Exact Solutions with Lie Symmetry Analysis for Nano-Ionic Currents along Microtubules

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Abstract. In this work, the Lie symmetry analysis is tested by its applications to nano-solitons of ionic waves propagation along microtubules in living cells and nano-ionic currents of MTs.

1 Introduction

The main aim of this work is to study the executive equation defining nano-ionic currents along microtubules with the Lie symmetry analysis. This method is one of the most important and efficient techniques to find the exact solutions [1]-[7].

The nano ionic currents are elaborated in [8] take the form

$$\frac{L^2}{2}q_{xxx} + \frac{Z^{3/2}}{L}(\chi G_0 - 2\delta C_0)qq_t + 2q_x + \frac{ZC_0}{L}q_t + \frac{1}{L}(RZ^{-1} - G_0 Z)q = 0, \tag{1}$$

where $R = 0.34 \times 10^9 \Omega$ the resistance of the elementary rings (ER), $L = 8 \times 10^{-9}m$, $C_0 = 1.8\times10^{-15}F$ is the total maximal capacitance of the ER. $G_0 = 1.1\times10^{-13}S\,i$ is the conductance of pertaining nano-pores (NPS) and $Z = 5.5 \times 10^{10}\Omega$ is the characteristic impedance of the system. The parameter δ and χ describe the nonlinearity of ER capacitor and conductance of NPS in ER, respectively.

2 Lie Symmetry Analysis

We will now consider the one parameter group of point transformation of the form

$$x \to x + \varepsilon\xi(x, t, q) ,$$
$$t \to t + \varepsilon\tau(x, t, q) ,$$
$$q \to q + \varepsilon\phi(x, t, q) ,$$

where $\varepsilon < 1$ is a group parameter [9]. The vector field associated with the above group of transformations can be written as

$$V = \xi(x, t, q)\frac{\partial}{\partial x} + \tau(x, t, q)\frac{\partial}{\partial t} + \phi(x, t, q)\frac{\partial}{\partial q}. \tag{2}$$

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In order to study symmetries, from Eq. (1), it can be shown by a well established procedure as following

$$\xi = C_2,$$
$$\tau = C_1,$$
$$\phi = 0,$$

where C_1 and C_2 are constants. The Lie symmetry algebra of Eq. (1) is spanned by the following two generators.

$$V_1 = \frac{\partial}{\partial x},$$
$$V_2 = \frac{\partial}{\partial t}.$$

Now we consider the above mentioned vector fields for Eq. (1).

Case 1: For the generator $V_1 = \frac{\partial}{\partial x}$, the similarity variables are $\eta = t$, $q = f(\eta)$. Then from Eq. (1), we get the following reduction equation

$$\frac{Z^{3/2}}{L}(\chi G_0 - 2\delta C_0)ff' + \frac{ZC_0}{L}f' + \frac{1}{L}(RZ^{-1} - G_0 Z)f = 0. \tag{3}$$

Solution of this equation

$$f(\eta) = \frac{C_0}{Z^{1/2}(\chi G_0 - 2\delta C_0)} ProductLog[\frac{Z^{1/2}(\chi G_0 - 2\delta C_0)}{C_0}e^{\alpha\eta + C}], \tag{4}$$

where $\alpha = (RZ^{-1} - G_0 Z)/ZC_0$.

Case 2: For the generator $V_2 = \frac{\partial}{\partial t}$, the similarity variables are $\eta = x$, $q = f(\eta)$. Then from Eq. (1), we get the following reduction linear equation

$$\frac{L^2}{3}f''' + 2f' + \frac{1}{L}(RZ^{-1} - G_0 Z)f = 0. \tag{5}$$

Case 3: For the generator $V_3 = V_1 + V_2 = \partial/\partial x + \partial/\partial t$, we have $q = f(\eta)$ that $\eta = x - t$. If we substitute $q = f(\eta)$ into the governing equation, we obtain following NODE;

$$\frac{L^2}{3}f''' - \frac{Z^{3/2}}{L}(\chi G_0 - 2\delta C_0)ff' + (\frac{2L - ZC_0}{L})f' + \frac{1}{L}(RZ^{-1} - G_0 Z)f = 0. \tag{6}$$

Case 4: For the infinitesimal generator $V_4 = \frac{1}{\lambda}\frac{\partial}{\partial x} + \frac{1}{\rho}\frac{\partial}{\partial t}$, we have $q = f(\eta)$, $\eta = \lambda x - \rho t$ where λ and ρ are constants. If we substitute $q = f(\eta)$ into Eq. (1), so that

$$\frac{L^2}{2}\lambda^3 f''' - \frac{Z^{3/2}}{L}(\chi G_0 - 2\delta C_0)\rho ff' + (2\lambda - \frac{ZC_0}{L}\rho)f' + \frac{1}{L}(RZ^{-1} - G_0 Z)f = 0. \tag{7}$$

In this, we find the solution of Eq. (7) using the "Generalized Projective Riccati Equations Method" (GPREM) [10]. So, balancing f''' and ff' gives $N = 2$. Therefore

$$f(\eta) = A_0 + \sum_{i=1}^{2} \sigma^{i-1}[A_i\sigma(\eta) + B_i\nu(\eta)], \tag{8}$$

where A_j and B_j, $j = 0, 1, 2$ are constants.

In Eq. (8), $\sigma(\eta)$ and $\nu(\eta)$ satisfy the ODE

$$\sigma' = \epsilon\sigma(\eta)\nu(\eta), \tag{9}$$

$$\nu' = R + \epsilon\nu^2(\eta) - \mu\sigma(\eta), \epsilon = \pm 1, \tag{10}$$

where

$$\nu^2 = \epsilon(R - 2\mu\sigma(\eta) + \frac{\mu^2 + r}{R}\sigma^2(\eta), \tag{11}$$

r, R and μ are non zero constants.

So, if Eq. (8) are written in Eq. (7) for $\epsilon = -1$ and $r = 1$, we get solving the system of the algebraic equation.

Consequently, the solution of f(z) is obtained as following

$$f(z) = -\frac{3\lambda^2}{2\alpha_2\rho}\frac{\csc h\left[z\sqrt{\frac{-i}{\lambda\alpha_1}}\right]}{\left(1 + i\csc h\left[z\sqrt{\frac{-i}{\lambda\alpha_1}}\right]\right)} + \frac{2\lambda - i\lambda^2 - \rho\alpha_3}{\alpha_2\rho},$$

where $\alpha_1 = \frac{L^2}{2}\lambda^3$, $\alpha_2 = \frac{Z^{3/2}}{L}(\chi G_0 - 2\delta C_0)\rho$, $\alpha_3 = (2\lambda - \frac{ZC_0}{L}\rho)$ and finally we get $\alpha_4 = \frac{1}{L}(RZ^{-1} - G_0 Z)$.

3 Conclusion

In this work, the Lie symmetry analysis has been successfully applied to find the solution of the nonlinear partial differential equation which plays an important role in biology. Some of the equations obtained with the Lie symmetry are solved and we found new exact solutions for these equations. Also, while finding these new exact solutions, Generalized Projective Riccati method was applied to the reduced equation by Lie analysis.

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