

# Description of anisotropy of magnetic properties for chosen grades of electrical steels

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**Abstract.** Magnetic properties of electrical steels, either grain-oriented or non-oriented ones, are significantly affected by anisotropy. The paper considers two descriptions for the dependence: power losses versus angle. The first one is based on the theoretical studies on texture from materials science. The other one is purely phenomenological, and based on an equation of ellipse in a rotated coordinate system.

## 1 Introduction

The designers of magnetic circuits in electrical machines need more and more efficient and accurate design tools supporting the CAD process, what implies an increasing interest of engineering community in modelling of nonlinear phenomena occurring at the core. Anisotropy is a phenomenon that complicates an analysis. Above all, this applies to grain-oriented steel, used as a core material in transformers, but the so-called non-oriented (alternator) steel used in rotating electric machines also exhibits substantial anisotropy [1-4]. Usually, magnetic properties and dependencies are provided by the manufacturers of electrical steels solely for the rolling direction (for the angle 0°). Sometimes, the dependencies are also given either for the transverse direction (for the angle 90°) or for the mixed samples (combination of strips cut for two characteristic angles, measured with the Epstein Frame). The worst magnetic properties are exhibited for angles cut at 54-60°. Therefore, a simple interpolation scheme based on two characteristic directions, such as the one proposed by Jesenik et al. [5], may not always be useful.

In order to describe anisotropic magnetic properties of electrical steels several approaches are available. One method is based on the so-called co-energy concept [6-8]. If a hysteresis phenomenon is being neglected, the co-energy density per volume module is given with an expression

$$w'(\mathbf{H}) = \int_0^{\mathbf{H}} \mathbf{B}(\mathbf{H})d\mathbf{H} \quad (1)$$

The induction may be derived from (1) using (2).

$$\mathbf{B}(\mathbf{H}) = \text{grad}_{\mathbf{H}} w'(\mathbf{H}) \quad (2)$$

An important feature of the co-energy approach is its consistence with the laws of irreversible thermodynamics. The possibility to describe magnetisation curves for chosen

grades of grain-oriented and non-oriented electrical steels was verified in Refs. [7] and [3], respectively. At this point it should be recalled that the concept of co-energy is significant in the contemporary descriptions of electromagnetic energy conversion devices used for example in mechatronics, cf. e.g. [9-10] for details.

Some researchers with materials science background prefer to work with a description, which has its roots in the analysis of the so-called Orientation Distribution Functions (ODFs), commonly used in metallurgy for material characterisation [11-13]. The effective magnetic anisotropy of a polycrystalline sample should be obtained by averaging the grain anisotropy over the grain ODF, i.e. over the texture [14]. The magneto-crystalline energy, defined as an energy to rotate magnetic moments, averaged over the ODF and is given with the following expression

$$E(\alpha, \beta) = \frac{1}{9n_4\sqrt{\pi}} \left[ \frac{K_1}{5} + \frac{K_2}{55} \right] F_4(\alpha, \beta) + \dots \quad (3)$$

$$+ \frac{1}{13n_6\sqrt{\pi}} \frac{K_2}{231} F_6(\alpha, \beta)$$

where  $\alpha, \beta$  define the specimen direction in the reference frame,  $n_4$  and  $n_6$  are constants,  $n_4 = 0.64636$ ,  $n_6 = 0.359601$  [12,13],  $K_1$  and  $K_2$  are anisotropy constants, whereas texture functions  $F_4$  and  $F_6$  are defined as in (4) and (5), respectively [15]:

$$F_4(\alpha, \beta) = \frac{1}{\sqrt{2}} C_4^{11} P_4^0(\cos\alpha) \cos 2\beta + C_4^{13} P_4^4(\cos\alpha) \cos 4\beta \quad (4)$$

$$F_6(\alpha, \beta) = \frac{1}{\sqrt{2}} C_6^{11} P_6^0(\cos\alpha) + C_6^{12} P_6^2(\cos\alpha) \cos 2\beta + \dots \quad (5)$$

$$+ C_6^{13} P_6^4(\cos\alpha) \cos 4\beta + C_6^{14} P_6^4(\cos\alpha) \cos 6\beta$$

$C_4^{11}$  and  $C_4^{13}$  are parameters of texture, whereas expressions like  $P_4^q$  where  $q=0,2,4$  are Legendre's polynomials of the first kind, fourth order.

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For the angle  $\alpha = 90^\circ$  i.e. for the surface plane of the specimen, the relationship (3) yields an expression for a magnetic torque, which agrees considerably well with experimental data, as confirmed by several authors

$$M(\beta) = -\frac{dE(\beta)}{d\beta}. \quad (6)$$

Further simplification of the above-given expressions follows if one accounts facts that the anisotropy constant  $K_2$  is one order of magnitude lower than  $K_1$ . Then it is possible to neglect in the first approximation the second term in expression (3) (the one containing  $F_6$ ). Thus magneto-elastic energy may be written as proportional to fourth order texture function  $F_4(90^\circ, \beta)$  only, as suggested by Birsan and Szpunar [15]. From (3) and (4) a simplified relationship may be written

$$E(\beta) \approx E_1 \cos 2\beta + E_2 \cos 4\beta. \quad (7)$$

If the contribution of anisotropy constant  $K_0$  is accounted, which is a free term independent from the direction of the applied field, then it follows that magneto-elastic energy, as well as any physical quantity of interest, which is directly related to it, may be written as the function of three first ODF coefficients, i.e.

$$A = A_0 + A_1 \cos 2\beta + A_2 \cos 4\beta \quad (8)$$

In the formula (8)  $A$  may denote magneto-elastic energy, but also power losses, coercive field strength, permeability etc. The parameters  $A_i$ ,  $i = 0..2$  are given with the following auxiliary relationships obtained from measurements at the angles  $\beta = 0^\circ, 45^\circ, 90^\circ$

$$A_0 = 0.25 [A(0^\circ) + A(90^\circ) + 2A(45^\circ)], \quad (9)$$

$$A_1 = 0.5 [A(0^\circ) - A(90^\circ)], \quad (10)$$

$$A_2 = 0.25 [A(0^\circ) + A(90^\circ) - 2A(45^\circ)]. \quad (11)$$

From the above-given expressions it is evident that to predict angular variations for any magnetic quantity of interest, it is sufficient to carry out its values for three directions  $\beta = 0^\circ, 45^\circ, 90^\circ$ . In this sense, this procedure is similar to the Lankford parameter used for anisotropy evaluation in metallurgical processes [16,17].

Apart from these two above-discussed methods, which have solid foundations rooted in the theories of electromagnetism and materials science, in the literature there are also some other semi-empirical approaches, for instance [18-20]. The model proposed in the next section may be considered as a member of this class.

## 2 Model proposal

The equation of an ellipse in Cartesian coordinates and the formulas for rotating coordinate system by the angle  $\phi$  should be recalled. The ellipse equation is

$$\frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1, \quad (12)$$

or in the parametric form

$$x' = a \cos t, \quad y' = b \sin t, \quad (13)$$

where  $t \in (0; 2\pi)$ .

Rotation of the coordinate system is achieved by the transformation

$$\begin{cases} x = x' \cos \phi - y' \sin \phi \\ y = x' \sin \phi + y' \cos \phi \end{cases} \quad (14)$$

where  $\phi = 30^\circ$  (this is the complementary angle to the angle  $60^\circ$  shown in the Fig. 1). For simplicity of calculations it is assumed that the worst magnetic properties are at  $60^\circ$ .

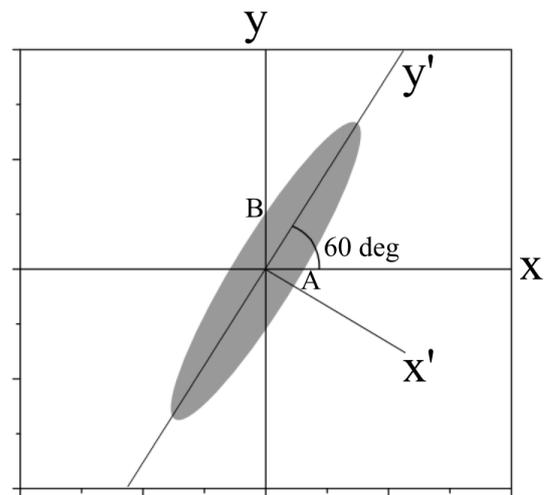


Fig. 1. The ellipse and the reference coordinate systems.

The required values for the prescribed direction may be easily found by means of interpolation. This approach should work for power losses at a constant induction level; for determination of field strength, the afore-described approach allows to determine just the magnitude of the vector  $\mathbf{H}$ ; thus, it is necessary to supplement it with the auxiliary dependence obtained from the assumed material behaviour for the rolling direction [7]. The problem shall be the subject of further studies.

An important advantage of the presented approach is that it requires just two values of the considered quantity (for example power loss) for the orthogonal directions  $0^\circ$  and  $90^\circ$ . These values determine the minor and the major axis of the ellipse. In this sense the approach is similar at the core to the co-energy based approach.

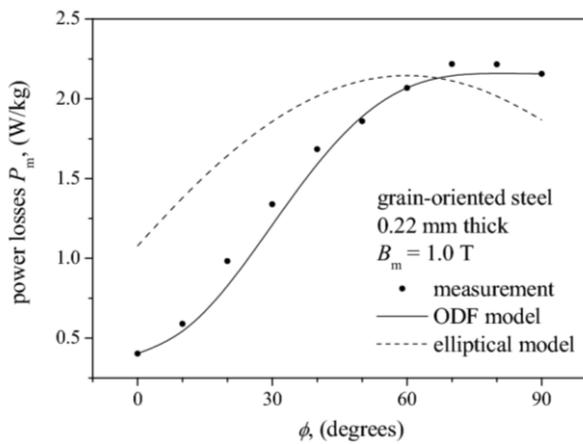
## 3 Modelling

### 3.1. Grain-oriented steel

In Ref. [15] Birsan and Szpunar provided a table concerning the measurements of power losses at 1.0 T for a fully processed (grain-oriented) SiFe steel, 0.22 mm thick. The values for the rolling and the transverse

directions were  $a = 0.183$  W/lb (0.404 W/kg) and  $b = 0.981$  W/lb (2.163 W/kg), respectively. As the measurements for the angle  $45^\circ$  were not available, it has been approximated as the average for  $40^\circ$  and  $50^\circ$ ,  $P_{45^\circ} = 0.5(0.766 + 0.846)$  W/lb = 0.806 W/lb, that is in SI units  $P_{45^\circ} = 1.777$  W/kg. This value is necessary for the ODF-based method. Values for parameters  $A_i$ ,  $i = 0 \dots 2$  from (9) – (11) have been determined. Next, required values for intermediate angles from (8) were defined.

For the other considered description (the rotated ellipse in the Fig. 1), power losses for an arbitrary angle were set by determining the magnitude of an auxiliary vector quantity, which perpendicular components were equal to values for the points A and B in the Fig. 1.



**Fig. 2.** The measured and modelled dependencies of losses vs. angle for grain-oriented steel.

Figure 2 depicts the modelling results for grain-oriented steel. It can be stated that the elliptical model describes the angular variation of losses only qualitatively for this grade of GO steel. The largest discrepancy between the measured and the modelled values occurs at  $\phi = 0^\circ$ ; it is equal to 181.7%. The averaged error value is 49.8%, whereas for the ODF-based method – 5.4% [in the latter case the largest error value (observed for the angle  $\phi = 20^\circ$ ) exceeded slightly 17.4%].

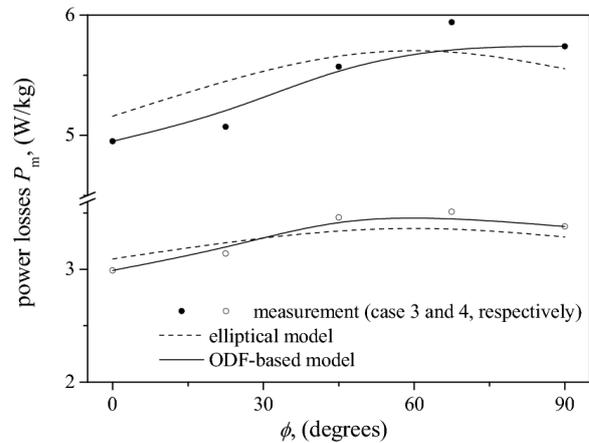
The paper by Shiozaki and Kurosaki [1] provided a discussion on the effect of processing technologies on the magnetic properties for typical non-oriented electrical steel sheets. The authors provided exemplary ODF figures and tried to establish the relationship between textures and typical figures of merit provided by the manufacturers: power losses at  $B = 1.5$  T,  $f = 50$  Hz and induction at  $H = 5000$  A/m,  $f = 50$  Hz. Technological processes considered by Shiozaki and Kurosaki are listed in the Table 1. Their detailed description is given in Ref. [1].

The processing conditions had a significant impact on the properties of the specimens, as could be inferred from the analysis of published measurement data. The problem of texture optimisation is of a great practical interest to metallurgists and materials science engineers,

cf. for example [21-25]. Simultaneously, it should be remembered that the magnetic properties of ready-made parts of magnetic circuits may usually be worse than the properties of steels themselves due to stresses introduced during subsequent material processing [26-28].

**Table 1.** Technological processes considered by Shiozaki and Kurosaki [1].

Case	Process
1.	single reduction, cold rolling process
2.	conventional temper-rolling process
3.	special temper-rolling process
4.	conventional hot band annealing and single reduction cold rolling process
5.	special hot band annealing and single reduction cold rolling process



**Fig. 3.** Measured and modelled dependencies of losses vs. angle for chosen non-oriented steels.

The prediction capabilities of the proposed elliptical model, for the five aforementioned specimens subject to different processing routes, have been checked. The results are given in the Table 2 and in the Fig. 3 (just two cases for better visibility). In the Table "max. error" denotes the maximum value  $\delta_{\%} = 100 \cdot |P_{meas} - P_{model}| / P_{meas}$ , whereas "average error" means that averaging has been carried out for all possible angles from the range  $\phi \in \langle 0^\circ, 90^\circ \rangle$ .

**Table 2.** Prediction errors for two considered models.

Process	ODF		elliptical	
	max. error	average error	max. error	average error
single reduction	1.1	0.3	2.6	1.5
conv. temper-rolling	1.2	0.3	3.7	1.7
spec. temper-rolling	3.4	1.1	7.7	4.2
conv. hot band annealing	1.4	0.6	3.9	3.3
spec. hot band annealing	0.3	0.1	6.0	2.9

It can be stated that the elliptical model yields slightly larger values of errors, yet it should be remembered that it requires less input information i.e. measurement data just for two directions. From data presented in the Table 1, it can be concluded that the ODF model might be particularly suited for engineering purposes. The elliptical model yields a relatively good qualitative and quantitative description of power losses vs. angle for the considered non-oriented steels.

## 4 Conclusions

In the paper two descriptions of anisotropic properties of electrical steels have been discussed in detail and their predicting accuracy was illustrated using some representative examples from the literature.

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