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On Some Complex Aspects of the (2+1)-dimensional Broer-Kaup-Kupershmidt System

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Abstract. The improved Bernoulli sub-equation function method is used in extracting some new exponential function solutions to the (2+1)-dimensional Broer-Kaup-Kupershmidt system. It is of vital effort to look for more solutions of the (2+1)-dimensional Broer-Kaup-Kupershmidt system, which are very helpful for coastal and civil engineers to apply the nonlinear water models in a harbor and coastal design. All the obtained solutions satisfied the (2+1)-dimensional Broer-Kaup-Kupershmidt system. The two- and three-dimensional shapes of all the obtained solutions in this paper are also presented. All the computations and the graphics plots in this study are carried out with the aid of the Wolfram Mathematica 9.

1 Introduction

Investigations of the solutions of various nonlinear evolution equations (NLEEs) have become of great important because of the roles the solutions play in our real life situations. Nonlinear evolution equations often used to describe complex aspects in the field of nonlinear sciences such as Biological sciences, chemistry, mathematical physics, engineering and physics. For the past decades, various scholars have displayed their different effort for seeking the solutions of such types of equations, many analytical methods have been developed for this task such as sine-Gordon expansion method [1–3], the Bell-polynomial method [4], the new generalized Jacobi elliptic function expansion method [5], the Exp-function method [6], the modified Exp-function method [7], the (G'/G)-expansion method [8–10], the sub equation method [11], the simplified Hirota's method [12], the simplest equation method [13], the modified simplest equation method [14–16], the improved (G'/G)-expansion method [17], the multiple exp-function algorithm [18], the Lie group analysis and symmetry reductions [19]. In general, various methods have been developed to explore the search of different types of solutions to different kind of NLEEs [20–26].

However, this study is devoted in using the improved Bernoulli sub-equation function method (IBSEFM) [27] to extract some new solutions to the Broer-Kaup-Kupershmidt system [28, 29]. The

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IBSEFM is designed by expanding the Bernoulli sub-equation function method [27]. The solutions to the Broer-Kaup-Kupershmidt system are assume to be very helpful to coastal and civil engineers in applying to the nonlinear water models in a harbor and coastal design [29]. Several efforts have been made to tackle the Broer-Kaup-Kupershmidt system this include the exp-function method [30], the improved mapping approach [31], the generalized modified CK direct method [32] and the the bifurcation theory of planar dynamical systems [33].

2 Analysis of the Method

In this section, we present the steps of the algorithm to be followed in exploring the search of the new solutions to the given NLEE.

Step 1. Let us consider the following nonlinear partial differential equation;

$$P(u,\ u_x u^2\ u_x,\ u_t,\ \ldots), \tag{1}$$

performing the wave transformation $u(x,\ t) = U(\eta)$, $\eta = x - ct$, reduces Eq. (1) to the following nonlinear ordinary differential equation (NODE);

$$Q(U,\ UU'\ U'',\ U^3,\ \ldots), \tag{2}$$

Step 2. Consider the following as a trial solution to Eq. (1);

$$U(\eta) = \frac{\sum_{i=0}^{n} a_i G^i}{\sum_{j=0}^{m} b_j G^j} = \frac{a_0 + a_1 G + a_2 G^2 + \ldots + a_n G^n}{b_0 + b_1 G + b_2 G^2 + \ldots + b_m G^m}, \tag{3}$$

according to the Bernoulli theory,

$$G' = wG + dG^M,\ w \neq 0,\ d \neq 0,\ M \in \mathbb{R} \setminus \{0, 1, 2\}, \tag{4}$$

where $F = F(\eta)$ is a Bernoulli differential polynomial. Substituting Eq. (4) into (2) produces an equation of polynomial $\Psi(G)$ function of G:

$$\Psi(G) = \rho_s G^s + \ldots + \rho_1 G + \rho_0 = 0. \tag{5}$$

We obtain the values of n, m and M by using the balancing technique.

Step 3. We get a system of equations by letting the summations of every coefficients of $\Psi(G)$ with the same power to zero. We obtain the values of the coefficients $a_0, \ldots, a_n$ and $b_0, \ldots, b_n$ by simplifying the system, using the Wolfram Mathematica.

Step 4. Solving the nonlinear Bernoulli differential equation (Eq. (4)), gives the following two cases based on w and d:

$$G(\eta) = \Big[\frac{-d}{w} + \frac{\epsilon}{e^{w(M-1)\eta}}\Big]^{\frac{1}{1-M}}, \quad w \neq d \tag{6}$$

$$G(\eta) = \left[\frac{(\epsilon - 1) + (\epsilon + 1) tanh\Big(\frac{w(1-M)\eta}{2}\Big)}{1 - tanh\Big(\frac{w(1-M)\eta}{2}\Big)}\right]^{\frac{1}{1-M}}, \quad w = d, \epsilon \in \mathbb{R}. \tag{7}$$

Using the complete discrimination of the polynomial of G, we solve the system of equations by using the Wolfram Mathematica 9 and obtain the various cases for the results of the coefficients. We obtain the exact solutions to Eq. (1) by putting the obtained values of the coefficients into the trial solution that is Eq. (3).

3 Application

Consider the following Broer-Kaup-Kupershmidt system [28, 29]:

$$\begin{aligned} u_{ty} - u_{xxy} + 2(uu_x)_y + 2v_{xx} &= 0, \\ v_t + v_{xx} + 2(uv)_x &= 0. \end{aligned} \tag{8}$$

Applying the wave transformation $u = U(\eta)$, $v = V(\eta)$, $\eta = x + y - ct$ on Eq. (8), yields the following NODE:

$$U'' - 2U^3 + 3cU^2 - c^2U = 0, \tag{9}$$

balancing the terms U'' and U^3, yields the following relations between m and n; $n + 1 = M + m$. Choosing $M = 3$, $m = 1$ and inserting in the obtained relation, gives $n = 3$.

Using $m = 1$, $n = 3$ along with the Eq. (3), we get the following equation:

$$U(\eta) = \frac{a_0 + a_1G(\eta) + a_2G^2(\eta) + a_3G^3(\eta)}{b_0 + b_1G(\eta)} = \frac{\Phi(\eta)}{\Omega(\eta)}, \tag{10}$$

differentiating Eq. (10) twice, we get the following equations;

$$U'(\eta) = \frac{\Phi'(\eta)\Omega(\eta) - \Phi(\eta)\Omega'(\eta)}{\Omega^2(\eta)}, \tag{11}$$

$$U''(\eta) = \frac{\Phi''(\eta)\Omega(\eta) - \Phi(\eta)\Omega'(\eta)}{\Omega^2(\eta)} - \frac{[\Phi(\eta)\Omega'(\eta)]'\Omega^2(\eta) - 2\Phi(\eta)[\Omega'(\eta)]^2\Omega(\eta)}{\Omega^4(\eta)}, \tag{12}$$

where $G' = wG + dG^3$, $w \neq 0$, $d \neq 0$. Substituting Eq. (11) and (12) into Eq. (9) and simplifying, we get an equation that involves the polynomials of G. We therefore collect the system of equations from that polynomial there by equating the summation of each coefficients of G that are having the same power to zero. We simplify the system of equations with the help of Wolfram Mathematica 9 and get the following cases of solutions;

Case-1.1: when $w \neq d$

$$a_0 = -2wb_0, a_1 = -2wb_1, a_2 = -2db_0, a_3 = -2db_1, c = -2w,$$

inserting the values of these coefficients into Eq. (10), gives:

$$\begin{aligned} u_{1.1}(x, y, t) = &-\frac{1}{b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}}}}\Bigg(\frac{2db_0}{\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}} \\ &+2wb_0 + \frac{2db_1}{\left(\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}\right)^{\frac{3}{2}}} + \frac{2wb_1}{\sqrt{\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}}}\Bigg), \end{aligned} \tag{13}$$

$$v_{1.1}(x,y,t) = -u_{1.1}(x,y,t)\Big(w + \frac{u_{1.1}(x,y,t)}{2} + \frac{\chi_1(x,y,t)}{2}\Big) - \frac{\chi_2(x,y,t)}{2}, \tag{14}$$

where $\chi_1(x,y,t) = \dfrac{\epsilon w b_1 e^{-2w(x+y+2wt)}}{\left(\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}\right)^{\frac{3}{2}}\left(b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}}}\right)}$

and

$$\chi_2(x,y,t) = \frac{1}{b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}}}}\left(\frac{4d\epsilon w b_0 e^{-2w(x+y+2wt)}}{\left(\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}\right)^2} + \frac{6d\epsilon w b_1 e^{-2w(x+y+2wt)}}{\left(\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}\right)^{\frac{5}{2}}}\right.$$
$$\left. + \frac{2\epsilon w^2 b_1 e^{-2w(x+y+2wt)}}{\left(\epsilon e^{-2w(x+y+2wt)} - \frac{d}{w}\right)^{\frac{3}{2}}}\right)$$

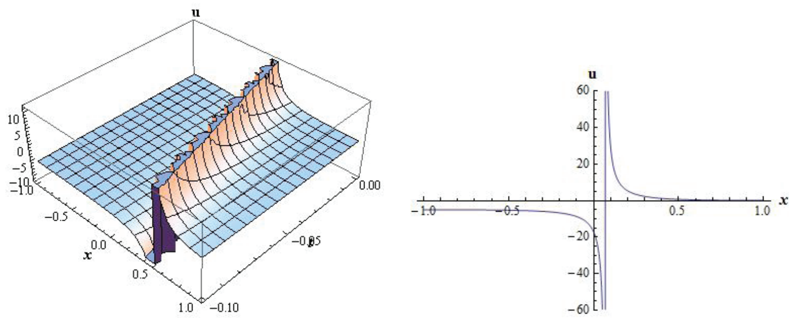

Figure 1. The 3-dimensional and 2-dimensional shapes of Eq. (13) by substituting these values $b_0 = 3$, $b_1 = 1.5$, $w = 2.5$, $d = 2$, $\epsilon = 3.3$, $y = 0.3$, $-1 < x < 1$, $-0.1 < t < 0$ and $t = 0.003$ for the 2-dimensional shape.

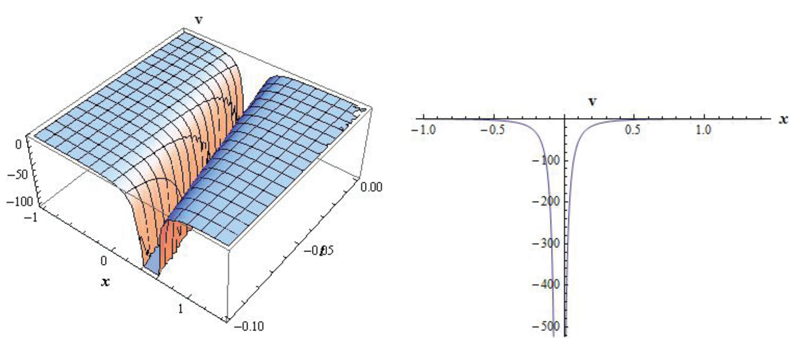

Figure 2. The 3-dimensional and 2-dimensional shapes of Eq. (14) by substituting these values $b_0 = 3$, $b_1 = 1.5$, $w = 2.5$, $d = 2$, $\epsilon = 3.3$, $y = 0.3$, $-1 < x < 1.42$, $-0.1 < t < 0$ and $t = 0.003$ for the 2-dimensional shape.

Case-2.1: when $w \neq d$

$$a_0 = 0, a_1 = 0, a_2 = -2db_0, a_3 = -2db_1, c = -2w,$$

inserting the values of these coefficients into Eq. (10), gives:

$$u_{2.1}(x,y,t) = -\frac{1}{b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}}}}\left(\frac{2db_0}{\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}} + \frac{2db_1}{\left(\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}\right)^{\frac{3}{2}}}\right), \tag{15}$$

$$v_{2.1}(x,y,t) = u_{2.1}(x,y,t)\left(w - \frac{u_{2.1}(x,y,t)}{2} - \frac{\chi_3(x,y,t)}{2}\right) - \chi_4(x,y,t), \tag{16}$$

where $\chi_3(x,y,t) = \frac{\epsilon w b_1 e^{-2w(x+y-2wt)}}{\left(\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}\right)^{\frac{3}{2}}\left(b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}}}\right)}$

and

$$\chi_4(x,y,t) = \frac{1}{b_0 + \frac{b_1}{\sqrt{\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}}}}\left(\frac{4d\epsilon w b_0 e^{-2w(x+y-2wt)}}{\left(\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}\right)^2} + \frac{6d\epsilon w b_1 e^{-2w(x+y-2wt)}}{\left(\epsilon e^{-2w(x+y-2wt)} - \frac{d}{w}\right)^{\frac{5}{2}}}\right)$$

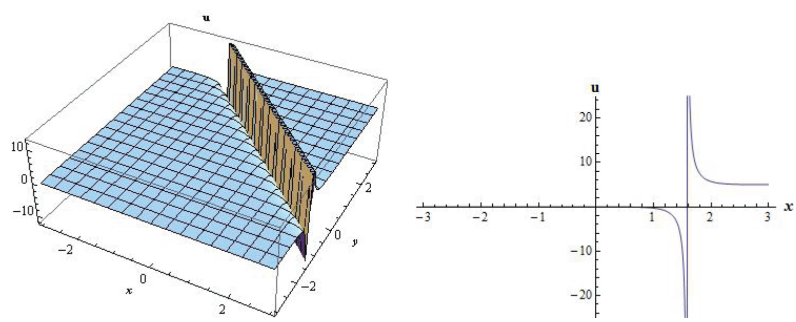

Figure 3. The 3-dimensional and 2-dimensional shapes of Eq. (15) by substituting these values $b_0 = 3$, $b_1 = 1.5$, $w = 2.5$, $d = 2$, $\epsilon = 3.3$, $t = 0.3$, $-3 < x < 3$, $-3 < y < 3$ and $t = 0.2$ for the 2-dimensional shape.

4 Conclusions

The improved Bernoulli sub-equation function method is used in this study to explore a search for the new solutions of the Broer-Kaup-Kupershmidt system and some new exponential function solutions are constructed. All the solutions obtained are verified to have satisfied the Broer-Kaup-Kupershmidt system. We also plot the three and two dimensional shapes of all the existing solutions in this paper. We performed all the computations and the graphics plot in this work with the aid of Wolfram Mathematica 9. We compare our results with the results obtained in [28, 29] and we observe that our results are indeed newly constructed solutions with some novel exponential function structures. From the results obtained, we observe that the improved Bernoulli sub-equation function method is a powerful mathematical tool that can be applied to the various nonlinear evolutions problems that arise in different fields of nonlinear sciences.

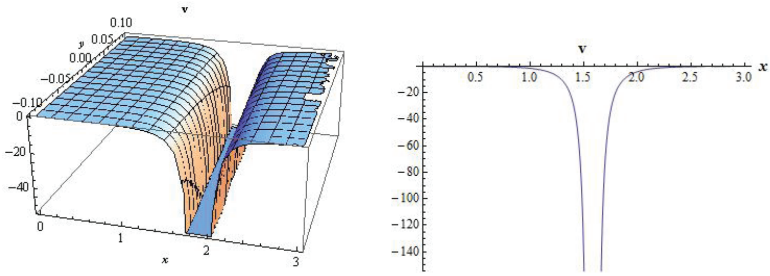

Figure 4. The 3-dimensional and 2-dimensional shapes of Eq. (16) by substituting these values $b_0 = 3$, $b_1 = 1.5$, $w = 2.5$, $d = 2$, $\epsilon = 3.3$, $t = 0.3$, $0 < x < 3$, $-0.1 < y < 0.1$ and $t = 0.2$ for the 2-dimensional shape.

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