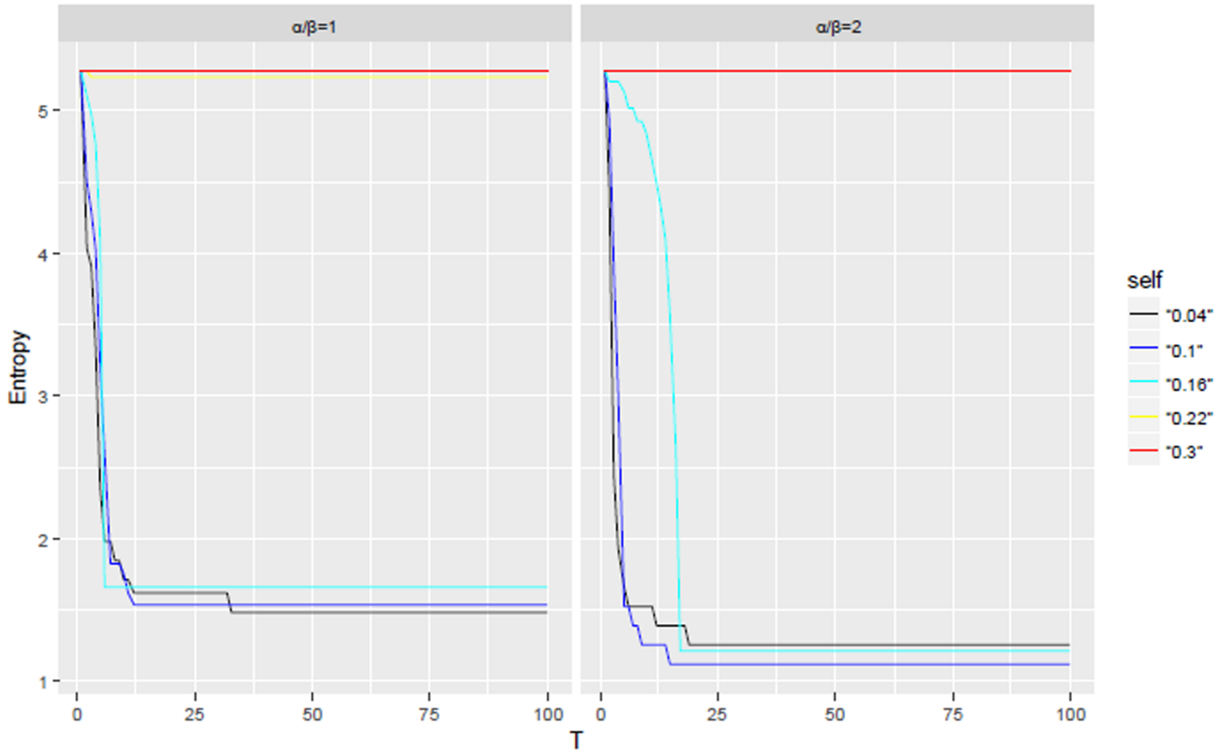

Figure 4. The relation between the changing of α and that of entropy during the game process when $self_i$=0.2, β=*1 or 2*.

The X-axis represents the steps of game and the Y-axis means the entropy of the topics propagated in the network in a particular step. Fig. 2 reveals that the smaller α is, the higher the entropy is. And the smaller α is, the later the entropy converges. Even when $\alpha=1$, the topics cannot be convergent. And this property is authentic no matter what β is. Similarly, Fig. 3 reveals that the larger β is, the entropy is. And the larger β is, the earlier the entropy converges.

3.2.2 self-perceived parameter

Subsequently, we compare the parameter $self_i$. As shown in Fig. 4, $self_i$ is inversely proportional to entropy, and the smaller $self_i$ is, the later the entropy converged. Even when $self_i \geq 0.22$, each individual will insist on their initial topics. This property occurs no matter what the parameters α, β are.

4 Conclusion and Future Directions

In this paper, a multi-topics diffusion model in social network based on evolutionary game theory is presented. Social network consists of nodes with personality, thus utility function and interact rules are proposed as close to reality as possible, considering memory, reputation and preference in social networks. Afterwards, a simulation experiment is carried out.

Our analysis reveals interesting insights into the nature of multi-topics diffusion. Multi-topics diffusion is strongly related with gregarious and information gain. If the participants prefer gregarious gain, the topics will be converged. But if the participants prefer information gain, the topics will be diversified. In addition, self-perceived will lead the topics diffusion significantly.

It is our hope that this paper will provide some new insights into the research of multi-topics diffusion in social network. To improve our model, several directions are proposed:

- Considering the initial network, variable network densities and structures can be studied. And the topology of network can be discussed, for example, the node's degree, clustering coefficient, assortative, communities and etc.
- The parameters α, β and $self_i$ is not a fixed value, each participant has its unique parameters. Then the different proportion of parameters value must lead to different topics diffusion, which is interesting to discuss.
- Government can be introduced in the game as a special participant, who has a different utility function.
- Considering the alterable topics makes the model more reality. In another word, the topics will not be fixed, the new topics will be produced and those who have little concentration will fade away.

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