ITM Web of Conferences **13**, 01004 (2017) DOI: 10.1051/itmconf/20171301004
CMES2017

Asymptotically Statistical Equivalent of Order α in Amenable Semigroups

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Abstract. In this study we introduce the concepts of asymptotically statistical equivalent functions of order α and strong asymptotically equivalent functions of order α defined on discrete countable amenable semigroups.

1 Introduction

The idea of statistical convergence was given by Zygmund [30] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [29] and Fast [13] and later reintroduced by Schoenberg [27] independently. Later on it was further investigated from the sequence space point of view and linked with summability theory by Altin et al. [2], Connor [1], Çinar et. al. [3], Çolak [4], Et et. al. ([8],[9],[10],[11],[12]), Fridy [14], Işık [15], Küçükaslan and Yılmaztürk [16], Mursaleen [20], Nuray and Rhoades ([22],[23],[24]), Salat [26], Şengül and Et [28] and many others.

The idea of statistical convergence depends upon the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of a subset $\mathbb{E}$ of $\mathbb{N}$ is defined by

$$\delta(\mathbb{E}) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_{\mathbb{E}}(k), \quad \text{provided that the limit texists.}$$

A sequence $x = (x_k)$ is said to be statistically convergent to L if for every $\varepsilon > 0$,
$\delta(\{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}) = 0$.

Marouf [19] introduced definitions for asymptotically equivalent sequences and asymptotic regular matrices. Patterson [25] extend these concepts by presenting an asymptotically statistically equivalent analog of these definitions and natural regularity conditions for nonnegative summability matrices.

Suppose that G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $w(G)$ and $m(G)$ denote the spaces of all real valued functions and all bounded real functions on G, respectively. $m(G)$ is a Banach space with the norm

$$\| f \|_\infty = sup\{|f(g)| : g \in G\}.$$

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Nomika [21] showed that, if G is a countable amenable group, there exists a sequence $\{S_n\}$ of finite subsets of G such that

i) $G = \cup_{n=1}^{\infty} S_n$,

ii) $S_n \subset S_{n+1,} \qquad n = 1, 2, 3, ...$

iii) $\lim_{n\to\infty} \frac{|S_n g \cap S_n|}{|S_n|} = 1, \qquad \lim_{n\to\infty} \frac{|gS_n \cap S_n|}{|S_n|} = 1, \qquad$ for all $g \in G$.

Here $|A|$ denotes the number of elements the set A.

Any sequence of finite subsets of G satisfying $(i), (ii)$ and (iii) is called a Folner sequence for G. For a detailed account of amenable semigroups one may refer to ([5],[6],[7],[17],[18])

2 Main Results

Definition 1 *Let α be any real number such that $0 < \alpha \leq 1$. We define the upper and lower Folner $\alpha-$density of the subset S of G by*

$$\overline{\delta}_\alpha(S) = \limsup_{n\to\infty} \frac{1}{|S_n|^\alpha} |\{g \in S_n : g \in S\}|$$

and

$$\underline{\delta}_\alpha(S) = \liminf_{n\to\infty} \frac{1}{|S_n|^\alpha} |\{g \in S_n : g \in S\}|$$

respectively. If $\overline{\delta}_\alpha(S) = \underline{\delta}_\alpha(S)$, then

$$\delta_\alpha(S) = \lim_{n\to\infty} \frac{1}{|S_n|^\alpha} |\{g \in S_n : g \in S\}|$$

is called Folner $\alpha-$density of S. For $\alpha = 1$, we have Folner density of $S \subset G$ which were defined an studied by Nuray and Rhoades ([22],[23],[24]).

Definition 2 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let α be any real number such that $0 < \alpha \leq 1$. The function $f \in w(G)$ is said to be statistically convergent to s of order α for any Folner sequence $\{S_n\}$ for G if*

$$\lim_{n\to\infty} \frac{1}{|S_n|^\alpha} |\{g \in S_n : |f(g) - s| \geq \varepsilon\}| = 0.$$

The set of all statistically convergent functions of order α will be denoted by $S^\alpha(G)$.

Definition 3 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, α be any real number such that $0 < \alpha \leq 1$. The function $f \in w(G)$ is said to be strongly summable of order α to s for any Folner sequence $\{S_n\}$ for G if*

$$\lim_{n\to\infty} \frac{1}{|S_n|^\alpha} \sum_{g\in S_n} |f(g) - s| = 0.$$

The set of all strongly summable functions of order α will be denoted by $w^\alpha(G)$.

Definition 4 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let* α *be any real number such that* $0 < \alpha \leq 1$. *Two nonnegative functions* $f, h \in w(G)$ *are said to be asymptotically statistical equivalent of order* α, *for any Folner sequence* $\{S_n\}$ *for G if, for every* $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{|S_n|^\alpha} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right| = 0.$$

In this case we write $f \overset{S^\alpha}{\sim} g$.

Definition 5 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let* α *be any real number such that* $0 < \alpha \leq 1$. *Two nonnegative functions* $f, h \in w(G)$ *are said to be strong asymptotically equivalent of order* α, *for any Folner sequence* $\{S_n\}$ *for G if, for every* $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{|S_n|^\alpha} \sum_{g\in S_n} \left| \frac{f(g)}{h(g)} - 1 \right| = 0.$$

In this case we write $f \overset{w^\alpha}{\sim} g$.

Theorem 6 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and suppose also that the parameters* α *and* β *are fixed real numbers such that* $0 < \alpha \leq \beta \leq 1$, *then* $f \overset{S^\alpha}{\sim} g$ *implies* $f \overset{S^\beta}{\sim} g$.

Proof. Proof follows from the inequality

$$\frac{1}{|S_n|^\beta} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right| \leq \frac{1}{|S_n|^\alpha} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right|.$$

From Theorem 6 we have the following. ■

Corollary 7 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let* α *be any real number such that* $0 < \alpha \leq 1$, *then* $f \overset{S^\alpha}{\sim} g$ *implies* $f \overset{S}{\sim} g$.

Theorem 8 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and suppose also that the parameters* α *and* β *are fixed real numbers such that* $0 < \alpha \leq \beta \leq 1$, *then* $f \overset{w^\alpha}{\sim} g$ *implies* $f \overset{w^\beta}{\sim} g$.

Proof. Omitted. ■

From Theorem 8 we have the following.

Corollary 9 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let* α *be any real number such that* $0 < \alpha \leq 1$, *then* $f \overset{w^\alpha}{\sim} g$ *implies* $f \overset{w}{\sim} g$.

Theorem 10 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and suppose also that the parameters* α *and* β *are fixed real numbers such that* $0 < \alpha \leq \beta \leq 1$, *then* $f \overset{S^\alpha}{\sim} g$ *implies* $f \overset{w^\beta}{\sim} g$.

Proof. Omitted. ■

From Theorem 10 we have the following results.

Corollary 11 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let α be any real number such that* $0 < \alpha \leq 1$*, then* $f \overset{S^{\alpha}}{\sim} g$ *implies* $f \overset{w^{\alpha}}{\sim} g$.

Corollary 12 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold and let α be any real number such that* $0 < \alpha \leq 1$*, then* $f \overset{S^{\alpha}}{\sim} g$ *implies* $f \overset{w}{\sim} g$.

Corollary 13 *Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, then* $f \overset{S}{\sim} g$ *implies* $f \overset{w}{\sim} g$.

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