

Some Prototype Results of the Symmetric Regularized Long Wave Equation Arising in Nonlinear Ion Acoustic Waves

*Erdem IŞIK*¹, *Hasan BULUT*², *Sibel Şehriban ATAS*^{2,*}

¹ Tunceli Vocational School, Munzur University, Tunceli, Turkey

² Faculty of Science, Department of Mathematics, Firat University, Elazig, Turkey

Abstract. In this manuscript, we consider the Bernoulli sub-equation function method for obtaining new exponential prototype structures to the Symmetric Regularized Long Wave mathematical model. We obtain new results by using this technique. We plot two- and three-dimensional surfaces of the results by using Wolfram Mathematica 9. At the end of this manuscript, we submit a conclusion in the comprehensive manner.

1. Introduction

In the recent years, the investigation of exact travelling wave solutions to nonlinear partial differential equations plays an important role in the study of nonlinear modelling physical phenomena. Also the study of the travelling wave solutions plays an important role in nonlinear science. A variety of powerful methods has been presented, such as Hirota's bilinear method[1], the inverse scattering transform [2], sine-cosine method [3], homotopy analysis method [4,5], variational iteration method [6-8]. We study different aspects of the Symmetric Regularized-Long-Wave equation (SRLW). The Symmetric Regularized-Long-Wave equation is a model for the weakly nonlinear ion acoustic and space-charge waves[9]. This equation was introduced by Seyler and Fenstermacher in[10]. Moreover, some important results have been submitted to the literature by various scientists [11,21]

The paper is organized as follows: In section 2, we describe the Bernoulli Sub-Equation function method (BSEFM). In section 3, we consider the following Symmetric

* Corresponding author: suzundag90@gmail.com

Regularized Long Wave equation [22] defined by

$$u_{tt} + u_{xx} + uu_{xt} + u_x u_t + u_{xxtt} = 0. \quad (1)$$

Moreover, in section 4 we give the physical interpretations and remarks of the solutions obtained by BSEFM. Also a comprehensive conclusion is given in section 5.

2. Fundamental Properties of the Method

An approach to the mathematical models including partial differential equations will be presented in this sub-section of paper. The steps of the this technique which is partially new modified can be taken as follows [10].

Step 1. We consider the following nonlinear partial differential equation (NLPDE) in two variables and a dependent variable u ;

$$P(u_x, u_t, u_{xt}, u_{xx}, \dots) = 0, \quad (2)$$

and take the travelling wave transformation

$$u(x, t) = U(\eta), \quad \eta = x - ct, \quad (3)$$

where $c \neq 0$. Substituting Eq.(3) in Eq.(2) yields a nonlinear ordinary differential equation (NLODE) as following;

$$N(U, U', U'', \dots) = 0, \quad (4)$$

where $U = U(\eta), U' = \frac{dU}{d\eta}, U'' = \frac{d^2U}{d\eta^2}, \dots$.

Step 2. Take trial equation of solution for Eq.(4) as following:

$$U(\eta) = \sum_{i=0}^n a_i F^i = a_0 + a_1 F + a_2 F^2 + \dots + a_n F^n, \quad (5)$$

and

$$F' = bF + dF^M, \quad b \neq 0, \quad d \neq 0, \quad M \in \mathbb{R} - \{0, 1, 2\}, \quad (6)$$

where $F(\eta)$ is Bernoulli differential polynomial. Substituting Eq.(5) along with Eq.(6) in Eq.(4), it yields an equation of polynomial $\Omega(F(\eta))$ of $F(\eta)$ as following;

$$\Omega(F(\eta)) = \rho_s F(\eta)^s + \dots + \rho_1 F(\eta) + \rho_0 = 0. \quad (7)$$

According to the balance principle, we can obtain a relationship between n and M .

Step 3. Let the coefficients of $\Omega(F(\eta))$ all be zero will yield an algebraic equations system;

$$\rho_i = 0, \quad i = 0, \dots, s. \quad (8)$$

Solving this system, we will specify the values of a_0, \dots, a_n .

Step 4. When we solve Eq.(6), we obtain the following two situation according to b and d ;

$$F(\eta) = \left[\frac{-d}{b} + \frac{E}{e^{b(M-1)\eta}} \right]^{\frac{1}{1-M}}, \quad b \neq d, \tag{9a}$$

$$F(\eta) = \left[\frac{(E-1) + (E+1) \tanh\left(\frac{b(1-M)\eta}{2}\right)}{1 - \tanh\left(\frac{b(1-M)\eta}{2}\right)} \right]^{\frac{1}{1-M}}, \quad b = d, \quad E \in \mathbb{R}. \tag{9b}$$

Using a complete discrimination system for polynomial, we obtain the solutions to Eq.(4) with the help of Wolfram Mathematica 9 programming and classify the exact solutions to Eq.(4). For a better understanding of results obtained in this way, we can plot two- and three-dimensional surfaces of solutions by taking into consideration suitable values of parameters.

3. Application

When we consider the travelling wave transformation and perform the transformation $u(x, t) = U(\eta), \eta = kx - ct$ which c is constant non-zero in Eq.(1), we obtain NLODE as following;

$$2(k^2 + c^2)U - ckU^2 + c^2k^2U'' = 0. \tag{10}$$

When we consider to Eq.(5) and Eq.(6) for balance principle between U'' and U^2 , we obtain the following relationship between n and M ;

$$2M = n + 2. \tag{11}$$

This relationship gives us some new different complex travelling wave solutions for Eq.(1).

Case 1: If we take as $n = 4$ and $M = 3$ in Eq. (11), we can write following equations;

$$U = a_0 + a_1F + a_2F^2 + a_3F^3 + a_4F^4, \tag{12}$$

$$U' = a_1bF + a_1dF^3 + 2a_2bF^2 + 2a_2dF^4 + 3a_2bF^3 + 3a_3dF^5 + 4a_4bF^4 + 4a_4dF^6, \tag{13}$$

and

$$U'' = b^2Fa_1 + 4bdF^3a_1 + 3d^2F^5a_1 + 4b^2F^2a_2 + 12bdF^4a_2 + 8d^2F^6a_2 + 9b^2F^3a_3 + 24bdF^5a_3 + 15d^2F^7a_3 + 16b^2F^4a_4 + 40bdF^6a_4 + 24d^2F^8a_4, \tag{14}$$

⋮

where $a_2 \neq 0, b \neq 0, d \neq 0$. When we put Eq.(12-14) in Eq.(10), we obtain a system of algebraic equations for Eq.(13). Therefore, we obtain a system of algebraic equations from

these coefficients of polynomial of F . Solving this system with the help of wolfram Mathematica 9, we find the following coefficients and solutions;

Case 1a. For $b \neq d$, it can be considered the following coefficients;

$$\begin{aligned}
 a_1 = 0, a_2 &= \frac{24\sqrt{2}dka_0}{\sqrt{a_0(a_0 - \sqrt{-16 + a_0^2})}}, a_3 = 0, \\
 a_4 &= 12d^2k^2 \left(a_0 + \sqrt{-16 + a_0^2} \right), \\
 c &= \frac{1}{4}k(a_0 + \sqrt{-16 + a_0^2}), b = \frac{\sqrt{a_0(a_0 - \sqrt{-16 + a_0^2})}}{4\sqrt{2}k}.
 \end{aligned}
 \tag{15}$$

Substituting these coefficients in Eq.(12) along with Eq.(9a), we obtain the following new exponential function solution for Eq.(1);

$$u_1(x,t) = a_0 + \frac{24a_0d\sqrt{2}e^{\frac{\rho(-4x+t(a_0+\sqrt{-16+a_0^2}))}{8\sqrt{2}}}}{\left(-4\sqrt{2}dk + e^{\varpi(-4x+t(a_0+\sqrt{-16+a_0^2}))} E k \rho \right)^2},
 \tag{16}$$

where a_0, d, k are real constants and $\rho = \sqrt{a_0(a_0 - \sqrt{-16 + a_0^2})}$, $\varpi = \frac{\rho}{8\sqrt{2}}$.

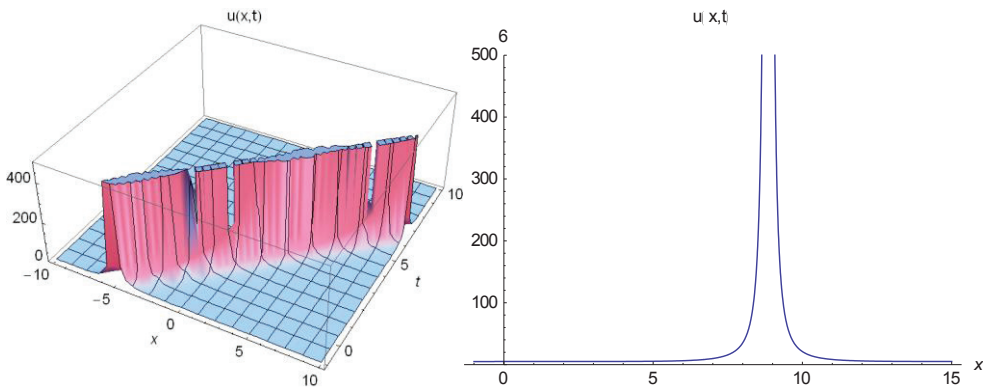


Figure 1. The 2D and 3D surfaces of Eq.(16) under the values of $a_0=5, k=0.3, d=50, E=0.8, -10 < x < 10, -1 < t < 10$ and $t=6$ for 2D surfaces.

Case 1b. Another coefficients for Eq.(1) and for $b \neq d$, it can be considered follows;

$$a_0 = \frac{2c}{k} + \frac{2k}{c}, a_1 = 0, a_2 = -24d\sqrt{c^2 + k^2}, a_3 = 0, \tag{17}$$

$$a_4 = 48cd^2k, b = -\frac{\sqrt{c^2 + k^2}}{2ck}.$$

When we regulate Eq.(12) under the terms of Eq.(9a,17), we find the other new exponential function solution for Eq.(1);

$$u_2(x,t) = \frac{2c}{k} + \frac{2k}{c} + \frac{48cd^2k}{\left(e^{\nu(-ct+kx)} E + \frac{2d}{\nu} \right)^2} - \frac{24d\nu ck}{2cdk + \nu ck e^{\nu(-ct+kx)} E}, \tag{18}$$

where c, d, k are real constants and $\nu = \frac{\sqrt{c^2 + k^2}}{ck}$.

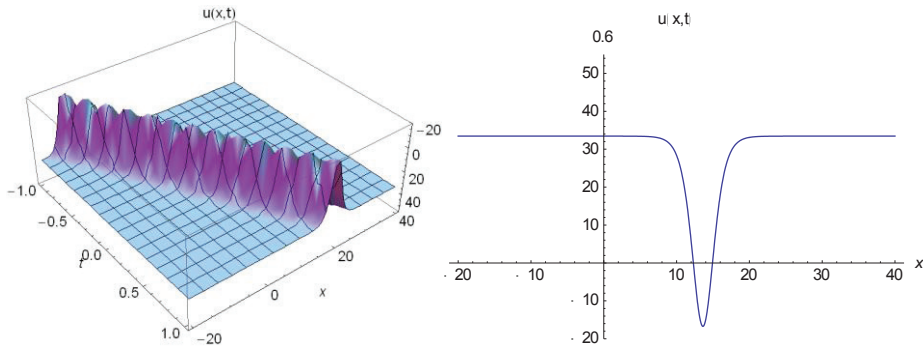


Figure 2. The 2D and 3D surfaces of Eq.(18) under the values of $c=5, k=0.3, d=50, E=0.8, -20 < x < 40, -1 < t < 1, t=0.6$ for 2D surfaces.

Case 1c. Another coefficients to the Eq.(1) for $b \neq d$, it can be considered as follows;

$$a_0 = \frac{8b^2c^2}{\sqrt{-1+4b^2c^2}}, a_1 = 0, a_2 = \frac{48bc^2d}{\sqrt{-1+4b^2c^2}}, a_3 = 0, \tag{19}$$

$$a_4 = \frac{48c^2d^2}{\sqrt{-1+4b^2c^2}}, k = \frac{c}{\sqrt{-1+4b^2c^2}}.$$

Considering these coefficients in Eq.(12) along with Eq.(9a,19), we gain other new complex exponential function solution to Eq.(1);

$$u_3(x,t) = \frac{8b^2c^2 \left(d^2 e^{4bc(-t+\kappa x)} + 4bde^{2bc(-t+\kappa x)} E + b^2 E \right)}{\sqrt{-1+4b^2c^2} \left(de^{2bc(-t+\kappa x)} - bE \right)^2}, \quad (20)$$

where b, c, d are real constants and $\kappa = \frac{1}{\sqrt{-1+4b^2c^2}}$.

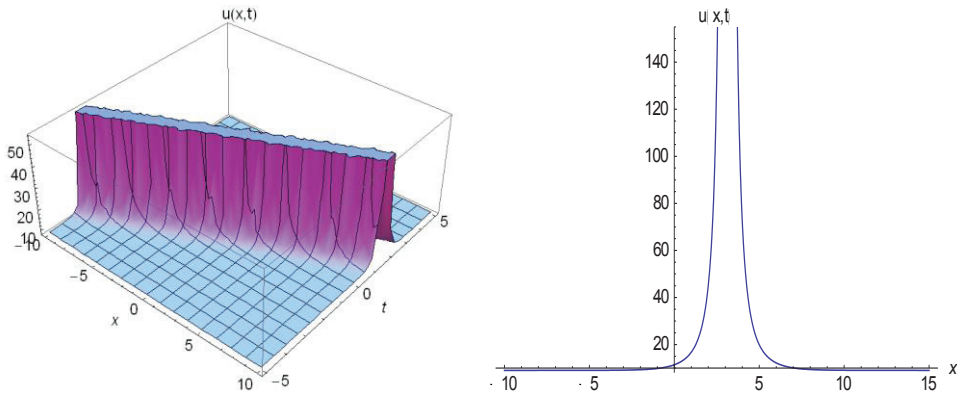


Figure 3. The 2D and 3D surfaces of real part of Eq.(20) under the values of $b=2, d=0.1, c=1.1, E=0.8, -10 < x < 10, -5 < t < 5, t=0.1$ for 2D surfaces.

4. DISCUSSION AND CONCLUSIONS

The solutions such as Eq.(16,18,20) obtained by using BSEFM are the new exponential function structures solutions for Eq.(1) when we compare with the paper submitted to the literature by [22]. It has been observed that all solutions have verified the Eq. (1) by using Wolfram Mathematica 9. To the best of our knowledge, the application of BSEFM to the Eq.(1) has not been previously submitted to the literature. The method proposed in this paper can be used to seek more travelling wave solutions of NLEEs because the method has some advantages such as easily calculations, writing programme for obtaining coefficients and so many. Eq.(18) can be rewritten as hyperbolic functions by using the fundamental properties of hyperbolic functions as follows;

$$u_1(x,t) = \frac{2c}{k} + \frac{2k}{c} + \frac{48cd^2k}{\left(Ee^{\nu(-ct+\kappa x)} + \frac{2d}{\nu} \right)^2} - \frac{24d(c^2+k^2)}{2cdk + Evkce^{\nu(-ct+\kappa x)}},$$

where c, d, k are real constants and $\nu = \frac{\sqrt{c^2+k^2}}{ck}$. When we consider the general properties of hyperbolic and exponential functions, we can rewrite $u_1(x,t)$ solution in the form of hyperbolic function as following.

$$u_1(x,t) = \frac{2c}{k} + \frac{2k}{c} + \frac{48cd^2k}{\left(\frac{E \pm E\sqrt{1-\operatorname{sech}^2(T)}}{\operatorname{sech}(T)} + \delta\right)^2} - \frac{24d(c^2+k^2)}{2cdk + \frac{\rho \pm \rho\sqrt{1-\operatorname{sech}^2(T)}}{\operatorname{sech}(T)}},$$

$$T = \frac{(-ct+kx)}{ck} \sqrt{c^2+k^2}, \delta = \frac{2cdk}{\sqrt{c^2+k^2}}, \rho = E\sqrt{c^2+k^2}.$$

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