

# Measurement system analysis for one-sided tolerance

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**Abstract.** Measurement system analysis is carried out in order to determine if a capability to perform measurements in terms of product and process control is sufficient, indicating that the type I and the type II appraisal errors probability are acceptable. Statistical analyses for measurement system evaluation presented in the literature and the industrial manuals are not applicable for all complex and unusual applications. Therefore, the purpose of this study was to develop a robust statistical analysis method for measurement system variability analysis, in terms of product control scenario applied to one-sided tolerance. In the hereby presented study, the authors presented the theoretical principles of statistical techniques for measurement variations evaluation. Subsequently, the formula of gauge repeatability and reproducibility in terms of lower specification limit was proposed. The research hypothesis was tested using the statistical analysis.

## 1 Significance and determinants of measurement systems quality

The quality of product and process has become one of the most significant elements of the successful management process in high volume manufacturing facilities. Therefore, measurement systems are used for control of manufacturing process variability and location, as well as a criterion for product acceptance.

Due to the measurements variability, the results, which are close to the specification limit, do not allow for a proper appraisal of product conformance in relation to the design tolerance limits. Consequently, in terms of statistical process control (SPC), the measurements variation has a negative impact on the evaluation of the process performance and stability; thus, the measurement system shall allow for a reliable evaluation of the manufacturing process statistical properties.

If the performance of measurement process is unknown, then a manufacturer may be exposed to bear significant costs due to the customer complaints, warranty returns, scrap increase and special causes in ongoing process monitoring applied throughout the SPC.

First of all, capability indexes are one of the main statistical techniques applied in SPC methodology. Next, as a product is defined by specification limits upon which allowable deviation is limited through tolerances [1], thus, the variability of the characteristic with respect to the specification limits is evaluated throughout Cp and Cpk indexes. The Cp index compares the product tolerance zone to the 6 standard deviations of the manufactured products distribution. The Cpk index takes also distribution location into account [2]. Finally, the effect of measurement system variability on the process capability index [3] is demonstrated in figure 1.

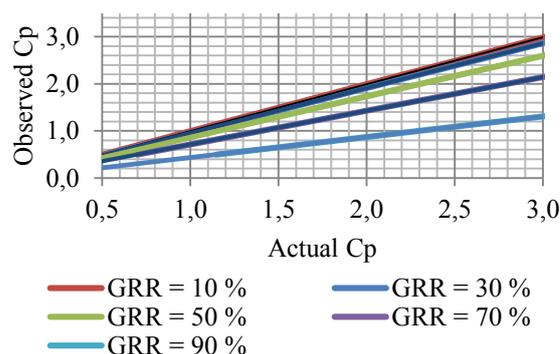


Fig. 1. Observed vs actual Cp value.

The importance of qualified measurement systems can be seen in figure 1. The repeatability and reproducibility of the measurement system has significant impact on the estimation of capability indices, thus, the actual capability index (Actual Cp) was introduced [3].

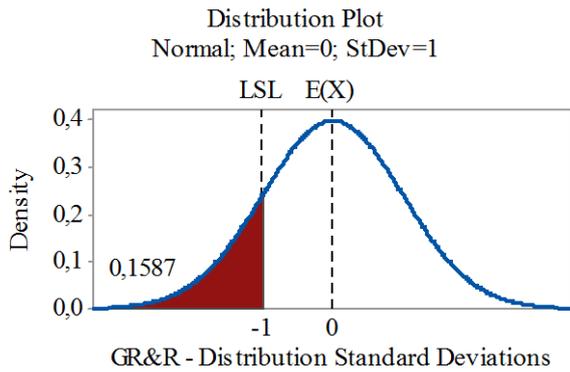
Respectively, based on collected measurements, the following crucial decisions are taken:

- To accept or reject (scrap or rework) manufactured product;
- To issue corrective action in a manufacturing system;
- To issue 100% control in running production;
- To qualify machine for production use.

The effect of excessive measurement system variability, in reference to the specification limit, is that the type I and type II classification errors are probable to occur. The probability of classification conforming part as non-conforming part is called type I error, producer's risk or false alarm.

The probability of type I error is shown in figure 2.

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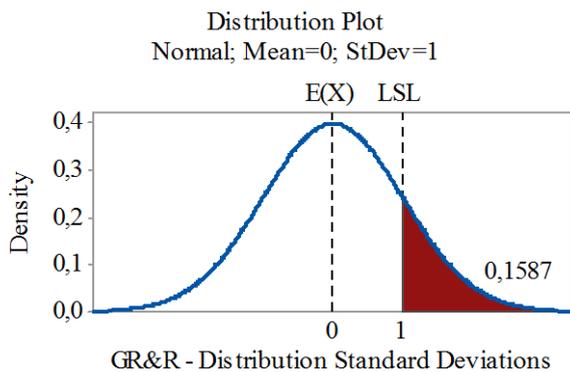


**Fig. 2.** The probability of type I error.

Probability chart demonstrated that in this particular scenario the likelihood of classification a conforming part as non-conforming is 0,1598. In this case, the variability of measurement process exposes the manufacturer to extra costs.

Respectively, the probability of classification non-conforming part as conforming is called miss rate, consumer's risk or type II error.

The probability of type II error is shown in figure 3.



**Fig. 3.** The probability of type II error.

Probability chart demonstrated that in this particular scenario, the likelihood of classification a non-conforming part as conforming is 0,1598. In this case, the variability of measurement process exposes the customer to the risk of utilizing part out of the specification.

Thereby, the measurement systems analysis has become a widely used method for the gauges approval for production use.

The purpose of the two main industrial manuals *MSA 4<sup>th</sup> Edition* and *VDA 5 Capability of Measurement Process* respectively is to [3-4]:

- Present guidelines for assessing the quality of measurement system without intention to be a compendium of analyses for all measurement systems;
- To summarize the requirements and procedures in order to gain a standardized model for the estimation of expanded measurement uncertainty.

The unusual application of product control scenario applied to one-sided tolerance is not discussed in both manuals. Therefore, the aim of this study was to develop a method for measurement system variability analysis in the case of product tolerated by a single lower (minimum) specification limit.

## 2 Methodical assumptions for management of measurement systems

In order to facilitate reading of this paper, the summary of essential terms which are used in this study is presented in the following section.

A gauge is any equipment used to assign a measured value to the material thing [4]. A measurement system is “the collection of gauges, standards, operations, methods, fixtures, software, personnel and assumptions used in measurement process” [4]. In general, the measurement system is a complete process used to obtain the measurement.

As the variation in any process comes from the many sources, therefore, measuring the same part does not result in identical measurements [5].

The main variability sources influencing the measurement process are as follows [6]:

- Operator;
- Measured object;
- Measurement method;
- Environment;
- Gauge;
- Fixture.

Measurement uncertainty is a “parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand” [7]. Discussed uncertainty is crucial if measurements are performed on a product with respect to a specification limit. If the measurement result, along with its uncertainty, does not exceed the specification limits, then the product can be evaluated as meeting the design criteria [8]. Each measurement is “biased, which means that measured value is never equal to true value” [9]. A true value is the value consistent with the definition of and observed measured characteristic [6]. The difference between estimating variation by measurement uncertainty and MSA methods is that MSA evaluates variation of the measurement system for product and process control [4].

Repeatability is “variation in measurements obtained with one measuring instrument when used several times by an appraiser while measuring identical characteristic on the same part” [10]. Consequently, reproducibility is “variation in the average of the measurements made by different appraisers using the same gauge when measuring a characteristic of a part” [10].

Gauge R&R (GRR) is “the combined estimate of measurement system repeatability and reproducibility” [10]. Studying the gauge variability determines the measurements variability and the part-to-part variation which is for instance the variability of measurements due to the significant part circularity [11].

A number of distinct categories is “the number of data categories that can be distinguished by the effective resolution of the measurement system and part variation from the observed process” [12].

Effective resolution is “the size of the data category when the total measurement system variation is considered” [12]. The Gaussian distribution properties are used to evaluate the above statistical parameters of the measurement systems.

### 3 Proposal of authorial changes in measurement system management methodology

According to the *MSA 4<sup>th</sup> Edition Manual*, the %GRR, when evaluating the quality of measurement process with respect to the product tolerance specification limits, is given by a formula (1) [4]:

$$\%GRR = 100 \cdot \frac{6 \cdot GRR}{T} \quad (1)$$

Where:

- $T = \text{tolerance zone} = USL - LSL$ ;
- $USL = \text{upper specification limit}$ ;
- $LSL = \text{lower specification limit}$ .

Therefore, to calculate %GRR when the product is tolerated by the single lower (minimum) specification limit, the authors proposed the formula (2):

$$\%GRR = 100 \cdot \frac{0,5 \cdot k \cdot GRR}{\bar{\bar{X}} - LSL} \text{ for } \bar{\bar{X}} > LSL \quad (2)$$

Where:

- $k = \text{coverage factor} = 6$ ;
- $\bar{\bar{X}} = \text{mean of all measurements}$ .

The coverage factor is to be further discussed in this paper. In the above formula, the measurement process variation is compared to the range between the lower specification limit and the average of all measured parts. The above formula is valid, if the mean of all measurements is greater than LSL.

The disadvantage of such an approach is that the capability of the measurement system is determined by the average of the manufacturing process, while the measurement system is evaluated with respect to tolerance specification limit.

Next, if the measurement result is in the area between LSL and half of the GRR distribution, then conformity with specification cannot be proved properly.

Moreover, a similar method has already been introduced in MiniTab 17 Statistical Software [13].

An alternative approach proposed by the authors to the following issue is that regardless the parts average value, the 3 standard deviations of measurement process (assuming the normality of measurement process and measured characteristic distribution) shall be smaller than the minimum average value of measured parts, which are representative to cover the entire manufacturing process.

The illustration of this scenario is demonstrated in figure 4. Probability chart shown in figure 4 demonstrated that in this case the likelihood of classification non-conforming part as conforming is 0,00135, thus, such probability of 3 standard deviations of measurement process is assumed by the authors as acceptable in accordance with the 6-sigma standard in quality engineering. Therefore, to evaluate GRR, when the product is tolerated by the single lower (minimum) specification limit, the authors proposed the formula (3):

$$P_{gk} = \frac{\min\{\bar{X}_1, \dots, \bar{X}_n\} - LSL}{0,5 \cdot k \cdot GRR} \geq 1 \text{ for } \min \bar{X}_i > LSL \quad (3)$$

Where:

- $k = \text{coverage factor} = 6$ ;
- $\min\{\bar{X}_1, \dots, \bar{X}_n\} - \text{minimum average value of the measured parts}$ .

The factor equal to 6 is sufficient to cover the probability of 0,9973 of a normal distribution. The graphical interpretation of the probability is shown in figure 4.

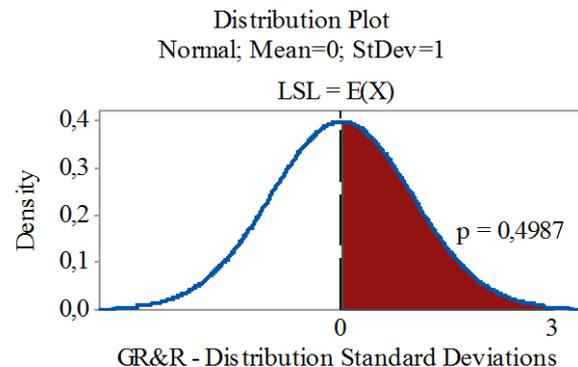


Fig. 4. The probability of type II error.

#### 3.1 Research assumptions

The numeric study has been conducted by measuring 10 representative parts 3 times by 3 appraisers. According to the *MSA 4<sup>th</sup> Edition Manual* the detailed procedure of GRR study is as follows:

- Obtain a sample of  $n > 10$  parts that represent the actual process variation;
- Calibrate the gauge;
- Measure the parts in random order three times for each part and appraiser combination;
- Analyze the results with respect to the acceptance criteria.

The acceptance criteria according to the manual are %GRR < 10% and a number of distinct categories (NDC) ≥ 5.

#### 3.2 Used methods and research tools

The analysis of variance (ANOVA) method was used to analyze the results. In the study, the variation components of the measurement process can be

distinguished in four categories (parts, appraisers, interaction between parts and appraisers, and measurement equipment).

The advantage of using analysis of variance method instead of average range method (ARM) is that estimation of variation components is more accurate, as well as more data related to the measurement process is provided (e.g. interaction between operators and parts).

On the other hand, the complexity of calculation as compared with ARM method is significant. Finally, the Gaussian probability distribution properties were used to evaluate formulas (2) and (3) respectively.

The statistical evaluation was performed using *MiniTab 17* software.

#### 4 The results of applying the proposal on a selected example

In this section, authors presented the numeric results of gauge repeatability and reproducibility study for illustration of proposed formulas applicability and statistical validation. Collected data are shown in table 1.

**Table 1.** Measurement data for the numerical example.

Appraisal		A		
Trial		1	2	3
Part	1	15,572	15,568	15,568
	2	15,610	15,613	15,610
	3	15,609	15,610	15,608
	4	15,627	15,629	15,631
	5	15,609	15,612	15,608
	6	15,549	15,552	15,550
	7	15,555	15,557	15,555
	8	15,600	15,604	15,602
	9	15,572	15,572	15,573
	10	15,546	15,548	15,548
Appraisal		B		
Trial		1	2	3
Part	1	15,565	15,568	15,566
	2	15,612	15,611	15,613
	3	15,610	15,609	15,608
	4	15,630	15,628	15,626
	5	15,609	15,609	15,607
	6	15,551	15,550	15,550
	7	15,556	15,556	15,556
	8	15,602	15,603	15,602
	9	15,574	15,571	15,572
	10	15,548	15,547	15,548
Appraisal		C		
Trial		1	2	3
Part	1	15,568	15,568	15,566
	2	15,612	15,611	15,610
	3	15,609	15,609	15,609
	4	15,627	15,633	15,631
	5	15,608	15,607	15,611
	6	15,549	15,551	15,551
	7	15,556	15,555	15,556
	8	15,602	15,602	15,602
	9	15,571	15,573	15,573
	10	15,548	15,546	15,547

#### 5 Discussion of obtained results

Gauge repeatability and reproducibility analysis was carried out using collected data. The result of the study using the analysis of variance method is shown in table 2.

**Table 2.** Measurement systems GRR study results.

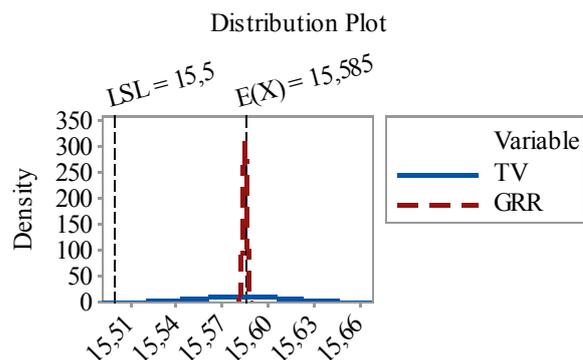
Source	$\sigma_{GRR}$
Total Gage R&R	0,0013706
Repeatability	0,0013706
Reproducibility	0
Operator	0
Part-To-Part	0,0298342
Total Variation	0,0298656
NDC	30

Subsequently, the %GRR was calculated using the proposed formula (2):

$$\bar{\bar{X}} = 15,5854$$

$$\%GRR = 4,82\%$$

The graphical analysis of formula (2) is presented in figure 5.



**Fig. 5.** % GRR with respect to LSL.

The analysis of the formula's (2) result shows that the variation of the measurement system, in comparison to the total variation (the combined estimate of the process and the gauge variation), equals 4,82%. This result is assumed as acceptable since according to the manual, the acceptance criterion is %GRR < 10%.

Subsequently, the number of distinct categories is greater than 5, which means that the measurement system can distinguish 30 number of groups from measurement data. This means that the measurement process is capable towards effective resolution, which is not necessarily required to be evaluated in analysed case study. However, if the effective resolution is not sufficient for the evaluation of the parts variability, it may be an input to improve the gauge and therefore lead to better process capability.

Moreover, with respect to the manufacturing system, if the process overall capability is greater or equal to 1,33, then the probability of type II error is ~ 0,001 assuming the %GRR equals 33%. The type II error probability in this case is shown in figure 6.

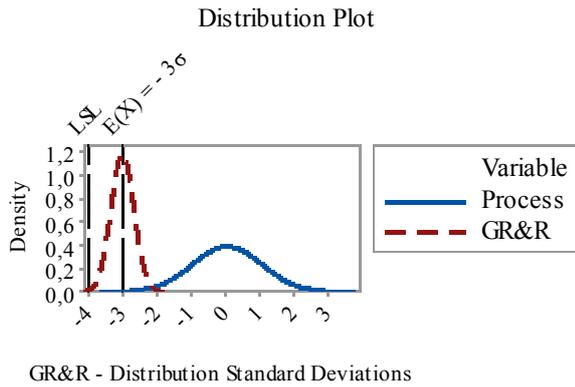


Fig. 6. % GRR impact on type II error with a capable process.

Subsequently, the  $P_{gk}$  was calculated using the proposed formula (3):

$$\min\{\bar{X}_1, \dots, \bar{X}_n\} = 15,547$$

$$P_{gk} = 11,51$$

The graphical analysis of  $P_{gk}$  index is presented in figure 7.

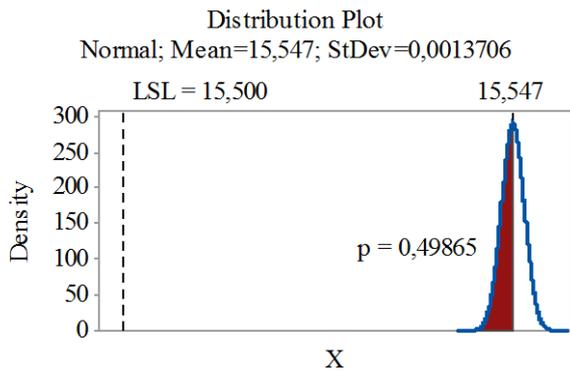


Fig. 7. % GRR with respect to LSL.

The analysis of the formula's (3) result shows that the percentile of 3 standard deviations of measurement process (which is equal to the probability of 49,865%) is over 11 times smaller than the zone between the minimum of observed parts average results. Therefore, the measurement process is considered as capable since  $P_{gk} \geq 1$ .

In a proper approach, a single index as the acceptance criterion for measurement system should not be taken. The final decision should consider different evaluations. For this reason, multiple studies should be used to determine the quality of measurements [14]. Therefore, if the overall process performance is less than 1, then the criterion for the product acceptance shall be the result in the conformance zone [15]. In this particular case, the conformance zone is the result  $x \in C_z$ , according to the formula (4):

$$C_z = \{LSL + 3\sigma_{GRR}; +\infty\}. \quad (4)$$

## 6 Conclusions

In the carried-out research authors have elaborated two robust, statistical methods for the gauge repeatability and reproducibility analysis, in terms of single lower specification limit scenario, as the additional technique of the analysis method presented in the *MSA 4<sup>th</sup> Edition Manual*.

The methods proposed by the authors are to be used for the evaluation of the measurement system variability and effective resolution regarding the product control. The elaborated formulas use statistical properties of studied parts.

Since in the studied scenario there is no upper specification limit, the elaborated formula (2) takes the mean of the studied parts as the upper specification limit for the tolerance zone determination. It may not be fully effective, as the quality of measurement system becomes dependent on the process location. Thus, the alternative approach was elaborated and presented in the formula (3). This method takes the minimum value of the parts average as the upper specification limit and compares the variation of the measurement process to the zone between the lower specification limit and calculated minimum value. The advantage of such method is that assuming the measured parts representative to the process 6 standard deviations, it prevents the manufacturer from the type II error.

Nevertheless, if the process capability is insufficient, the minimum mean value of measured parts may be smaller than lower specification limit, and then the method cannot be used. Therefore, the conformance zone based on the gauge repeatability and reproducibility was elaborated. Hence, the authors have referred to the type II probability error as the additional criterion in the acceptance of the complete measurement results.

Consequently, the proposed formulas for the GRR were used in the numeric study following the *MSA 4<sup>th</sup> Edition Manual* procedure, using the analysis of variance method.

Afterwards, the numeric study results were evaluated using the statistical properties of the Gaussian distribution. Additionally, the graphical interpretation of each formula was presented.

Moreover, the presented methods are applicable for all measurement systems which are normally distributed.

In addition, the influence of the measurement process variability on the process evaluation (capability index, control charts) and product qualification (type II, type II errors) was described.

Finally, the most important definitions used in this paper were discussed, including such terms as gauge, measurement system, measurements variability sources, measurements uncertainty, gauge repeatability and reproducibility, measurement system qualification criteria.

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