Unknown parameter identification of mobile heating source by using the sensitivity of sensor network

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Abstract. This communication focuses on an inverse problem of the physical phenomena which can be described by the partial differential equation. Especially, an unknown parameter identification process of the heat conduction problem in the thermal domain related to mobile heating source tracking is investigated. To do this, an iterative minimization of a quadratic cost-function based on the conjugate gradient method is widespread since this algorithm provides regularization properties for an estimation of the trajectory of a mobile heating source. Furthermore, another identification method is proposed by using the conjugate gradient method combined with the sensitivity problem of the sensor network. A quasi-online adaptation of this numerical method aims to identify the unknown characteristics with a small delay after the required temperature measurements by selecting the better sensors locations, this could also reduce the computational time. This approach offers the opportunity to move mobile sensors in order to increase the overall sensitivity and thus to improve the quality of the identification which is a shorter delay and better accuracy.

Keywords: heat equation, inverse problem, modeling, parametric identification, partial differential equation.

1 Introduction

In engineering science, it is necessary to model many physical phenomena or technical system which can be modeled by partial differential equation (PDE) in order to predict and to act on their effect. However, to determine the value of the parameters is not easy when building their mathematical models. To do this, an inverse problem can be formulated as follows: find the unknown parameters such that the simulated output provided by the resolution of direct problem are closed to the measurements provided by sensors. It is usual to investigate such inversion as a minimization problem where a quadratic cost-function has to be minimized. Especially, we use the heat conduction problem which is also described by a parabolic partial differential equation such as an example. The goal aims to identify the trajectory of the mobile heating source. It is well known that inverse heat conduction problems are ill-posed in Hadamard sense and that it is still possible to find accurate solutions using regularization techniques [1]. Iterative minimization based on conjugate gradient method (CGM) is known as a stable algorithm for this case. Stabilizing effect during the iterative minimization is highlighted in [2] and it is shown that this method acts like a sequential filtering mechanism capable of rejecting random perturbations in measurements during the identification process.

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In this research context, the quasi online identification was proposed and developed in [3], it is based on successive time intervals for identification which slide on the total time horizon in order to take into account the continuous updating of observations. The disadvantage was necessary to use all of sensors. A sensors positioning method is investigated in order to improve it. To do this, several mobile pointwise sensors are considered. Thus sensitivity functions are determined throughout the process and are analysed in order to determine the better position for each sensor. Such methodology has been developed in [4] for a set of fixed sensor in order to select a subset of relevant sensors among a large number of candidate sensors. The main problem of mobile sensors positioning is to avoid observations redundancy and to deal with the constraints and conflicts related to the trajectories. In order to illustrate the proposed approach a demonstrator is built in the laboratory aided by the simulated experimentation. A numerical software dedicated to quasi online identification based on conjugate gradient method is implemented. Adaptation to sensors positioning is discussed considering this validated numerical tool.

The content of this article consists of four following parts. In the next part, physical system context and mathematical model are briefly described. Thus direct problem is stated. In the third part, the methodology of inverse problem (identification of the trajectory) is proposed and quasi online identification method using the sensitivity problem of sensor network is presented. Some numerical results of this method is considered in order to discuss about sensors positioning in the fourth section. The concluding remarks are presented in the last section.

2 Problematic

Let us consider that a small mobile heating source is moving on a plane surface. The main objective is to determine the source trajectory considering temperatures measurements provided by five mobile sensors. In the following several elements of the experimental device are described. Then partial differential equations system is proposed in order to provide an efficient numerical predictive tool.

A mobile heating source is moving on a square plate. In this study, the hypothesis of a 2D geometry was considered in order to reduce the computing times. This assumption is valid only if heat transfers in the plate thickness are neglected. Then, a thin metallic plate (with a high thermal conductivity) is considered. Aluminum plate was chosen (square plate with a size equal to 3m) and horizontally set on a support providing insulation.

2.1 Physical system context

To construct an experimental model for verifying the method proposed in this paper, we assume that there is one heat source S moving on the surface of a square aluminum sheet $\Omega \in \mathbb{R}^2$ of lateral dimension L and thickness e described as shown in Figure 1. The boundary of the studying domain is denoted by $\partial\Omega \in \mathbb{R}$. The variable space of the system $(x, y) \in \Omega = \left]-\frac{L}{2}, +\frac{L}{2}\right[\times \left]-\frac{L}{2}, +\frac{L}{2}\right[$ are measured in meter and the variable time $t \in \mathrm{T} = \left[0, t_f\right]$ is measured in second. This metal sheet is heated by a heat source having the thermal flux density functions $\phi(t)$ in Wm^{-2} which are assumed to be a homogeneous disk D with center $I\left(x(t), y(t)\right)$ and radius r. The temperature distribution function of a metal plate is $\theta(x, y, t)$ in Kelvin is a continuous function in space and time. Assuming that the values of parameters of a system used to construct the experimental model $\Psi\left(\Omega, \rho, c, \lambda, h, \phi(t), I(t), \theta_0\right)$ are known and listed in Table 1 with the unit of measure of the quantities in the unit of measure in the International System of Units (SI).

In particular, the metal plate is heated by a heat source traveling on its surface (Cartesian coordinate system xOy) to allow us to investigate the heat transfer on the surface and inside the plate. At the same time, the heat density densities of the sources are given by the function $\phi(t) = \phi_{\max} \exp\left(-(t-\alpha)^2/\beta^2\right)$ in (W/m^2) with $\alpha = 600$ and $\beta = 175$. The expression of the total thermal power density function of source $\Phi(x, y, z; t)$ is used to heat the experimental metal plate as follows:

$$\Phi(x, y, z; t) = \begin{cases} \phi(t) & if\,(x, y, z) \in D(I(t), r) \\ \\ 0 & otherwise \end{cases} \tag{1}$$

This expression can be represented continuously and differentially as a function of the component density functions in time variables and in spatial coordinates as follows:

$$\Phi(x, y, z, t) = \frac{\phi(t)}{\pi} arccotan\left(\eta\sqrt{(x - x(t))^2 + (y - y(t))^2} - r\right) \tag{2}$$

The regularization parameter $\eta \in \mathbb{R}^+$ has been chosen to describe with precision the heat flux discontinuity. The time interval $\left[0, t_f\right]$ can be divided into N_t segments and defined using piecewise continuous linear functions: $\left[0, t_f\right] = \bigcup\limits_{i=0}^{N_t-1} [t_i, t_{i+1}]$ with $t_i = \tau i$ and a discretization step defined by $\tau = t_f/N_t$. In order to avoid losing of generality, the orbital equation of all positioning of heat source $I(x(t), y(t), z(t))$ were re-established as discrete functions linearly and rewritten using basic triangle function $s^i(t)$ with $i = 0, 1, \ldots, N_t$:

$$s^i(t) = \begin{cases} 1 + t/\tau - i & if & t \in [t_{i-1}, t_i] \\ 1 - t/\tau + i & if & t \in [t_i, t_{i+1}] \\ 0 & otherwise & \end{cases} \tag{3}$$

Then, the density function of heat flow is expressed as follows $\phi(t) = \sum\limits_{i=0}^{N_t} \phi^i s^i(t) = \left(\overline{\phi}\right)^{tr} \overline{s(t)}$ and the equation of motion of the heat source is expressed as follows $x(t) = \sum\limits_{i=0}^{N_t} x^i s^i(t) = (\overline{x})^{tr} \overline{s(t)}$ and $y(t) = \sum\limits_{i=0}^{N_t} y^i s^i(t) = (\overline{y})^{tr} \overline{s(t)}$. It denotes that "$tr$" is the symbol of the transposition matrix.

To evaluate the reliability of the proposed mathematical model to simplify the heat transfer equation into two-dimensional space, a set of heat sensors are fixed on the metal plate in order to collect temperature data from the sensor locations during the experiment. Or, the temperature value on the sheet metal is heated by each heat source recorded by sensors. Furthermore, to assess the effect of errors during the measurement process, it is assumed that the temperature collected from the sensors has been affected by the noise. These disturbances are followed by Gaussian probability distributions $\mathcal{N}(\mu, \sigma^2)$ with mean $\mu = 1$ and standard deviation $\sigma = 0$.

2.2 Mathematical modelling of studied system

Temperature $\theta(x, y, t)$ is described by the following partial differential equation system as follow where density ρ in kgm^{-3}, specific heat capacity c in $Jkg^{-1}K^{-1}$, thermal conductivity λ in $Wm^{-1}K^{-1}$, natural heat convection coefficient h in $Wm^{-2}K^{-1}$ and ambient temperature θ_0 in $K(Kevin)$ are assumed to be constant.

The direct problem of this system is expressed by equation (4), when the source term $\dfrac{\Phi(x,y;t)-2h(\theta(x,y;t)-\theta_0)}{e}$, expressed in Wm^{-3}, is relevant if the thickness e is small enough. In the studied configuration, according to the heat fluxes range and to the parameters taken into account in the studied case, a previous numerical study has shown that the 2D model is valid in comparison with the 3D domain.

$$\begin{cases} \forall (x,y,t) \in \Omega \times T & \rho c \dfrac{\partial \theta(x,y,t)}{\partial t} - \lambda \Delta \theta(x,y,t) = \dfrac{\Phi(x,y;t) - 2h(\theta(x,y,t) - \theta_0)}{e} \\ \forall (x,y) \in \Omega & \theta(x,y;0) = \theta_0 \\ \forall (x,y,t) \in \partial\Omega \times T & -\lambda \dfrac{\partial \theta(x,y,t)}{\partial \overrightarrow{n}} = 0 \end{cases} \tag{4}$$

Usually, such mathematical problem is solved using numerical method such as the finite element method [5][6][7] and a numerical software such as Comsol Multiphysics TM interfaced with Matlab® [8][9]. The previous direct problem is well-posed since few noises on parameters Ψ introduce small disturbances on state estimation $\theta(x,y,t)$. Several illustrations for direct problem solution are shown in the following figures for a mobile source on a small area of the plate (about 4 square meters, see figure 1).

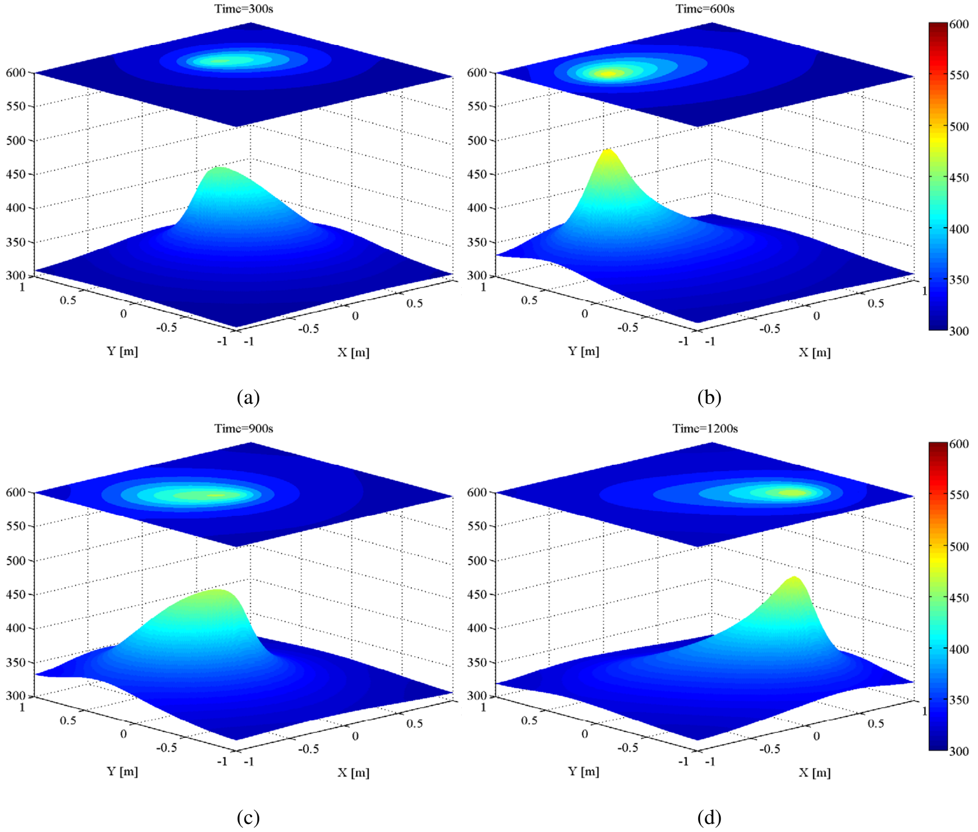


Figure 1. Spatial evolution of temperature on the plate generated by heat source

Temperatures are observed by a virtual network of 4 mobile sensors located on the plate. These "observed" temperatures are arbitrarily disturbed by a Gaussian noise. Once a parameter is unknown, mathematical formulations of the inverse problem considered in order to identify the unknown parameters are proposed in the following section.

3 Methodology of inverse problem

3.1 Inverse problem

Let us consider in the following that the source trajectory $\{x_I(t), y_I(t)\}$ is not known and has to be identified. In such an aim sensors measurements are available: $\hat{\theta}_m(t)$ provided by sensors C_m (for m=1 to 5). Even if observations are provided by pyrometers on a small surface, sensors C_m are assumed to be pointwise.

Inverse problem can be expressed as follows: Find $\{x_I(t), y_I(t)\}$ such that $J(\theta, I) = \frac{1}{2}\sum\limits_{m=1}^{5}\int\limits_{0}^{t_f}\left(\theta(C_m, t; I) - \hat{\theta}_m(t)\right)^2 dt$ is minimum with *theta* $(x, y; t)$ solution of (3).

Without loss of generalities, a discrete formulation is considered;

Find $\overline{I} = (\overline{x}_I, \overline{y}_I)$ such that $J\left(\theta, \overline{I}\right) = \frac{1}{2}\sum\limits_{m=1}^{5}\sum\limits_{j=1}^{n}\left(\theta\left(C_m, t^j; \overline{I}\right) - \hat{\theta}_m^j\right)^2$ is minimum with $\theta(x, y; t)$ solution of (3).

where n is related to the time sample for sensors measurements. CGM is implemented to identify the unknown parameters [10][11]. This algorithm requires iterative resolution of three well-posed problems: the direct problem (3) to calculate the cost-function $J\left(\theta, \overline{I^k}\right)$ and estimate the quality of the estimate $\overline{I^k}$ at iteration k; the sensitivity problem to calculate the descent depth (in the descent direction); the adjoint problem to determine the gradient of the cost-function $J\left(\theta, \overline{I}\right)$ and thus to define the next descent direction [12][13].

In the following, the mathematical formulation of the problems will be presented in detail in order to calculate the intermediate parameters of the identification method.

3.1.1 Sensitivity problem

Let us consider the variation of temperature $\delta\theta(\omega)$ with $(\omega) = (x, y, t)$ induced by a variation of the total heating flux noted: $\Phi^+(x, y; t) = \Phi(\omega) + \varepsilon\delta\Phi(\omega)$ with $\delta\theta(\omega) = \lim\limits_{\varepsilon\to 0}\left(\frac{\theta^+(\omega) - \theta(\omega)}{\varepsilon}\right)$.

It's combined with the equation (4), the sensitivity problem is described by the following system:

$$\begin{cases} \rho c\dfrac{\partial\delta\theta(x,y,t)}{\partial t} - \lambda\Delta\delta\theta(x,y,t) = \dfrac{\delta\Phi(x,y,t) - 2h\delta\theta(x,y,t)}{e} & \forall(x,y,t)\in\Omega\times T \\ \delta\theta(x,y;0) = 0 & \forall(x,y)\in\Omega \\ -\lambda\dfrac{\partial\delta\theta(x,y,t)}{\partial\vec{n}} = 0 & \forall(x,y,t)\in\partial\Omega\times T \end{cases} \tag{5}$$

where: $\xi(\omega) = \sqrt{(x - x_I(t))^2 + (y - y_I(t))^2}$

$$\begin{aligned} \delta\Phi(\omega) &= \frac{\partial\Phi(\omega)}{\partial x_I(t)}\delta x_I(t) + \frac{\partial\Phi(\omega)}{\partial y_I(t)}\delta y_I(t) \\ &= \frac{\eta\phi(t)}{\pi\xi(\omega)\left(1 + \eta^2(\xi(\omega) - r)^2\right)}\left((x - x_I(t))\delta x_I(t) + (y - y_I(t))\delta y_I(t)\right) \end{aligned} \tag{6}$$

The sensitivity problem solution $\delta\theta(\omega)$ is useful to calculate the descent depth γ^{k+1} for each iteration, such as:

$$\overrightarrow{\Phi}^{k+1} = \overrightarrow{\Phi}^{k} - \gamma^{k+1}\overrightarrow{d}^{k+1} \tag{7}$$

Descent depth γ^{k+1} minimizes the criterion $J\left(\theta;\overrightarrow{\Phi}^{k+1}\right)$: $[\gamma^{k+1} = \underset{\gamma\in}{\text{Argmin}}\, J\left(\theta;\overrightarrow{\Phi}^{k+1}\right)$ or $\dfrac{\partial J\left(\theta;\overrightarrow{\Phi}^{k} - \gamma^{k+1}\overrightarrow{d}^{k+1}\right)}{\partial\gamma^{k+1}} = 0$. This implies that descent depth calculated at each iteration is:

$$\gamma^{k+1} = \frac{\int\limits_0^{t_f}\sum\limits_{n=1}^{N_c}\left(\theta_{C_n}\left(t;\overrightarrow{\Phi}^{k+1}\right) - \hat{\theta}_{C_n}(t)\right)\delta\theta\left(C_n,t;\overrightarrow{\Phi}^{k}\right)dt}{\int\limits_0^{t_f}\sum\limits_{n=1}^{N_c}\left(\delta\theta\left(C_n,t;\overrightarrow{\Phi}^{k}\right)\right)^2 dt} \tag{8}$$

In order to solve the sensitivity problem, the descent direction $\overrightarrow{d}^{k+1}$ $k \in {}^{6}\mathbb{N}_t$ has to be known. In such an aim the cost function gradient has to be computed *(according to the following problem).*

3.1.2 Adjoin problem

In order to determine the gradient $\nabla\overrightarrow{J} = \left(\dfrac{\partial J}{\partial x_I};\dfrac{\partial J}{\partial y_I}\right)$ with $\forall i = 1, 2, ..., N_t$ at each iteration, a Lagrangian formulation $\ell(\theta(\omega),\Phi,\psi)$ is introduced such that:

$$\begin{aligned}\ell(\theta(\omega),\Phi,\psi) &= J(\theta(\omega),\Phi)\\ &+\int\limits_0^{t_f}\int\limits_\Omega\left[\rho c\frac{\partial\theta(\omega)}{\partial t} - \lambda\Delta\theta(\omega) - \frac{\Phi(\omega)}{e} + \frac{2h(\theta(\omega)-\theta_0)}{e}\right]\psi d\Omega dt\end{aligned} \tag{9}$$

If $\theta(\omega)$ is solution of the heat equation (4) then $\ell(\theta(\omega),\Phi,\psi) = J(\theta(\omega),\Phi) \to \delta\ell(\theta(\omega),\Phi,\psi) = \delta J(\theta(\omega),\Phi)$. When $\psi(x,y;t)$ is fixed, we denote $d\theta(\omega) = \theta_{C_n}\left(\omega;\overrightarrow{\Phi}^{k+1}\right) - \hat{\theta}_{C_n}(\omega)$, the Lagrangian variation can be written as:

$$\begin{aligned}&\delta\ell(\theta(\omega),\Phi,\psi)\\ &= \delta J(\theta(\omega),\Phi) + \int\limits_0^{t_f}\int\limits_\Omega\left[\rho c\frac{\partial\delta\theta(\omega)}{\partial t} - \lambda\Delta\delta\theta(\omega) - \frac{\delta\Phi - 2h(\delta\theta(\omega))}{e}\right]\psi(\omega)d\Omega dt\\ &= \int\limits_0^{t_f}\int\limits_\Omega\sum\limits_{n=1}^{N_c} d\theta(t)\,\delta\theta(\omega)\,\delta_D(C_n)\,d\Omega dt + \int\limits_0^{t_f}\int\limits_\Omega\rho c\frac{\partial\delta\theta(\omega)}{\partial t}\,\psi(\omega)\,d\Omega dt\\ &-\int\limits_0^{t_f}\int\limits_\Omega\lambda\Delta\delta\theta(\omega)\,\psi(\omega)\,d\Omega dt + \int\limits_0^{t_f}\int\limits_\Omega\frac{2h\delta\theta(\omega)}{e}\psi(\omega)\,d\Omega dt - \int\limits_0^{t_f}\int\limits_\Omega\frac{\delta\Phi}{e}\,\psi(\omega)\,d\Omega dt\end{aligned} \tag{10}$$

Let us denote by $E(\omega) = \sum\limits_{n=1}^{N_c} d\theta(\omega)\,\delta_D(x(t) - x_{C_n}(t))\,\delta_D(y(t) - y_{C_n}(t))$ with $\delta_D(x - x_{C_n})\,\delta_D(y - y_{C_n})$ the Dirac distribution related to sensor $C_n(x_{C_n}, y_{C_n})$. Then it comes:

$$\begin{aligned}\delta\ell(\theta(\omega),\Phi,\psi) &= \int\limits_0^{t_f}\int\limits_\Omega\left[E(\omega) - \rho c\frac{\partial\psi(\omega)}{\partial t} - \lambda\Delta\psi(\omega) + \frac{2h\psi(\omega)}{e}\right]\delta\theta(\omega)\,d\Omega dt\\ &+\rho c\int\limits_\Omega\delta\theta\left(x,y;t=t_f\right)\psi(\omega)\,d\Omega + \lambda\int\limits_0^{t_f}\left[\delta\theta(\omega)\nabla\psi(\omega)\right]_{\partial\Omega}dt - \int\limits_0^{t_f}\int\limits_\Omega\delta\Phi\frac{\psi(\cdot)}{e}\,d\Omega dt.\end{aligned} \tag{11}$$

Since $\frac{\partial \ell\left(\theta\left(x,y;t\right),\Phi,\psi\right)}{\partial \theta\left(x,y;t\right)}\delta\theta\left(\cdot\right)=0 \quad \forall \delta\theta\left(x,y;t\right)$, then the terms multiplied by $\delta\theta\left(x,y;t\right)$ have to be equal to zero, then it is necessary that the adjoin function $\psi\left(x,y;t\right)$ is solution of the following problem:

$$\begin{cases} \rho c\dfrac{\partial\psi(x,y,t)}{\partial t}+\lambda\Delta\psi(x,y,t)=E(x,y,t)+\dfrac{2h\psi(x,y,t)}{e} & \forall(x,y,t)\in\Omega\times T \\ \psi(x,t;t_f)=0 & \forall(x,y)\in\Omega \\ -\lambda\dfrac{\partial\psi(x,y,t)}{\partial\vec{n}}=0 & \forall(x,y,t)\in\partial\Omega\times T \end{cases} \tag{12}$$

Finally, when $\psi(x,y,t)$ is solution of the previous problem then: $\delta\ell\left(\theta\left(x,y,t\right),\Phi,\psi\right)=\frac{\partial\ell\left(\theta\left(x,y,t\right),\Phi,\psi\right)}{\partial\Phi}\delta\Phi=-\int\limits_0^{t_f}\int\limits_\Omega\delta\Phi\frac{\psi\left(x,y,t\right)}{e}d\Omega dt=\delta J\left(\theta\left(x,y,t\right),\Phi\right)$.
After several mathematical developments which cannot be presented due to the limited number of pages of this communication, cost function gradients can be formulated as follows:

$$\begin{aligned} \nabla J_{x_I^i} &= -\int\limits_0^{t_f}\int\limits_\Omega \frac{\eta\phi(t)}{\pi}\frac{(x-x_I(t))s^i_{x_I}(t)}{\pi\xi(x,y,t)\left(1+\eta^2\left(\xi(x,y,t)-r\right)^2\right)}\frac{\psi(x,y,t)}{e}d\Omega dt \\ \nabla J_{y_I^i} &= -\int\limits_0^{t_f}\int\limits_\Omega \frac{\eta\phi(t)}{\pi}\frac{(x-y_I(t))s^i_{y_I}(t)}{\pi\xi(x,y,t)\left(1+\eta^2\left(\xi(x,y,t)`-r\right)^2\right)}\frac{\psi(x,y,t)}{e}d\Omega dt \end{aligned} \tag{13}$$

The descent direction can be estimated at each new iteration $k+1$ from the previous gradient formula, as follow:

$$\overrightarrow{d}^{k+1}=-\overrightarrow{\nabla J}(\theta;\Phi^k)+\frac{\left\|\overrightarrow{\nabla J}(\theta;\Phi^k)\right\|^2}{\left\|\overrightarrow{\nabla J}(\theta;\Phi^{k-1})\right\|^2}\overrightarrow{d}^k \tag{14}$$

where $\|\cdot\|$ is the Euclidean norm.
In the following section, the application of the online identification method based on the iterative regularization method (CGM) considering a network of fixed sensors is numerically implemented in order to identify the flux density as well as the trajectory of two sources.

3.2 Identification method based on CGM using sensitivity of sensor network

In recent works [3] authors have investigated the interest of CGM adaptation to quasi on line identification. Indeed, the main inconvenient for CGM is the convergence time which can be very important according to the problem complexity. Several strategies have been tested and compared to the reference strategy which is the offline approach: identification algorithm is launched when all the measurements are available. Quasi online identification is based on time windows for observation which are sliding according to the identification process.

- constant offset with constant time window size;
- adaptive overlap with constant time window size;
- time window size related to a priori information;
- adaptive time window size and constant offset;
- adaptive time window size and adaptive overlap.

In the configuration studied in [3] the last strategy has provided identification result in a computational time divided by 200 in comparison with the reference strategy (offline identification). From the results obtained from the method of choice of sensors in offline mode proposed above, we develop another method of choice of sensors in quasi-online mode. Near-line identification is performed using the adaptation of the conjugate gradient method (predictive with adaptive window size) using the on-line sensor selection algorithm.

Algorithm of selectioning sensor method based on the sentivity problem	
Step 1	- initialisation of parameter: k, L^2
	- time interval $\tau_m = [\tau_m^-, \tau_m^+]$
Step 2	- solve the sensitivity problem
	- extract values of $\theta(C_k, t)$
Step 3	- calculate $L_k^2 = \|\delta\theta\,(C_k, t)\|$
	- choose a sensor having the largest value of L^2
	- remove this sensor in the list
Step 4	- update index $k = k + 1$
	- ***if*** $(k > k_{max})$
	$\rightarrow$ end of algorithm
	- return to step 1.

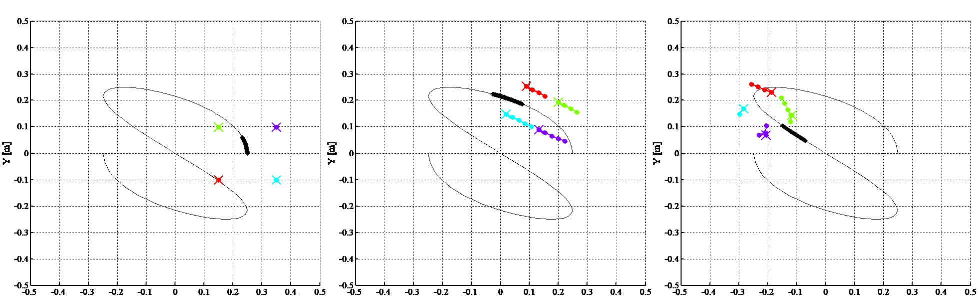
This algorithm allows us to select the most relevant sensors over the time interval $\tau_m = [\tau_m^-, \tau_m^+]$ in which to deploy the robot-sensors. In addition, the online identification method makes it possible to "continue" during the experiment the mobile sources. This method relies on finding the most sensitive sensors by maximizing the following standard over slippery time intervals τ_m:

$$L_k^2 = \|\delta\theta\,(C_k, t)\| = \sqrt{\sum_{\tau_m} (\delta\theta\,(C_k, t))^2\,(\forall k)} \tag{15}$$

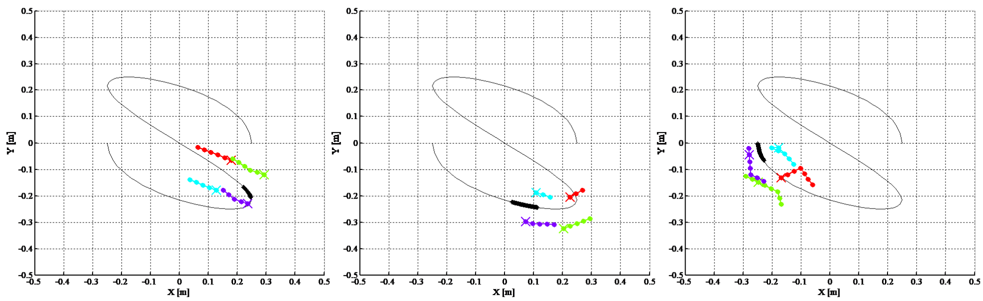
First of all, it is necessary to determine the time interval using the method of Determination of the interval proposed and to initialize the parameters of the algorithm. By comparing these sensor standard values, the most sensitive sensors are selected. These are the optimal positions (in the sense of sensitivity) for robot-sensors in the next interval of the identification window. The application of this online sensor selection algorithm makes it possible to develop optimal sensor deployment strategies.

4 Numerical results and discussion

By applying proposed identification method based on CGM for an inverse problem combined with the sensitivity of sensor network, we can build an example to identify trajectory of a mobile heat source. After realizing the algorithm of this method, we obtained some results of identification process (see figure 2).



(a) Time interval 0 to 71 (b) Time interval 240 to 321 (c) Time interval 776 to 847

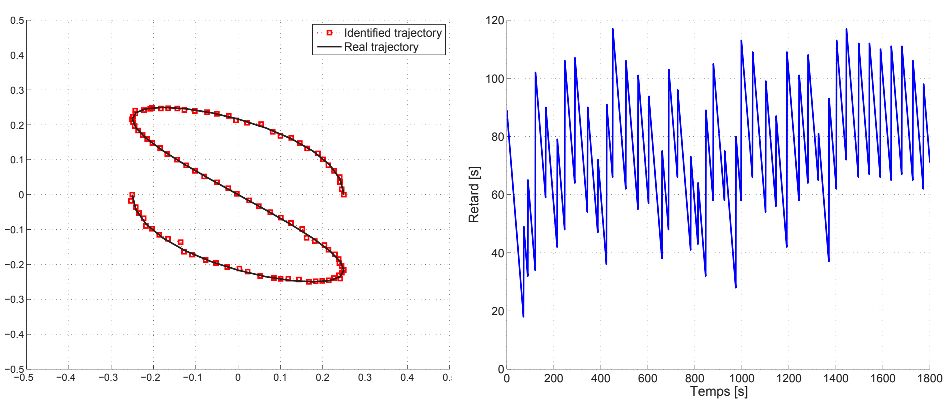


(d) Time interval 0 to 71 (e) Time interval 240 to 321 (f) Time interval 776 to 847

Figure 2. Results of identification process

In these above figures, a set of four better sensors were chosen in order to track the movement of the heat source according to its trajectory. These sensors provide the measured temperatures for an acquisition which is used in the proposed identification method based on CGM. It allows identifying the trajectory of this mobile heat source. It is now necessary to identify for each source the position every 15 seconds.

This represents an experiment of 1800 seconds is 121 parameters to identify. The acquisition of temperatures at different points on the plate will be carried out by a pack of 4 mobile sensors.



(a) Identified trajectory (b) Time of delay

Figure 3. Results of identified trajectory and time of relay

In figure 3, the figure (a) shows the identified trajecroty of the mobile heat source by using the proposed method. While the duration of the campaign is 1800 seconds, the estimation procedure converged in 1886 seconds, 86 seconds after the end of the experiment, the average identification delay is 85 seconds. We also obtain the following results (identification time, average temperature residues, the standard deviation of residues):

Table 1. Results of quasi-online identification process

$t_{identification}$ (s)	$\mu_{residues}$ (K)	$\sigma_{residues}$ (K)
1886	-0.19	0.64

5 Conclusion and outlooks

In this paper, a quasi-online unknown parameter identification method is proposed by using an iterative minimization based on the conjugate gradient method. Considering a mobile radiative heating source on a plane metallic surface, several mobile sensors are used in order to determine the unknown source trajectory. This ill-posed inverse problem is solved quasi-online by means of adapted iterative regularization method by minimizing a quadratic cost-function. This iterative regularization technique was used for an inverse problem in the thermal domain. Furthermore, the sensitivity problem of the sensor network was investigated to propose a method to choose the better position of each sensor. With the numerical results of this research, some future prospects are related to developing the whole methodology and to proposing the identification of new strategies for sensor moving and source tracking (in order to take into account redundancy and several heating sources). A long-term goal is the management of a fleet of embedded sensors, e.g. in the automobile, in order to track polluting cloud in a city.

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