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A Note on Sturm-Liouville Problem for Difference Equations

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Abstract. In this study, Sturm-Liouville delay difference equation is considered with initial conditions as follows

$$-\Delta^2 x(n-1)+q(n)x(n-k)=\lambda x(n),$$
$$x(1)=x(2)=1,$$

where $k\in\mathbb{Z}^+$, n is a finite integer. Sum representation is found and from here, behaviors of eigenfunctions are investigated for different k values by illustrating with graphs and tables.

1 Introduction

Difference equations is discrete analogue of differential equations. So, some special type differential equations' discrete counterparts are analyzed paralelly. Sturm-Liouville problems is one of these examples. Sturm-Liouville problem has an important role because of a lot of applications in mathematical physics. Sutrm-Liouville differential and also difference equations were studied by variable authors [1-6]. Oscillation criteria and asymptotic behaviours of delay difference equations were studied by [7-14].

While generally, Sturm-Liouville difference equations are handle with a constant potential function, we handle it as a variable in our studies [3,4]. For the first time, we consider Sturm-Liouville delay difference equation.

In this study, our aim is to investigate behaviors of eigenfunctions for Sturm-Liouville delay difference initial value problem, defined as follows

$$-\Delta^2 x(n-1)+q(n)x(n-k)=\lambda x(n), \tag{1}$$
$$x(1)=x(2)=1, \tag{2}$$

where $k\in\mathbb{Z}^+$, n is a finite integer, Δ is the forward difference operator, λ is the positive spectral parameter, $q(n)$ is a real valued potential function.

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2 Preliminaries

Definition 1. [15] Casoratian is defined by

$$w(n)=\begin{bmatrix} x_1(n) & x_2(n) & \dots & x_r(n) \\ x_1(n+1) & x_2(n+1) & \dots & x_r(n+1) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(n+r-1) & x_2(n+r-1) & \dots & x_r(n+r-1) \end{bmatrix},$$

where $\sum_{i=a}^{b-1} x(i)\Delta y(i)=\left[x(i)y(i)\right]^{b}_{a}-\sum_{i=a}^{b-1}\Delta x(i)y(i+1).$ are solution functions.

Theorem 1 [17] (*Wronskian-Type Identity*) Let x and y be a solutions of (1). Then,

$$W[x,y](n)=\left[x(n)\Delta y(n-1)-y(n)\Delta x(n-1)\right]$$

is a constant.

Theorem 2 [15] (**Summation by parts**) If $a<b$, then

$$\sum_{i=a}^{b-1} x(i)\Delta y(i)=\left[x(i)y(i)\right]^{b}_{a}-\sum_{i=a}^{b-1}\Delta x(i)y(i+1).$$

Theorem 3 [15] *If* $y(n)$ is an indefinite sum of $x(n)$, then

$$\sum_{i=a}^{b-1} y(i)=x(b)-x(a).$$

3 Main Results

In this study, our aim is to investigate behaviors of eigenfunctions for Sturm-Liouville initial value delay difference problem, defined as follows

$$-\Delta^2 x(n-1)+q(n)x(n-k)=\lambda x(n), \tag{1}$$

$$x(1)=x(2)=1, \tag{2}$$

where $k\in\mathbb{Z}^{+}$, n is a finite integer Δ is the forward difference operator, λ is the positive spectral parameter, $q(n)$ is a real valued potential function.

Theorem 4 The representation of solution of Sturm-Liouville initial value delay difference problem (1)-(2) is found as follows;

$$x(n)=\frac{\sin 2\theta-\sin\theta\left(1+q(1)x(1-k)\right)}{\sin\theta}\cos n\theta+\frac{-\cos 2\theta+\cos\theta\left(1+q(1)x(1-k)\right)}{\sin\theta}\sin n\theta$$
$$-\frac{1}{\sin\theta}\sum_{i=1}^{n-1} q(i)x(i-k)\sin(n-i)\theta\,,$$

where $\lambda=2-2\cos\theta,\theta\neq k\pi,\ \sum_{i=a}^{b}.=0,$ if $a>b$.

Proof It is straightforward to obtain the representation of solution by variation of parameters method [15,16]. It is seen that the representation of solution holds for the problem (1)-(2) by writing into place.

3.1 Applications

Application 1 Let $q(n)=\frac{1}{n^{3/2}}, \lambda=1,$ and corresponding eigenfunctions for *k=0* and *k=1* are $x_1(n)$ and $x_2(n)$ respectively, then we can obtain datas following

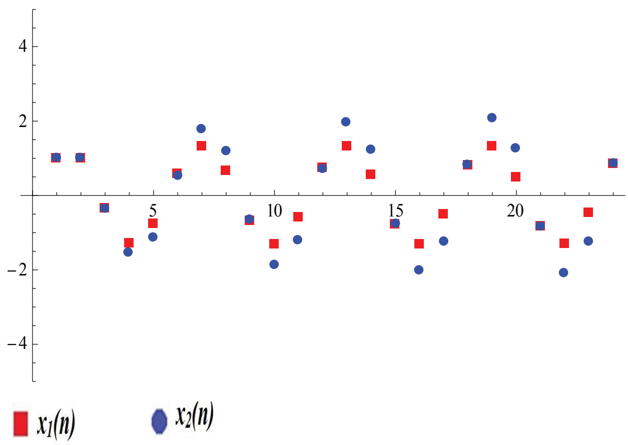

Fig. 1. Comparison of datas in Table 1

Table 1

n	$x_1(n)$	$x_2(n)$
1	1	1
2	1	1
3	−0.35	−0.35
4	−1.28	−1.54
5	−0.77	−1.14
10	−1.31	−1.88
11	−0.59	−1.22
22	1.3	−2.1
23	−0.46	−1.25
24	0.84	0.86

Application 2 Let $q(n)=\frac{1}{n^{3/2}}, \lambda=1,$ and corresponding eigenfunctions are $x_1(n)$ and $x_2(n)$ for *k=0* and *k=2* respectively, then we can obtain datas following

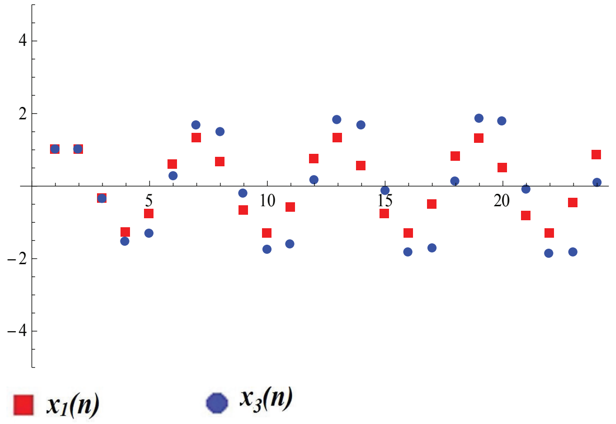

Fig. 2. Comparison of datas in Table 2

Table 2

n	$x_1(n)$	$x_3(n)$
1	1	1
2	1	1
3	−0.35	−0.35
4	−1.28	−1.54
5	−0.77	−1.31
10	−1.31	−1.75
11	−0.59	−1.6
22	1.3	−1.88
23	−0.46	−1.81
24	0.84	0.06

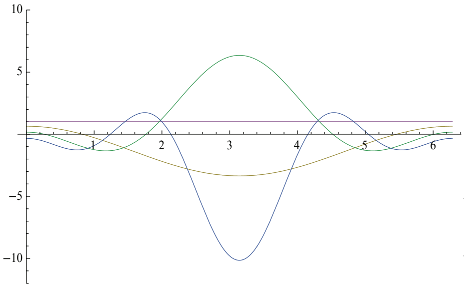

Fig. 3. $x_1(n)$, n=1,2,3,4

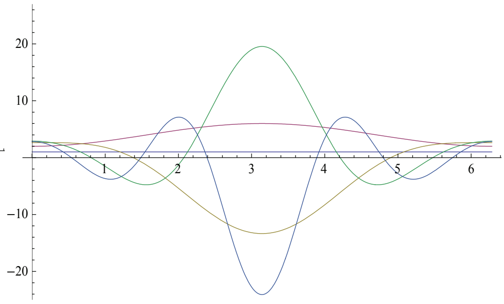

Fig.4. $x_2(n)$, n=1,2,…,5

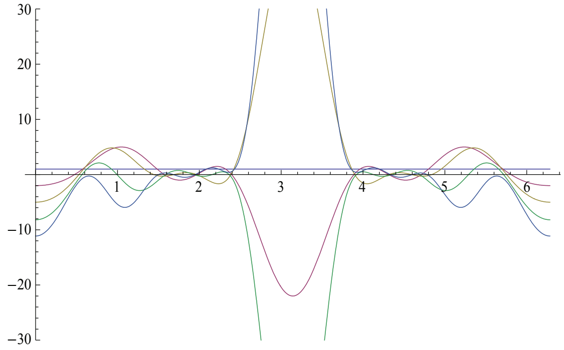

Fig. 5. $x_3(n)$, n=1,2,…,5

Conclusion

In this study, we investigated the behaviors of eigenfunctions for different k values and obtained some numerical results. We showed the results by tables and figures. Obtained results show that the eigenfunctions are shifted for different k values.

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