CICT: New Eyes on Computational Competence in Computational Science

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Abstract. Science does not exists to enlighten people's minds only. It mainly exists to show the educated way from quanta to qualia. And that way starts from computational competence. In previous papers published elsewhere, we have already shown that traditional $\boldsymbol{Q}$ Arithmetic can be regarded as a highly sophisticated open logic, powerful and flexible bidirectional formal language of languages, according to "Computational Information Conservation Theory" (*CICT*) new perspective. This new awareness can offer competitive approach to guide more effective and convenient algorithm development and application to arbitrary multiscale (AMS) biomedical system modeling and simulation. An articulated example on function computational modelling is presented and compared to standard, well-known and traditional approach. Results are critically discussed.

1 Introduction

In the past decades, we learned how traditional human-made system can be quite fragile [1] to unexpected perturbation, because statistics by itself can fool you, unfortunately. In fact, they were developed and used under the implicit assumption of classic model completeness. Traditional models of Systems Theory were intended to completely represent aspects of phenomena and processes, such as the motion of a pendulum or the operation of an amplifier. They concern the phenomena in their temporal and spatial completeness [2]. The current successful extraction of scientific knowledge from experimentation is based on computer modeling and simulation mostly. Classically, the possible incompleteness in the modeling is assumed as having a provisional or practical nature as being still under study and because there is no theoretical reason why the modeling can not be complete.

Concepts and approaches regarding contexts and processes for which systems modeling can not be conceptually exhaustive have been already introduced in the literature [3]. We recall, first of all, fuzzy sets and fuzzy logic [4,5] for which, however, completeness has merely a probabilistic nature. It is matter of classical, computable probability. Nevertheless, there are phenomena, which must be modeled regarding their coherence being a crucial systemic theme with respect to their completeness or comprehensiveness as considered by Logical Openness [6] and computational information conservation theory (*CICT*) [7] for arbitrary multiscale (AMS) [8] system issues. Therefore, we must consider contexts and processes for which modelling through the use of systems is incomplete since related to only some properties, as well as those for which such modeling is theoretically incomplete as in the case of processes of emergence and for approaches considered by the Second Systemics [9]. As an example, in advanced biomedical applications [10, 11], sociology and social cybernetics [12], to ascertain complex causality reliably is always problematic, because the usual external observations always reveal superficial reasons only; they cannot reveal deep, concealed reasons [11, 13]. Furthermore, it would be interesting to consider how theoretical incompleteness, incomplete modelling, i.e., not exhausted by using individual models, of processes and phenomena should be explored as a conceptual coexistence of different approaches not so much with the purpose of exhausting, but to conceptually represent the structural dynamics of becoming [14].

At the 15th IEEE International Conference on Cognitive Informatics & Cognitive Computing, the author presented a paper introducing neuromorphic ALS (anticipatory learning system) [15], where he suggested that to achieve reliable system intelligence outstanding results, current computational system modeling and simulation has to face and to overcome two orders of issues at least, as soon as possible:

1- To minimize the traditional limitation of current digital computational resources that are unable to capture and to manage even the full information content of a single Rational Number $\boldsymbol{Q}$ exactly, leading to information dissipation and opacity.

2- To develop stronger, more effective and reliable correlates by correct arbitrary multiscale (AMS) [8] modeling approach to complex system.

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To address effectively the two above modeling issues, we need a transdisciplinary approach first, to become deeply aware that human beings and most current advanced computational and measurement systems formal representations have quite a limited capacity to capture and to extract reliable information from noisy sources or generators traditionally. This is mostly due to the representation ambiguity at the core of traditional and current computational mathematics [16].

2 Symbolic vs. Operational Number Representation

In computational mathematics, computer algebra (CAL), also called symbolic computation or algebraic computation, is a scientific area that refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects. Algebra started with methods of solving problems in arithmetic. The numerical symbols consisted probably of strokes or notches cut in wood or stone, and intelligible alike to all nations. For example, one notch in a bone represented one animal, or person, or anything else. The Egyptians paid attention to geometry and numbers, and the Phoenicians to practical arithmetic, book-keeping, navigation, and land-surveying. Numerical notation's distinctive feature, i.e. symbols having local as well as intrinsic values (arithmetic), implies a state of human civilization at the period of its invention. At the beginning of CAL, circa 1970, when the long-known algorithms were first put on computers, they turned out to be highly inefficient [17]. Therefore, a large part of the work of the researchers in the field consisted in revisiting classical algebra in order to make it effective and to discover efficient algorithms to implement this effectiveness. A typical example of this kind of work is the computation of polynomial greatest common divisors (GCD), which is required to simplify fractions. Surprisingly, the classical Euclid's algorithm turned out to be inefficient for polynomials over infinite fields, and thus new algorithms needed to be developed. The same was also true for the classical algorithms from linear algebra.

Although, properly speaking, CAL should be a subfield of scientific computing, they are generally considered as distinct fields because scientific computing is usually based on numerical computation with approximate floating point numbers, while symbolic computation emphasizes exact symbolic computation with expressions containing variables that have no given value and are manipulated as symbols, hence the name "symbolic computation" (SC). SC has also been referred to, in the past, as "symbolic manipulation", "algebraic manipulation", "symbolic processing", "symbolic mathematics", or "symbolic algebra", but these terms, which also refer to non-computational manipulation, are no more in use for referring to computer algebra.

CAL is widely used to experiment in mathematics and to design the formulas that are used in numerical programs. It is also used for complete scientific computations, when purely numerical methods fail, like in public key cryptography or for some non-linear problems. As numerical software are highly efficient for approximate numerical computation, it is common, in CAL, to emphasize on exact computation with exactly represented data. Such an exact representation implies that, even when the size of the output is small, the intermediate symbolic data generated during a computation may grow in an unpredictable way. This behavior is called "expression swell." To obviate this problem, various methods are used in the representation of the data, as well as in the algorithms that manipulate them.

The usual numbers systems used in numerical computation are either the floating point numbers and the integers of a fixed bounded size, that are improperly called integers by most programming languages. None is convenient for CAL, because of the expression swell. Therefore, the basic numbers used in CAL are the integers of the mathematicians, commonly represented by an unbounded signed sequence of digits in some base of numeration, usually the largest base allowed by the machine word. These integers allow to define the rational numbers $\boldsymbol{Q}$, which are irreducible fractions of two integers.

Computer arithmetic (CAR) is a branch of computer engineering that deals with methods of representing integers and real values (e.g., fixed- and floating-point numbers) in digital systems and efficient algorithms for manipulating such numbers by means of hardware circuits or software routines. Programming an efficient implementation of the arithmetic operations to obtain a reliable numeric representation is a hard task usually. In fact, a current intriguing point is that, although there are multiple models for the integer numbers, they all will agree on the definition of computable functions. However, current real number computation does not have these properties. Traditional scientific computation, based on classic CAR, uses specified fixed-length finite representations (related to scientific notation) of real numbers, and so can achieve only limited precision, can make errors in comparisons, and can even be unstable over rounds of conversion to and from corresponding decimal representation. Amazingly, whether an extended Turing machine model or a real-number computation model is appropriate for scientific computation is still an open topic of discussion. Therefore, most free computer symbolic algebra systems and some commercial ones, like Maple (software), use the GMP library, which is thus a de facto standard [18].

Number theory or, in older usage, arithmetic is a branch of pure mathematics devoted primarily to the study of the integers. It is sometimes called "The Queen of Mathematics" because of its foundational place in the discipline. In fact, by arithmetic, Plato meant, not arithmetic in our sense, but the science which considers numbers in themselves, in other words, what we mean by the Theory of Numbers. The word "arithmetic" is used by the general public to mean "elementary calculations"; it has also acquired other meanings in mathematical logic, as in Peano arithmetic, and computer science, as in floating point arithmetic). The use of the term arithmetic for number theory regained some ground in the second half of the 20th century, arguably in part due to French influence. In particular, "arithmetical" is preferred as an adjective to "number-theoretic." Arithmetic combinatorics (AC) is a field in the intersection of number theory, combinatorics, ergodic theory and harmonic analysis. Arithmetic combinatorics is about combinatorial computation and estimates associated with arithmetic operations (addition, subtraction, multiplication, and division). "Additive combinatorics" is the special case when only the operations of addition and subtraction are involved [19]. Number theorists study prime numbers as well as the properties of objects made out of integers (e.g., rational numbers) or defined as generalizations of the integers (e.g., algebraic integers). Questions in number theory are often best understood through the study of analytical objects (e.g., the Riemann zeta function) that encode properties of the integers, primes or other number-theoretic objects in some fashion (analytic number theory). One may also study real numbers in relation to rational numbers, e.g., as approximated by the latter (Diophantine approximation). In numeric representation of rational number $\boldsymbol{Q}$, rational proper quotient is represented by infinite repetition of a basic digit cycle, called "reptend" (the repeating decimal part) [20].

According to *CICT* new perspective, the first repetition of basic digit cycle corresponds to the first full scale interval where number information can be conserved completely, and *CICT* calls it "representation fundamental domain" (RFD) [21]. In this case the new computational representation of rational number $\boldsymbol{Q}$, is called "OpeRational" (OR) Representation, just to remember that *CICT* is able to conserve Rational Number $\boldsymbol{Q}$ full information content, much better than past computational approaches [22]. It is even possible to show a new point of view for number processing, where the complex upper half-plane can naturally emerge by the interplay of the coupled inversion of two counter oriented upper quarter planes. That representation can be even read as the self-reflexion of a reciprocal conformal relation of an "Outer Symbolic Representation" (OSR) to its corresponding reflected fundamental "Inner OpeRational Representation" (IOR) and vice-versa, where D in $\boldsymbol{Z}$, $A = 1/D$ and $D = 1/A$, and $DA = AD = \overline{0}1.\overline{0}$, for positive oriented upper quarter-space (right upper quarter plane) and $DA = AD = -\overline{0}1.\overline{0}$, for negative oriented upper quarter-space (left upper quarter plane) [21]. The relationship of these two counter-oriented upper quarter-spaces can be thought as a reciprocally projective conformal correspondence to be the first operative example in literature for explicit self-reflexive oriented numeric representation space [23]. This space can define a powerful reference framework to develop OR numerical applications for maximum information conservation (i.e. minimal information entropy generation and information dissipation) [24]. For the sake of simplicity, we consider main properties of one oriented upper quarter-space only, because the second one is just its reflection with respect a vertical axis, so LTR (Left-To-Right) representation turns into RTL (Right-To-Left) representation and vice-versa. A direct representation turns into additive complemented representation and vice-versa, when needed. In virtue of this relationship, the same information content can get two different numerical representations in direct space (DS) and reciprocal space (RS), across the outer universe-inner universe (OU-IU) boundary (Fig.1) which can resonate to each other freely, at the same time, with respect to an inversive reference boundary element (i.e., centered unitary segment in 1-d, unitary circumference in 2-d, unitary sphere in 3-d, unitary ipersphere in 4-d, etc.) between an n^{th}-dimensional "Outer Space" representation and a correspondent "Inner Space" representation: an n^{th}-dimensional OSR to its resonating IOR and vice-versa.

Therefore, the well-posed arithmetic structural description is self-defined by its functional closures. Naturally, we have two different types of closures: the linear and the exponential one [25]. This new awareness opens the way to new computational competence in computational science.

3 New Eyes on Computational Science

In the previous section we saw that current computer computation must be either symbolic or approximate. Nevertheless it can be shown that computer computation can use either an "approximated approximation", as in the past approaches, or "exact approximation" representation system, as proposed by *CICT* [21]. To achieve exact approximation computational number representation, logic must be described in terms of "closure spaces." From traditional information modeling point of view, the main focus is on the DS representation only (Euclidean space). Nevertheless, according to *CICT* ODR approach [26], to grasp the full information content of phenomenic reality, DS is just half of the OU human representation (sharable representation) and its co-direct space (CS) is the other half, the DS natural closure. Coupled to the OU is the IU human representation (subjective representation), composed by the RS and its natural closure, the reciprocal co-space (RC), or the DS dual. DS and CS are the coupled, complementary, asymptotic components of the fundamental, irreducible dichotomy of human OU representation (Fig. 1) [16, 27].

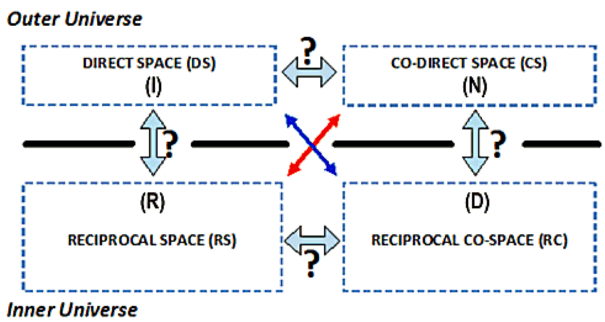

Fig. 1. The fundamental blocks of the *CICT* Outer Universe-Inner Universe Boundary approach (IOU Diagram) for full computational information conservation [16, 27] (see text).

This fundamental representation is based on two root conceptual components: unfolded information (linear sharable information that can be communicated in a formal way by media) and folded information (complex subjective information that cannot be communicated by any traditional media) [28]. According to *CICT*, this is the minimum framework to capture and to conserve information efficiently. Therefore, the best traditional DS representation can capture at maximum 25% only of the full information available theoretically. Obviously, considering these additional relationships increases the sophistication of the representation system.

On the other side, we should consider the relation between our continuous perception of the OU and the discrete representation of it. Human perception and representation of our universe can be mapped as in Fig. 1, where the encoding process is carried out by human affectors (our biological sensors) and the decoding process is done by human effectors (our biological actuators). Human biological transducers, by which we acquire information on the outer world interacting with it, are intrinsically discrete. This means that our perception of continuous shapes is just an illusion created by our brain. From this ground we can infer that an illusion of continuity can be achieved with discrete supports, without even noticing any difference, maintaining thus a maximum representation coverage property. The discrete approach reveals to be, in this sense, highly convenient because it strongly decreases the computational cost and the complexity of the system for representation modeling. According to *CICT*, in arithmetic the structure of closure spaces (across the Outer-Inner Universe boundary) is self-defined by Natural Numbers Reciprocal Space (RS) representation (Fig. 1). Furthermore, the four fundamental components of the IOU diagram (DS, CS, RS, DS) can be mapped directly to the four fundamental transformations of the Piaget-Klein four-group (I, N, R, D) to achieve predicative and reasoning competence [27].

4 Modeling OpeRational Example

The rich operative scenario offered by combinatorial modular group theory is full of articulated solutions to information processing problems. *CICT* framework is quite flexible and can be used under two major operational representation schemes: formal recurrence sequence (FRS) and formal power series (FPS) presentation, respectively. *CICT* can be presented and used in term of recurrence relation. In this case, rational geometric series can be seen as simple recurrence sequences in a wider recurrence operative framework where all algebraic recurrence sequences of any countable higher order include all the lower order ones and they can be optimally mapped to rational number system $\boldsymbol{Q}$ operational representations (OR) and generating functions (OECS, Optimized Exponential Cyclic Sequences [16]). Basic *CICT* result can be even presented in term of classic either monopolar or bipolar power series, or in general by formal power series, to show the close relationships to classic and modern control theory approaches for causal continuous-time and discrete-time linear systems [21].

Usually, from a system modeling computational perspective, all approaches that use a top-down (TD) AMS point of view allow for starting from an exact global solution of analytic solution families, which offers a shallow local solution computational precision to real specific needs (in other words, from global to local perspective overall system information is not conserved, as misplaced precision leads to information dissipation, ambiguity, confusion and system decoherence [8], [29]). This is main due to the limitations of traditional numeric computational resources as presented in previous section 2. For this reason, usually further analysis and validation (by probabilistic and stochastic methods) is always necessary to work out a localized computational solution of any practical value, in real application. A local discrete solution is achieved and computationally approximated as the last step in their line of model reasoning, that started from an overall continuous system approach (from continuum to discreteness). In this way, statistical and applied probabilistic theory became the core of classic scientific knowledge in engineering applications at system macroscale level. They were applied to all branches of human knowledge under the "continuum hypothesis" (CH) assumption, reaching highly sophistication development, and a worldwide audience. Many classic "Science 1.0" researchers and scientists up to scientific journals assume it is the ultimate language of science and it is the traditional instrument of risk-taking. Unfortunately, the "probabilistic veil" can be very opaque computationally, in a continuum-discrete arbitrary multi-scale (AMS) environment [1].

As a matter of fact, under the discreteness hypothesis (DH) with a discrete support of arbitrary precision, increasing the representation precision, we can arrive to an illusion of function continuity, offering quite interesting computational savings over the classic CH approach. Traditionally, the discreteness approach, developed under the DH assumption, in specific scientific disciplines, has been considered in peculiar application areas only. It has been further slowly developed by a few specialists and less understood by a wider audience. Unfortunately, over the centuries, the above two large scientific research areas (CH based and DH based) have followed separate mathematical development paths with no or quite little, inconsistent synergic coupling. That is the main reason why quantum field theory (QFT) modeling applications are still mostly overlooked by traditional scientific and engineering researchers for AMS system modeling, from system nano-microscale to macroscale [30].

Taking advantage of fundamental power series and polynomial system properties awareness, as achieved by *CICT* [26], here, we like to show how the concept of numerical precision can be used to control the transition from discrete to continuous in function modeling. Traditionally, in computational science, the precision of a numerical quantity is a measure of the detail in which the quantity is expressed. This is usually measured in bits, but sometimes in decimal digits. It is related to precision in mathematics, which describes the number of digits that are used to express a practical value. *CICT* uses the precision of numerical quantity to model the discreteness to continuum function transition quite easily. Our computational example has been programmed in MATLAB® language. In this way, it is possible to verify and validate through numerical computation the final results.

4.1. Example Definition

For the purpose of generating a discrete power series related example, we define the required specification as an exercise to be solved and then present the solution. Let us define the function presentation interval in the [0; 10] span and a LTR vector with a linear, fixed step Δx. This will be the axis of abscissae of the presentation window. Then, we define the values of the axis of the ordinates as the fraction of this LTR vector and a second vector (its reflection with respect to the vertical axis) in the RTL direction.
For example, if we fix the step $\Delta x = 1.0$, the representation window step is equal to an arbitrary unitary step value. We define the two vectors as follows:

$$LTR = \{0,1,2,3,4,5,6,7,8,9,10\}$$
$$RTL = \{10,9,8,7,6,5,4,3,2,1,0\} \quad , \qquad (01)$$

then we compute the ration of their components with the same index respectively. We obtain:

$$values = \frac{LTR}{RTL} = \left\{\frac{0}{10},\frac{1}{9},\frac{2}{8},\frac{3}{7},\frac{4}{6},\frac{5}{5},\frac{6}{4},\frac{7}{3},\frac{8}{2},\frac{9}{1},\frac{10}{0},\right\} = \left\{0,\frac{1}{9},\frac{1}{4},\frac{3}{7},\frac{2}{3},1,\frac{3}{2},\frac{7}{3},4,9,\infty\right\}$$

This are the starting function model (FM) values of our example. Now, we formulate four operative requests as follow.

Requests:

A. Find a parameter "a" to describe this envelope in the form a^x and compare it to the previously generated values. Plot the results.

B. Increase the number of steps (precision) in the same abscissae interval [0; 10] until you have the appearance of a continuous envelope in a plot.

C. Transform the steps and the axes to make the plot of this function appear linear.

D. Adjust the precision steps to obtain a distribution of the values in the axis of the ordinates that resembles the appearance of a continuous envelope in the entire abscissae interval [0; 10], with a convenient amount of computation for each tract on the axis of the abscissae.

4.2. Results and discussion

First, it is useful to observe the shape of the discrete envelope of this FM. It looks like an exponential-like-shaped function (Fig. 2).

4.2.1 Point A Result

At starting, we keep the step equal to the initial FM, step $\Delta x = 1$, and plot the result of the values, with respect to the axis of abscissae previously defined. Observing Fig. 2, first we should notice that the tenth value is not showed. This is due to the fact that the infinite value obtained in the tenth position is too far away from the previous ninth value: plotting it, would stop us from seeing the real envelope clearly.

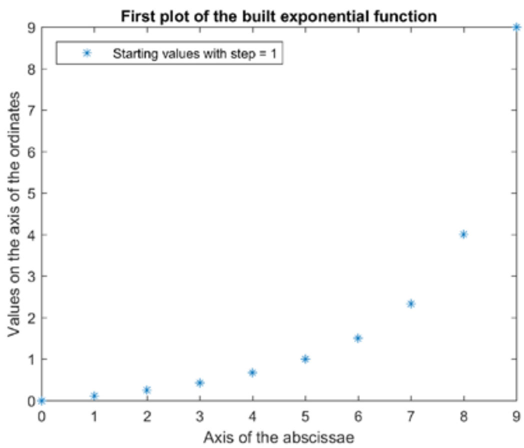

Fig. 2. Plot of the discrete exponential-like-shaped FM with unitary step.

Considering, anyway, the fact that the infinite value is present, we can identify a numerical symmetry at point (5, 1). Translating ideally the origin at this point could simplify the resolution of the Point A request. In fact, we know that the required model has the shape of an exponential function a^x and we know that it is number symmetrical at point (5, 1). Thus, once translated to the origin, we can express this numeric symmetry as $(x+5) = a^x(x-5)$. This brings us to conclude that our parameter a is equal to:

$$a = \left(\frac{x+5}{x-5}\right)^{-x} \qquad (02)$$

It is a variable parameter function of x. In order to verify this relation, we can plot the values obtained with the model against the LTR/RTL values of our starting envelope of Fig. 2. The result is shown in Fig. 3 and it is clear that the two sets of points overlap, thus the model appears accurate. Obviously, the FM is fed with an input interval [-5; 5] respecting the assumed position of the origin, and then translated into the initial [0; 10] interval for the comparison.

For a better understanding of the FM properties, we can compare it with respect to a power of ten envelope and to a discrete natural exponential shape. In Fig. 4, initial FM, with step $\Delta x = 1$, is compared to the discrete natural exponential function and the 10^n behavior, evaluated only in the considered LTR points.

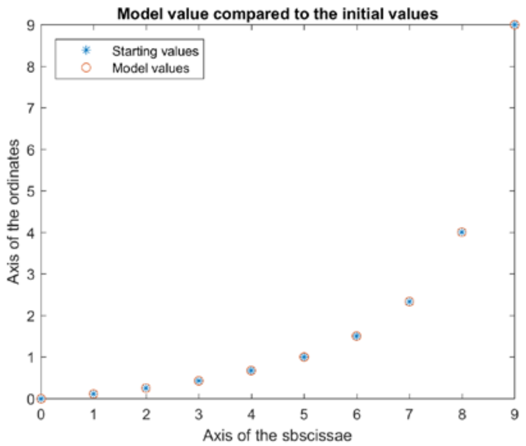

Fig. 3. Comparison between the initial FM model values and the model with the identification of parameter "a" as from eq.(02), with unitary step (see text).

Obviously, the 10^n values diverge larger from FM values, especially toward the *LTR* end of the representation window. The exponential values, instead, are closer to initial FM. Therefore, we can say that the initial FM is a discrete representation of a natural exponential function. The advantages of this representation are many: with a discrete approach it is possible to decide the level of precision at which we want to stop, with respect to the used support. The right level of precision with respect to the used representation support resolution allows to obtain the illusion of continuity in the final FM representation, with a reduced computational effort in comparison to the usual continuous approach.

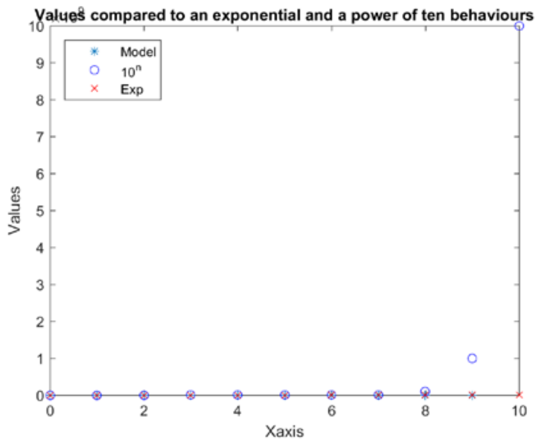

Fig. 4. Comparison of the initial FM (Model) generated values with two well-known discrete function behaviors (see text).

Furthermore, with the discrete power series approach, at each point it is possible to compute a complement to a reference value (see Section 3 for CS and RC issues), while a traditional continuous approach does not contemplate this opportunity (it offers an approximated approximation only).

4.2.2 Point B Result

For dealing with the request of obtaining the illusion of continuity, first it is possible to create a function that can be used to obtain values of the FM and the starting system as the step changes. This function, called *Ex1*, accepts as input the value of the step and returns three discrete arrays: one describing the common axis of the abscissae and the other two describing the axis of ordinates. In this way, the differences at arbitrary step values can be computed. We use *Ex1* with different values from $\Delta x=10^{-1}$ to $\Delta x=10^{-5}$ noticing that every time the step is decreased, the illusion of continuity increases and widens from the value 0.0 towards the right end of the considered representation window. The points are closer, on the axis of the ordinates, at the left beginning of the representation window and become sparse at the right end. Fig. 5 shows this behavior with step Δx = 0.001 = 10=$^{-3}$. Looking at the plot, the illusion of continuity with this step is achieved until the last interval [9; 10]. The value x = 9,97 is the point on the axis of the abscissae (derived from the MATLAB® plot with the "data cursor" function) from which we start to see again the single dots of discrete values.

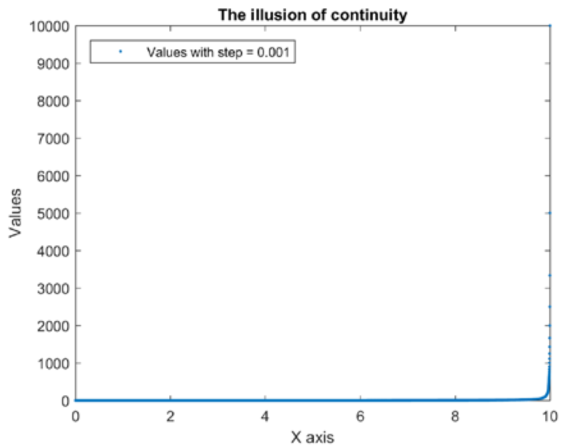

Fig. 5. The FM illusion of continuity for step Δx = 0.001 (see text).

The theoretical infinite value point is reached at x = 10, but also it slows the computation and it does not allow to reach the illusion of continuity for the last values. To solve this limitation, we should consider for the "Point D request" to vary the number of computed values in between every subset of the interval. The expected behavior is the need of higher steps in the firsts sub-intervals, while decreasing steeply towards the last ones. In this way the computation can be greatly reduced with respect to the continuous approach, and we can reach a better (or more far) illusion of continuity. The solution to this problem is presented in the following subsections.

4.2.3 Point C Result

A linear function is characterized a linear relation on the axis of the abscissae and of the ordinates. Thus, the simplest solution for this request is plotting the values against the very same vector instead of the initial [0; 10] interval. This results in a linear behavior, that put into evidence the sparse points towards the right end of the representation window. The limitation on the illusion of the continuity, as you can see in Fig. 6, is much more evident with the linear behavior.

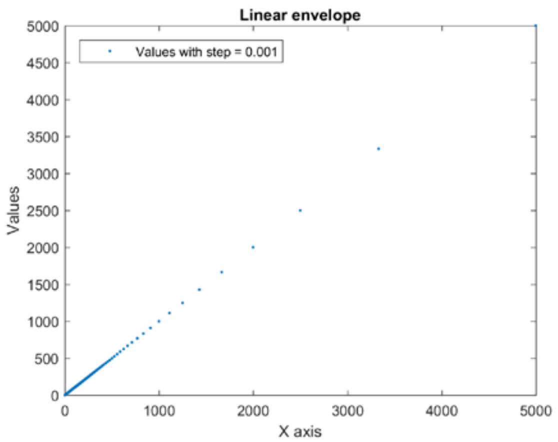

Fig. 6. Linear function representation of the FM with step Δx = 0.001 (see text).

This happens because the abscissae values become sparser with respect to the previous interval. In this configuration, unless limiting the interval to a smaller value (in this example, near to x = 400), it will be impossible, without further computation, to reach the illusion of a continuous plot. Anyway, with any linear behavior, a simple interpolation would be enough for the reconstruction of every missing point in the sparse regions.

4.2.4 Point D Result

As introduced in previous Point B subsection, in order to reduce the computational burden at each interval, while reaching the illusion of a continuous function, one approach can be adjusting the steps in each sub-interval inside the range [0; 10]. For this purpose, we create a different function called *Ex1b*. It is possible even to obtain a slightly more versatile function, allowing the user to define in which range he wants to create the exponential function. Therefore *Ex1b* accepts as input the value of this "range", defining the initial interval in [0; *range*]. Following the same approach of previous function *Ex1*, *Ex1b* defines three vectors. First, it generates an array that is not linear for the axis of the abscissae: based on previous remarks, it decreases the distance between the points as it proceeds to the right end of the interval. It is possible to define a general rule that is experimentally verified to give the illusion of a continuous curve along the entire representation interval. We can decrease the step between the points (from the starting 10^{-3} to the small value of 10^{-10}) every 4/5 of the remaining span, starting from the entire range and reaching the few points remaining near to the "range" value. This distribution of points compensates for the rarefaction effect due to the exponential shape of our initial FM. Then, the initial system values are computed in the same way implemented in *Ex1*: the results are shown in Fig. 7.

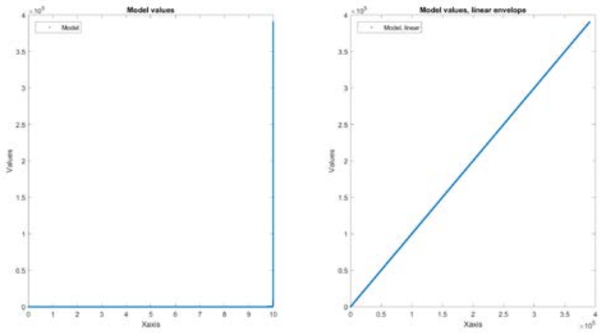

Fig. 7. FM with the illusion of continuity, both for the exponential and the linear behavior. Both graphics contain 204,000 points (see text).

We can notice that the function actually seems described by a continuous line, with respect to the resolution of the screen used (3200x1800 pixels), but with a far smaller computational effort with respect to the traditional approach. In fact, with a linear subdivision of the X axis as in Fig. 6, a plot with a range [0; 10] and a precision step of 10^{-10}, suitable for a continuous appearance at the end of the range, would require 10^{11} points. Instead, thanks to the discrete approach presented here, the final plot is constituted by just 204,000 points, with the same visual result. In other words, we obtain the reduction ratio RR = 99.9999796 % for representation information content savings.

When comparing Fig. 3, Fig. 5, and Fig. 7 we can notice that the envelope of the FM changes shape when increasing the number of points used, arriving to a theoretical orthogonal shape for precision → ∞. This is due to the construction rules of the FM itself, which start dividing sequentially the values with respect to the X axis points. The final result, with an envelope similar as much as possible to the continuous line, visually, starts growing towards infinite very close to the right end of the representation window, and the envelope becomes closer to the ideal 90° angle (reaching the ideal orthogonality). In fact, at x = 9.99 we can see that the plot reaches a value of one thousand, while until x = 9.9 it remains under a hundred. In the first example instead, with just 10 values, we see a clear growing trend already at x = 2.0. Fig. 5 has a behavior that is in the half, with 10,000 points and a visible growing behavior after x = 9.5.

Furthermore, if we focus on the final envelope (Fig. 7 on the left) between the abscissae values 9.0 and 10.0, and we superimpose it to the initial 10.0 values shown in Fig. 2, we can notice that the behavior is the same. In Fig. 8 we show this comparison. On the left you can see the initial values showed in the interval [0; 9] (because the last value, corresponding to 10.0 is ∞), while on the right subplot there is the thickened line of the model, zoomed in between abscissae [9.92; 10.00] that reveals the same envelope with the illusion of continuity.

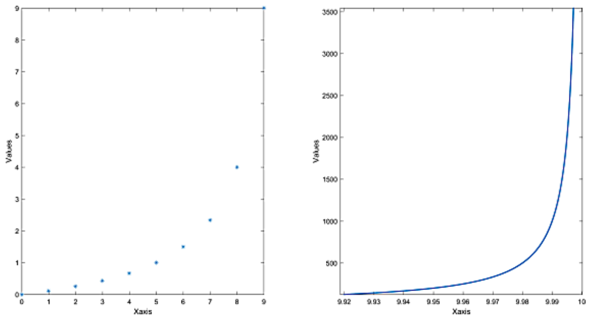

Fig. 8. Comparison of the initial values (on the left) and the corner of the thickened line (on the right).

We can conclude that, once reached the illusion of continuity, we obtained the maximum coverage on the used representation support for the function in the assigned representation window. Nevertheless, in order to achieve the minimum information content of the FM, we should assure that the number of abscissae points that represent the function are the minimum possible. In this example, instead, even if the total number of points was significantly reduced, obtained by function *Ex1b*, we cannot declare that this is the minimum information content of the FM that can be achieved, even if we obtained the reduction ratio RR = 99.9999796 % for representation information content savings!

5 Conclusion

In previous papers published elsewhere [7, 8, 10, 11, 15, 16, 21, 23, 24, 25, 26, 27, 28, 29, 30] we have already shown that traditional ***Q*** Arithmetic can be regarded as a highly sophisticated open logic, powerful and flexible bidirectional formal language of languages, according to "Computational Information Conservation Theory" (*CICT*) new perspective. We need to become deeply aware that human beings and most current advanced computational and measurement systems formal representations have quite a limited capacity to capture and to extract reliable information from noisy sources or generators traditionally. This is mostly due to the representation ambiguity at the core of traditional and current computational mathematics. This new awareness can offer competitive approach to guide more effective and convenient algorithm development and application to arbitrary multiscale (AMS) system modeling and simulation in advanced biomedical, sociology and social cybernetics applications. For *CICT* the well-posed arithmetic structural description is self-defined by its functional closures. This new awareness opens the way to new computational competence in computational science. In this paper we presented an articulated modeling OpeRational (OR) example to show how function model visual continuity appearance illusion can be achieved with big computational saving. Specifically we show how 99.9999796 % representation information content saving is obtained on a 3200x1800 pixels representation support (video screen). Science does not exists to enlighten people's minds only. It mainly exists to show the educated way from quanta to qualia. And that way starts from computational competence first.

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