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Sequences with Random Indice in Classical Banach Space

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Abstract. In this study we define sequences with random indice in classical Banach space and analyzing it's some properties. After then we define differences sequence space on sequences with random indice. The difference operation is based on indice, and the generated difference sequence has a free stepwise. And here an important definition is the embedding space. Consequently we showed that l_∞ is a embedding space to c.

1 Introduction

Throughout the study ω, l_∞, c, c_0 denote the spaces of all, bounded, convergent and null sequences, respectively. Kızmaz introduced the different sequence spaces on space A and later was generalized by Et and Çolak generalized different sequence spaces respectively $\Delta(A)$, $\Delta^r(A), r \geq 1$. These defined sequence spaces are often used in analysis, and the norm defined on them is usually supremum norm as follows Where convergence is defined according to norm and special sum of sequence. Let's say a few important following by Lorentz

$$\|a\| = sup_n |a_n| \,, a \in A \tag{1}$$

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$$\hat{c} = \left\{ a \colon lim_n \frac{1}{n} \sum\nolimits_{k=1}^{n} a_{k+m} < \infty \; uniformly \; in \; m \right\} \tag{2}$$

By Maddox, strongly almost convergent to a number L,

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$$[\hat{c}] = \left\{a: \frac{1}{n}\sum\nolimits_{k=1}^{n} |a_{k+m} - L| \to 0 \text{ uniformly } in\ m \text{, as } n \to \infty\right\} \quad (3)$$

The following relationship exists between these sequence spaces

$$c \subset [\hat{c}] \subset \hat{c} \subset l_\infty \quad (4)$$

By Nanda

$$[\hat{c}, p] = \left\{a: \frac{1}{n}\sum\nolimits_{k=1}^{n} |a_{k+m} - L|^{p_k} \to 0 \text{ uniformly in } m \text{, as } n \to \infty\right\} \quad (5)$$

$$[\hat{c}, p]_0 = \left\{a: \frac{1}{n}\sum\nolimits_{k=1}^{n} |a_{k+m}|^{p_k} \to 0 \text{ uniformly in } m \text{, as } n \to \infty\right\} \quad (6)$$

$$[\hat{c}, p]_\infty = \left\{a: sup_{n,m} \frac{1}{n}\sum\nolimits_{k=1}^{n} |a_{k+m}|^{p_k} < \infty\right\} \quad (7)$$

Where $p = (p_k)$ positive reel sequence.

By M. Et and R. Çolak

$$A(\Delta^r) = \{a: \Delta^r a \in A\} \quad (8)$$

Were $\Delta^r x_k = \sum_{v=0}^{r} (-1)^r \binom{r}{v} a_{k+v}$. In addition the following sequence spaces have been defined by nodulus function f,

$$[\hat{c}, f, p](\Delta^r) = \left\{a: \frac{1}{n}\sum\nolimits_{k=1}^{n} [f(|\Delta^r a_{k+m} - L|)]^{p_k} \to 0\ uniformly\ in\ m, as\ n \to \infty\right\} \quad (9)$$

$$[\hat{c}, f, p]_0(\Delta^r) = \left\{a: \frac{1}{n}\sum\nolimits_{k=1}^{n} [f(|\Delta^r a_{k+m}|)]^{p_k} \to 0\ uniformly\ in\ m, as\ n \to \infty\right\} \quad (10)$$

$$[\hat{c}, f, p]_\infty(\Delta^r) = \left\{a: sup_{n,m} \frac{1}{n}\sum\nolimits_{k=1}^{n} [f(|\Delta^r a_{k+m}|)]^{p_k} < \infty\right\} \quad (11)$$

Where modulus f is a function from $[0, \infty)$ to $[0, \infty)$ such that

i) $f(x) = 0$ if and only if $x = 0$

ii) $f(x + y) \le f(x) + f(y)$ for $x, y \ge 0$

iii) f is increasing

iv) f is a continuous the right at 0.

The related features, about above spaces, are shown in the following three theorems,

Theorem 1. The spaces $[\hat{c}, f, p](\Delta^r)$, $[\hat{c}, f, p]_0(\Delta^r)$ and $[\hat{c}, f, p]_\infty(\Delta^r)$ are linear spaces on the complex numbers.

Theorem 2. The spaces $[\hat{c}, f, p](\Delta^r)$, $[\hat{c}, f, p]_0(\Delta^r)$ are subspaces to $[\hat{c}, f, p]_\infty(\Delta^r)$ linear space.

Theorem 3. The set A' is a convex subset of linear space A then $\Delta^m(A') \subset \Delta^m(A)$ is a convex subset.

2 Sequences with Random Indice

$\xi = \{X_t : t \geq 0\}$ be a continuous time parameter stochastic process with state space $S = \{1, 2, \cdots\}$. Every X_t is a random variable on the probability space $(\Omega, \Im, P)$. Let be define sequence as following

$$(a, \xi) = \left(a_{X_t}\right), t \geq 0 \tag{12}$$

Where the realization sequence $(a, \xi(\omega)) = \left(a_{X_{t(\omega)}}\right)$ is equal to (a) or subsequence of (a) with every realization $X_t(\omega)$ for arbitrary $\omega \in \Omega$. We defined special condition $(a) = (a_{X_{t(\bar{\omega})}})$. Every realization $\left(a_{X_{t(\omega)}}\right) \subset (a)$ for $\bar{\omega} \neq \omega$. Let we define following set and following spaces

$$\{a, \xi, \Omega\} = \left\{a_{X_{t(\omega)}} : \omega \in \Omega \right\} \tag{13}$$

$$\{A, \xi, \Omega\} = \left\{a_{X_{t(\omega)}} : \omega \in \Omega\, , a \in A\right\} \tag{14}$$

$$\{A, \xi\} = \left\{a_{X_t} : a \in A\right\} \tag{15}$$

Where it easily seen that $\{A, \xi, \Omega_1\} \subset \{A, \xi, \Omega_2\}$ for $\Omega_1 \subset \Omega_2$.

Definition: Let A and B sequence space with $A \subset B$. B is defined a embedding space to A if following equation is hold $\{A, \xi\} = \{B, \xi, \bigcap_{i=1}^{\infty} \Omega_i\}$, for every $\Omega = \bigcup_{i=1}^{\infty} \Omega_i$ and $\Omega \supset \Omega_1 \supset \Omega_2 \ldots$

The following theorem is a correctness the definition,

Theorem 1. The space l_∞ is a embedding space to c.

Proof: It's advisedly that $c \subset l_\infty$. For proof we must shown that $\{c, \xi\} = \{l_\infty, \xi, \bigcap_{i=1}^{\infty} \Omega_i\}$. For every $a \in l_\infty$, there is a $k_0 \geq 1$ and $\omega \in \Omega_{k_0}$ such that $a_{X_{t(\omega)}} \in \left\{l_\infty, \xi, \Omega_{k_0}\right\}$. For the sequence a is a bounded there is a at least one limit point. In this case the sequence $a_{X_{t(\omega)}}$ has a one limit point such that $a_{X_{t(\omega)}} \in \{c, \xi\}$. Otherwise the sequence $a_{X_{t(\omega)}}$ has a one limit point for $k > k_0$. So that $a_{X_{t(\omega)}} \in \{c, \xi\}$. As a result $\{c, \xi\} = \{l_\infty, \xi, \bigcap_{i=1}^{\infty} \Omega_i\}$. Then the space l_∞ is a embedding space to c. Proof is complete.

Let us some of important theorems for space $\{A, \xi, \Omega\}$

Theorem 2. If A is a Banach space with norm $\|\quad\|$ then $\{A, \xi, \Omega\}$ is a Banach space.

Proof: Let be $(e_n) \subset A$ is a Cauchy sequence then $e_n \to e \in A, n \to \infty$ fort this $(A, \|\quad\|)$ Banach space. Such that $e_{X_t}, e_{nX_t} \in \{A, \xi, \Omega\}, n \geq 1$, following inequality hold

$$\left\|e_{nX_t} - e_{X_t}\right\| \leq \|e_n - e\| \tag{16}$$

The proof is complete.

Theorem 3. If A is a linear space on $\mathbb{C}$ complex numbers then $\{A, \xi, \Omega\}$ is a linear space.

Proof: The proof is easily seen that $(\lambda a + \mu b)_{X_{t(\omega)}} = \lambda a_{X_{t(\omega)}} + \mu b_{X_{t(\omega)}}, \lambda, \mu \in \mathbb{C}, a, b \in A$.

Important Result: The following sequence spaces is a linear space on $\mathbb{C}$

$$\{\hat{c}, \xi, \Omega\} = \left\{a_{X_t(\omega)}: n^{-1}\sum_{k=1}^{n} a_{k+X_t(\omega)} < \infty, n \to \infty, uniformly\ \omega \in \Omega\right\} \quad (17)$$

$$\{[\hat{c}], \xi, \Omega\} = \left\{a_{X_t(\omega)}: n^{-1}\sum_{k=1}^{n} \left|a_{k+X_t(\omega)} - L\right| \to 0, n \to \infty, uniformly\ \omega \in \Omega\right\} \quad (18)$$

$$\{[\hat{c}, p], \xi, \Omega\} = \left\{a_{X_t(\omega)}: n^{-1}\sum_{k=1}^{n} \left|a_{k+X_t(\omega)} - L\right|^{p_k} \to 0, n \to \infty, uniformly\ \omega \in \Omega\right\} \quad (19)$$

$$\{[\hat{c}, p]_0, \xi, \Omega\} = \left\{a_{X_t(\omega)}: n^{-1}\sum_{k=1}^{n} \left|a_{k+X_t(\omega)}\right| \to 0, n \to \infty, uniformly\ \omega \in \Omega\right\} \quad (20)$$

$$\{[\hat{c}, p]_\infty, \xi, \Omega\} = \left\{a_{X_t(\omega)}: sup_{n,\omega} n^{-1}\sum_{k=1}^{n} \left|a_{k+X_t(\omega)}\right|^{p_k} < \infty\right\} \quad (21)$$

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