Two-exponential model of ferroimpurities extraction during magnetic inspection of these impurities content in various media

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Abstract. Illustrative examples of process media for which the key mass-operational characteristics of magnetophoretic control of ferroimpurities (represented in test semilogarithmic coordinates) are not subject to the basic (one) exponential model are presented and analyzed. The concept is described and implemented of a two-exponential model that assumes the desuperposition of the key control characteristic (for two subfractions of ferroimpurities). The possibilities are demonstrated of performing practical mass calculations of ferroimpurities in a process media sample and corresponding values of their concentration in this media.

1 Introduction. The main provisions of (one)-exponential model of the step-by-step extraction of ferroimpurities

Many liquid and free-flowing media in industry (fuel and lubricants, raw materials in the production of glass and ceramics, molding mixture for casting molds, food products, etc.) contain a fraction of impurities of almost any kind, as inclusions in the form of ferroparticles. The metal (structural steels) wear and corrosion in various equipment are the main sources of their formation and entry into operating environments.

To ensure the necessary quality (purity) of certain media, which in turn requires professional choice and application of appropriate purification methods and devices, the target content of ferroimpurities is of fundamental importance. Such control is performed via various methods (depending on specific media), including the polyoperational magnetophoresis method [1-4]. In particular, in this case a sample of the liquid media is passed through a sectional magnetic analyzer with the number of sections-operations n or n operations of "magnetic scanning" of a granular medium thin layer [1,2]. This approach leads to one of the conceptual provisions of the physical control model of this type: the analytic form appears of the decrease in mass m of ferroimpurities, extracted from the sample while operations, desirable for the extrapolation analysis, and this form is usually an exponential one [1-4]:

$$m = A \cdot \exp\left(-k \cdot n\right), \qquad (1)$$

where A and k – empirical parameters. Then, based on this actual section of experimentally obtained mass-operational dependence at the disposal of the experimenter (operator), it is possible to set the parameter values A and k.

In addition to the exponential regularity (1) itself, the conceptual provisions of the corresponding physical model of this control method include the possibility of an experimental calculation of the total mass of released ferroimpurities. Thus, in analytical terms, the dependence (1) is discrete, representing a quantitative series, the terms of which, beginning with the first one $m_1=A\cdot\exp(-k\cdot 1)$, decrease in a geometric progression with the geometric ratio $q=\exp(-k)$. Therefore, given the functional extrapolation (in order to obtain objective predictive data) beyond the actual experimental operations of magnetophoresis, they can also be summed up to any amount value of n, up to $n\to\infty$.

Hence, a reliable determination of the total (both, actually separated and residual) mass of ferroimpurities of the analyzed medium sample amount to finding the sum $m_{1\ldots\infty}$ of an infinite number of terms ($n\to\infty$) of the given progression, i.e.

$$m_{1\ldots\infty} = \sum_{n=1}^{\infty} m_n = \frac{m_1}{1-q} = \frac{A}{\exp k - 1}. \qquad (2)$$

Noticed, when using the calculated expression (2) we are taking into account not only the mass values of the actually separated ferroparticle (as a result of n operations of their magnetophoretic separation), but also the data of the residual mass of such particles, which has not been separated yet from the analyzed medium sample after actual n operations. To calculate $m_{1\ldots\infty}$ according to (2), it is only necessary to use the values of empirical parameters A and k previously established (as a result of performing magnetophoresis operations). It should be noted that in the absence of such an original experimental-analytical approach, the obtaining of such total ferroimpurities mass ($m_{1\ldots\infty}$) is hardly possible: it could be obtained only under hypothetically unlimited

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(not feasible in practice, of course) number of magnetophoresis operations.

It is clear, the established value of the sediment total mass $m_{1...\infty}$ of ferroimpurities in the analyzed medium volume V and mass M is the key for determining the true values of the desired ferroimpurities concentration $c = m_{1...\infty} / V$ and/or its analogue – mass fraction of these ferroimpurities: as $c = m_{1...\infty} / M$, including in %, i.e. $c = (m_{1...\infty} / M) \times 100\%$.

2 Simulation results of magnetic inspection characteristics atypical behavior (two-exponential model). Desuperposition concept

In terms of the above-mentioned conceptual position of the basic physical model (polyoperational magnetophoresis), the experimentally obtained dependence of the separated masses of ferroimpurities particles m from the ordinal number of magnetophoresis operations n should correspond to the regularity (1). Testing the statement of such correspondence is easy to implement, in particular, with the convenient semi-logarithmic coordinates, where this dependence should become quasilinear.

At the same time, along with numerous data [1-4], which showed the specified correspondence, a number of data revealed, which obviously do not correspond to the regularity (1). Among them – the data of polyoperational magnetophoresis, obtained for the molding mixture, sand sugar, feldspar samples. Thus, the expanded mass-operational dependences for these media (Fig.1*a*, 2*a*, 3*a*), being represented with semilogarithmic coordinates (Fig. 1*b*, 2*b*, 3*b*), are not quasi-linearized (by a single line), and therefore do not follow the classic exponent (1).

Herewith, it cannot go unnoticed that for such atypical mass-operational dependences, as if having a break in semilogarithmic coordinates (Fig. 1*b*-3*b*), the propensity to quasilinearization (and, hence, to exponential form) clearly preserves their tail sections. This circumstance directly indicates the presence in the composition of the ferroimpurities fraction (each of the media analyzed here) of a relatively wide range of coarseness and/or magnetic susceptibility, i.e. the presence of the ferroimpurities "subfractions". For example, a similar fact ("break" of a specific ξ-characteristic) was noted for [5] in magnetic-filtration purification of liquids from ferroparticles of different kinds with a change in the length of the magnetic-capture zone.

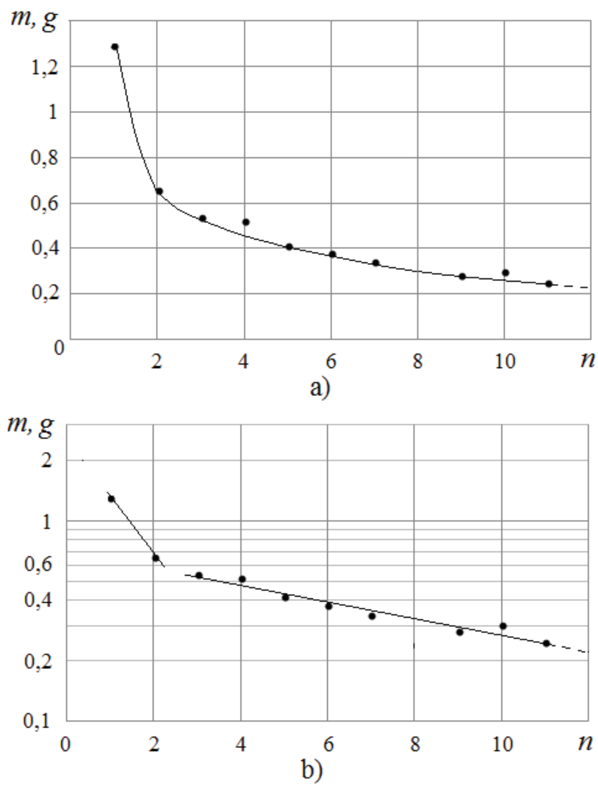

Fig. 1. Dependence of the operational (separated through polyoperational magnetophoresis) fraction mass of the molding mixture ferroimpurities; *a* and *b*) – in ordinary and semilogarithmic coordinates. Judging from the fact that the data in figure "*b*" are not quasilinearizable, this dependence is the result of the probable superposition of the ferroimpurities subfractions operational masses.

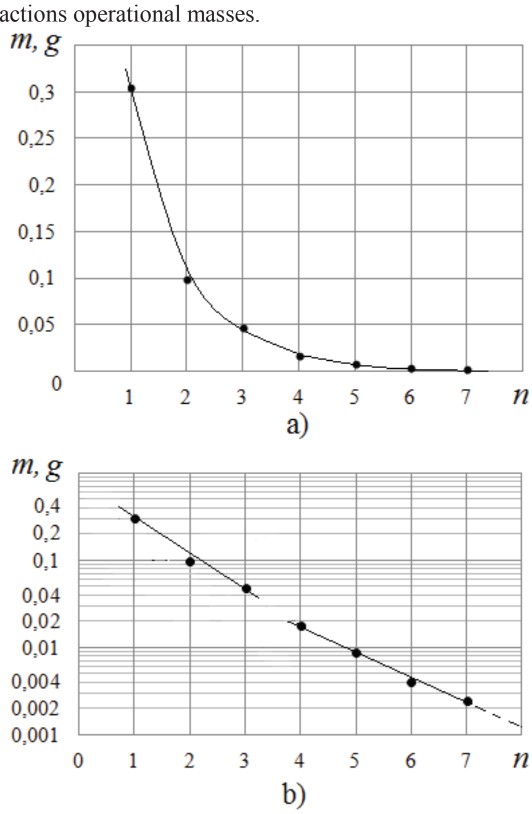

Fig. 2. Same as in Fig. 1, but for the sand sugar ferroimpurities.

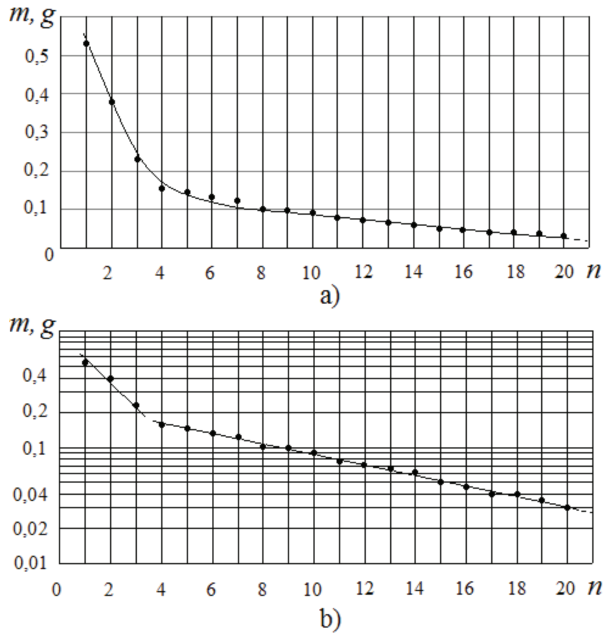

Fig. 3. Same as in Fig. 1, but for the feldspar ferroimpurities.

Consequently, same as above [5], it is expedient to resort to a justified quasi-separation of impurities ferroparticles, for example, into two subfractions, which differ considerably from one another, in the whole fraction of the ferroimpurities. This follows from Fig.1*b*-3*b* in terms of indirect indications, namely, by the presence of two distinct sections of the mass-operational dependence, which is, please note, characteristic of not an "alternating", but simultaneous precipitation of both subfractions. The first subfraction is conditionally easily extracted (*A*), consisting of larger particles and/or possessing certain magnetic properties. The second subfraction is conditionally difficult to extract (*B*), consisting of particles with greater degree of dispersion and/or possessing comparatively (in comparison with the first one) deteriorated magnetic properties.

Hence, same as in [5], it will be fair not to completely abandon the exponential model, but its in-depth analysis. Herewith, it is enough to assume that such an atypical dependence (Fig.1*b*-3*b*) – is the result of a superposition, for example, of two exponential types (characteristic of the magnetophoretic extraction of each of the two subfractions). Then, it is appropriate to raise the question of these "autonomous" dependencies legalization, i.e. question of a peculiar desuperposition of the resulting dependence (some of them are shown in Fig. 1-3).

To develop the corresponding physical model (in order to obtain the necessary calculation dependences) of the two-exponential model, in essence, the independence principle should be used, here – the principle of independent (from each other) magnetophoretic behavior of the ferroparticles subfractions *A* and *B*. Then, we can state the obvious fact that for each of the ferroparticles subfractions *A* and *B*, similar to (1), the individual characteristics (m_A and m_B) of the exponential operational masses loss are valid:

$$m_A = A_A \cdot \exp(-k_A \cdot n),$$

$$m_B = A_B \cdot \exp(-k_B \cdot n), \quad (3)$$

in this case, the alternative to the superposition expression (1) for the operational mass *m* is the detailed (as $m = m_A + m_B$) expression:

$$m = m_A + m_B = A_A \cdot \exp(-k_A \cdot n) + A_B \cdot \exp(-k_B \cdot n), \quad (4)$$

where A_A and A_B, k_A and k_B – the corresponding empirical parameters for *A*–subfraction and *B*–subfraction. Obtaining their values (as the required initial step in the implementation of the proposed desuperposition concept) is as follows.

One or another experimental operational dependence of *m* on *n* (Fig. 4, shaded points), which is subject, as mentioned above, to the partial linearization (for the tail part) in the semi-logarithmic coordinates used here, must be decomposed using graph-analytic method into dependency components (by fractions *A* and *B*). The "break point" (for $n=[n]$) of the experimentally obtained dependence (Fig. 4, shaded points) can serve as a reference mark for such a decomposition.

Obviously, the area of this dependence at $n>[n]$ (Fig. 4, shaded points), i.e., namely the tail section, which is subject to quasilinearization, is practically characteristic of the fraction *B* only – under conditions of almost complete absence of fraction *A* (separated to $n=[n]$). In order to fully identify the component *B* the actual section *B* follows the dependence of *m* on *n* (its tail section in Fig. 4: solid line, shaded points) to extrapolate to the region of small values of *n* (dashed line, ushaded points).

To identify the component *A* (at $1 \leq n \leq [n]$ – see Fig. 4), a simple mutual subtraction of the corresponding ordinates is performed of the actual main here section (*A*+*B*) of *m* on *n* dependence and extrapolated section *B* (dashed line *B*). This component (*A*) in Fig. 4 is also illustrated by the dashed line *A* (unshaded enlarged points).

Extrapolation of the identified dependences *A* and *B* (dashed lines in Fig. 4) to the formal area up to $n \to 0$ directly provides values A_A and A_B, and the necessary calculations using these obtained values A_A and A_B, expressions (3) and specific dependences *A* and *B* data in Fig. 4 allow finding also the values of the parameters k_A and k_B. Of course, using the corresponding graph-analytical identification data for dependencies *A* and *B* (Fig. 4), we can obtain similar results for determining the values A_A, A_B, k_A and k_B using one of the known programs, in particular, Excel.

On the basis of (3), similarly to (2), it is also possible to obtain calculated expressions for the total masses of the ferroparticles of each of the subfractions:

$$(m_{1\ldots\infty})_A = \frac{A_A}{\exp k_A - 1},$$

$$(m_{1\ldots\infty})_B = \frac{A_B}{\exp k_B - 1}. \quad (5)$$

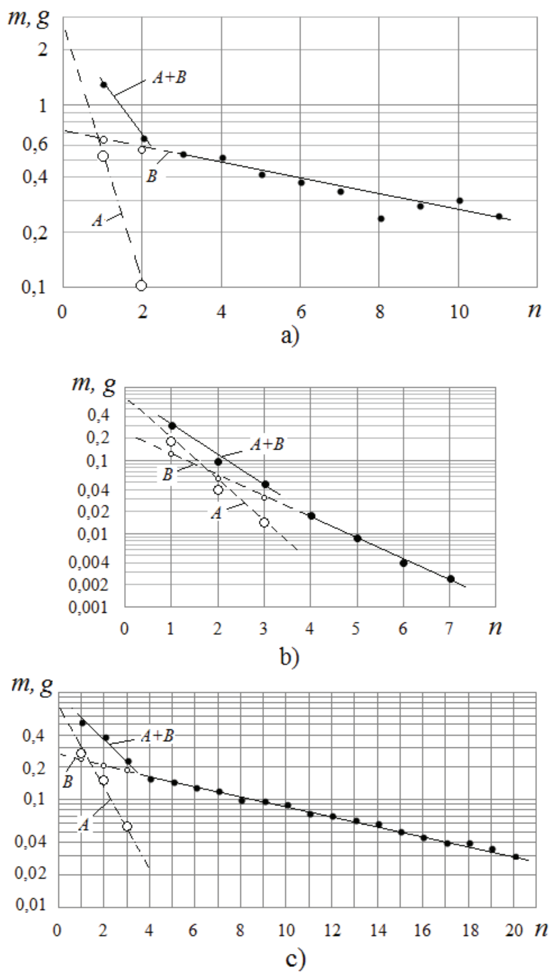

Fig. 4. Illustration of graph-analytic desuperpositions of the mass-operational dependencies shown in Fig. 1-3 (shaded points), on individual mass-operational dependencies for *A*–subfraction (ushaded enlarged points) and *B*-subfraction (unshaded reduced points first, and then, shaded ones): *a*) molding mixture, *b*) sand sugar, *c*) feldspar.

Hence, based on the concept of desuperposition proposed here, an alternative to the superposition expression (2) for the total mass $m_{1\ldots\infty}$ of ferroimpurities contained in the medium, should be the detailed (by subfractions *A* and *B*) expression:

$$m_{1\ldots\infty} = \left(m_{1\ldots\infty}\right)_A + \left(m_{1\ldots\infty}\right)_B = \frac{A_A}{\exp k_A - 1} + \frac{A_B}{\exp k_B - 1} \quad (6)$$

An important consequence of the desuperposition concept considered herein is that it becomes possible to determine individual share value of each subfraction in the general fraction of ferroimpurities:

$$\lambda_A = (m_{1\ldots\infty})_A / [(m_{1\ldots\infty})_A + (m_{1\ldots\infty})_B], \quad \text{x}100\% \quad (7)$$

$$\lambda_B = (m_{1\ldots\infty})_B / [(m_{1\ldots\infty})_A + (m_{1\ldots\infty})_B] = (1-\lambda_A) . \text{x}100\% \quad (8)$$

moreover – clear quantitative values (identified by the experimental data, with their inherent difference parameters), in addition to the qualitative ones declared at the stage of this concept definition.

Corresponding values of the magnetophoretic control parameters of the conventionally easily extracted (*A*) and difficult-to-extract (*B*) fractions of the ferroimpurities are given in the table.

Table 1. The parameters of polyoperational magnetophoretic control obtained for the subfractions *A* and *B* of the ferroimpurities fraction (as shown in Fig. 1-4, as a result of desuperposition).

Sample of analyzed media	Values of parameters			
	A_A, g; A_B, g	k_A; k_B	$(m_{1\ldots\infty})_A+(m_{1\ldots\infty})_B=$ $= m_{1\ldots\infty}$, g	λ_A, %; λ_B, %
Molding mixture	2,6; 0,7	1,63; 0,094	0,63+7,1= **=7,73**	8,2; 91,8
Sand sugar	0,8; 0,21	1,37; 0,64	0,273+0,234 **=0,507**	53,8; 46,2
Feldspar	0,7 0,27	0,84 0,12	0,532+2,118 **=2,65**	20; 80

3 Conclusion

The main provisions are declared of a well-proven exponential model of the mass-operational (prone to linearization in semi-logarithmic coordinates) characteristics of magnetophoretic control of ferroimpurities in liquid and free-running media. Attention is drawn to those characteristics that are not typical, i.e. formally do not follow the exponential model. It has been established (and shown by an appropriate theoretical-experimental data analysis) that an atypical form of such characteristics is not a result of removing from the exponential regularity of the operational masses loss of the extracted (under polyoperational magnetophoresis) ferroimpurities. It is caused by a superposition of at least two exponential regularities – for two mutually different subfractions of ferroimpurities. The concept is proposed of desuperposition of the atypical mass-operational characteristic of the magnetophoretic control of ferroimpurities resulting in particular (for each of the subfractions) exponents, particular expressions for the total masses and, accordingly, information on the fractional presence of each subfraction.

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