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On Intuitionistic Fuzzy 2-Metric Spaces

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Abstract In this study, we first recall the notion of an intuitionistic fuzzy 2-metric space and fundamental definitions with several illustrative examples. Then we define the notion of $\delta-$chainable space and $(\delta,\lambda)-$uniform locally contractive mapping between intuitionistic fuzzy 2-metric spaces. After that, by using the proposed concepts, we obtain a few fixed point theorems of self-mappings defined on this spaces.

1 Introduction

Fuzzy set theory which is one of the most important and useful branch of mathematics, was introduced by Zadeh [13] in 1965. The range of applications of this theory is wide and starting from artificial intelligence and also touching on a lot of scientific disciplines such as engineering, economics, etc. Many researchers have obtained a lot of interesting results and applications concerning the fuzzy set theory. Kramosil and Michalek [5] described the concept of fuzzy metric space applying the idea of fuzziness to the clasical notions of metric and metric spaces. Park [8] defined the concept of intuitionistic fuzzy metric space and proved some well-known theorems of metric spaces for this spaces.

The notion of 2-metric which was proposed in Euclidean space by the area function, was initiated by Gähler [2]. In 2002, Sharma [11] gave the definition of fuzzy 2-metric space and obtained some common fixed point theorems. Mursaleen and Lohani [6, 7] launched the notion of intuitionistic fuzzy 2-metric space which is the generalization of the intuitionistic fuzzy metric space. They studied some topological properties of this spaces such as convergence, completeness, completion. Bakry [1] studied the existence and uniqueness of a common fixed point theorem in complete intuitionistic fuzzy 2-metric spaces. Shrivastava et al. [12] gave the definition of the weak compatible mappings in intuitionistic fuzzy 2-metric spaces and investigated a common fixed point theorems with the help of these mappings.

2 Preliminaries

Definition 2.1. [10] If a binary operation $*\ (\diamond)$ on the interval $[0,1]$ satisfies the following conditions, then $*\ (\diamond)$ is said to be a t-norm (t-conorm):
(a) $*\ (\diamond)$ is associative and commutative.
(b) $*\ (\diamond)$ is continuous.

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(c) $x * 1 = x$ $(x \diamondsuit 0 = x)$ for all $x \in [0,1]$.
(d) $x * y \leq z * u$ $(x \diamondsuit y \leq z \diamondsuit u)$ whenever $x \leq z$ and $y \leq u$ for all $x, y, z, u \in [0,1]$.

Definition 2.2. [2, 4] Let $X \neq \emptyset$ be a set. A function $d: X \times X \times X \to \mathbb{R}$ is called a $2-$metric if d satisfies the following conditions :
(i) to each pair of points $x, y \in X$ $(x \neq y)$, there is a point $z \in X$ such that $d(x, y, z) \neq 0$,
(ii) $d(x, y, z) = 0$ when at least two of x, y, z are equal,
(iii) $d(x, y, z) = d(x, z, y) = d(y, z, x)$ for all $x, y, z \in X$,
(iv) $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)$ for all $x, y, z, w \in X$.
The ordered pair (X, d) is called a $2-$metric space when d is a $2-$metric on X. We may think of this as meaning that $d(x, y, z)$ is a family of distance-like functions of x and y indexed by $z \in X$.

Example 2.3. [9] Let $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\}$. The mapping $d: X \times X \times X \to [0, \infty)$ defined by

$$d(x,y,z) = \begin{cases} 1, & x, y, z \text{ are distinct and } \left\{\frac{1}{n}, \frac{1}{n+1}\right\} \subseteq \{x, y, z\} \\ 0, & \text{otherwise} \end{cases}$$

for all $x, y, z \in X$, is a 2-metric on X.

Definition 2.4. [1, 6] Let $X \neq \emptyset$ be an arbitrary set, $*$ be a continuous t-norm, $\diamondsuit$ be a continuous t-conorm and $\mu, \nu : X \times X \times X \times [0, \infty) \to [0,1]$ be the mappings. If the following conditions are satisfied, then a 5-tuple $(X, \mu, \nu, *, \diamondsuit)$ is called an intuitionistic fuzzy 2-metric space or shortly if2ms:
(IFM1) $\mu(x, y, z, t) + \nu(x, y, z, t) = 1$.
(IFM2) $\mu(x, y, z, 0) = 0$.
(IFM3) $\mu(x, y, z, t) = 1$ for all $t > 0$ when at least two of x, y, z are equal.
(IFM4) $\mu(x, y, z, t) = \mu(x, z, y, t) = \mu(y, z, x, t) = \mu(z, x, y, t)$.
(IFM5) $\mu(x, y, z, t_1 + t_2 + t_3) \geq \mu(x, y, u, t_1) * \mu(x, u, z, t_2) * \mu(u, y, z, t_3)$.
(IFM6) $\mu(x, y, z, \cdot): [0, \infty) \to [0,1]$ is left continuous.
(IFM7) $\nu(x, y, z, 0) = 1$.
(IFM8) $\nu(x, y, z, t) = 0$ for all $t > 0$ when at least two of x, y, z are equal.
(IFM9) $\nu(x, y, z, t) = \nu(x, z, y, t) = \nu(y, z, x, t) = \nu(z, x, y, t)$.
(IFM10) $\nu(x, y, z, t_1 + t_2 + t_3) \leq \nu(x, y, u, t_1) \diamondsuit \nu(x, u, z, t_2) \diamondsuit \nu(u, y, z, t_3)$.
(IFM11) $\nu(x, y, z, \cdot): [0, \infty) \to [0,1]$ is right continuous.
for all $x, y, z \in X$ and $t, t_1, t_2, t_3 > 0$. The values $\mu(x, y, z, t)$ and $\nu(x, y, z, t)$ may be interpreted as the degrees of nearness and non-nearness that the area of triangle enlarged x, y, z with respect to t.

Example 2.5. Let (X, d) be a 2-metric space, $a * b = ab$ and $a \diamondsuit b = min\{1, a + b\}$. Let $\mu_d, \nu_d : X^3 \times [0, \infty) \to [0,1]$ be two mappings defined by

$$\mu_d(x,y,z,t) = \begin{cases} \frac{t}{t + d(x,y,z)}, & t > 0 \\ 0, & t = 0 \end{cases}, \qquad \nu_d(x,y,z,t) = \begin{cases} \frac{d(x,y,z)}{t + d(x,y,z)}, & t > 0 \\ 1, & t = 0 \end{cases}$$

for all $x, y, z \in X$. Then $(X, \mu_d, \nu_d, *, \diamondsuit)$ is an if2ms and said to be the standard if2ms induced by the 2-metric d. Hence, we have an if2ms when a 2-metric d is given.

Definition 2.6. [6] Let (x_n) be a sequence in a given if2ms $(X, \mu, \nu, *, \diamondsuit)$.
(1) $(x_n) \subseteq X$ is said to converge to $x \in X$ if $\lim_{n \to \infty} \mu(x_n, x, z, t) = 1$ and $\lim_{n \to \infty} \nu(x_n, x, z, t) = 0$ for all $z \in X$ and $t > 0$. It is denoted by $\lim_{n \to \infty} x_n = x$ or $x_n \to x$.
(2) $(x_n) \subseteq X$ is called a Cauchy sequence if $\lim_{n,m \to \infty} \mu(x_n, x_m, z, t) = 1$ and $\lim_{n,m \to \infty} \nu(x_n, x_m, z, t) = 0$ for all $z \in X$ and $t > 0$.

(3) If every Cauchy sequence in if2ms $(X,\mu,\nu,*,\diamond)$ is convergent, then this space is called complete.

Note 2.7. [1] From now, we will assume that $(X,\mu,\nu,*,\diamond)$ is an if2ms with the condition (IFM12) $\lim_{t\to\infty}\mu(x,y,z,t)=1$ and $\lim_{t\to\infty}\nu(x,y,z,t)=0$ for all $x,y,z\in X$.

Lemma 2.1. If (x_n) is a sequence in a given if2ms $(X,\mu,\nu,*,\diamond)$, then the following inequalities hold for all $z\in X, t>0$ and $p>0$:

$$\mu(x_n,x_{n+p},z,t)\geq\mu\left(x_n,x_{n+1},x_{n+p},\frac{t}{2(p-1)+1}\right)*\mu\left(x_{n+1},x_{n+2},x_{n+p},\frac{t}{2(p-1)+1}\right)$$
$$*\dots*\mu\left(x_{n+p-2},x_{n+p-1},x_{n+p},\frac{t}{2(p-1)+1}\right)*\mu\left(x_n,x_{n+1},z,\frac{t}{2(p-1)+1}\right)$$
$$*\dots*\ \mu\left(x_{n+p-1},x_{n+p},z,\frac{t}{2(p-1)+1}\right)*\mu\left(x_{n+p-1},x_{n+p},z,\frac{t}{2(p-1)+1}\right)\text{ and}$$
$$\nu(x_n,x_{n+p},z,t)\leq\nu\left(x_n,x_{n+1},x_{n+p},\frac{t}{2(p-1)+1}\right)\diamond\nu\left(x_{n+1},x_{n+2},x_{n+p},\frac{t}{2(p-1)+1}\right)$$
$$\diamond\dots\diamond\nu\left(x_{n+p-2},x_{n+p-1},x_{n+p},\frac{t}{2(p-1)+1}\right)\diamond\nu\left(x_n,x_{n+1},z,\frac{t}{2(p-1)+1}\right)$$
$$\diamond\dots\diamond\nu\left(x_{n+p-1},x_{n+p},z,\frac{t}{2(p-1)+1}\right)\diamond\nu\left(x_{n+p-1},x_{n+p},z,\frac{t}{2(p-1)+1}\right)$$

3 Fixed Point Results

Definition 3.1. [3] Let $(X,\mu,\nu,*,\diamond)$ be an if2ms and T be a self-mapping on $(X,\mu,\nu,*,\diamond)$. If there is $k\in(0,1)$ such that

$$\mu(Tx,Ty,z,kt)\geq\mu(x,y,z,t)\ and\ \nu(Tx,Ty,z,kt)\leq\ \nu(x,y,z,t)$$

for all $x,y,z\in X, t>0$, then T is called a contractive mapping. (k is called the fuzzy contractive constant of T.)

Theorem 3.1. [3] Let $(X,\mu,\nu,*,\diamond)$ be a complete if2ms and T be a self-mapping on $(X,\mu,\nu,*,\diamond)$. If T is a contractive mapping, then T has a unique fixed point in X.

Definition 3.2 Let $(X,\mu,\nu,*,\diamond)$ be an if2ms and T be a self-mapping on $(X,\mu,\nu,*,\diamond)$.

(i) T is said to be continuous at $x_0\in X$ if for all $x_n\to x_0$ implies $Tx_n\to Tx_0$.

(ii) Let $\delta>0$ and $0<\lambda<1$. T is called (δ,λ)-uniform locally contractive if

$$\mu(x,y,z,t)>1-\delta\ \Rightarrow\ \mu(Tx,Ty,z,t)\geq\mu\left(x,y,z,\frac{t}{\lambda}\right)$$
$$\nu(x,y,z,t)<\delta\Rightarrow\ \nu(Tx,Ty,z,t)\leq\ \nu\left(x,y,z,\frac{t}{\lambda}\right)$$

for all $x,y,z\in X$ and $t>0$. Clearly, every (δ,λ)-uniform locally contractive mapping T is continuous.

Example 3.3. Let $(X,\mu,\nu,*,\diamond)$ be an if2ms and $T:(X,\mu,\nu,*,\diamond)\to(X,\mu,\nu,*,\diamond)$ be given by $Tx=c$ for all $x\in X$ (c is a constant). Let $\delta>0$ and $0<\lambda<1$. Assume that $\mu(x,y,z,t)>1-\delta$ and $\nu(x,y,z,t)<\delta$ for all $x,y,z\in X$ and $t>0$. Therefore, we have that $1=\mu(Tx,Ty,z,t)\geq\mu\left(x,y,z,\frac{t}{\lambda}\right)$ and $0=\nu(Tx,Ty,z,t)\leq\nu\left(x,y,z,\frac{t}{\lambda}\right)$ for all $x,y,z\in X$ and $t>0$. Hence, T is a (δ,λ)-uniform locally contractive mapping.

Remark 3.4. In an if2ms $(X,\mu,\nu,*,\diamond)$, a contractive mapping can be considered as a $(1,\lambda)$-uniform locally contractive mapping.

Definition 3.5. An if2ms $(X,\mu,\nu,*,\diamond)$ is said to be metrically convex if for each $x,y,z\in X$, there is a $u\neq x,y,z$ for which $\mu(x,y,z,t)=\mu(x,u,z,t_0)*\mu(u,y,z,t_1)$ and $\nu(x,y,z,t)=\nu(x,u,z,t_0)\diamond\nu(u,y,z,t_1)$ where $t=t_0+t_1$ for all $t_0,t_1>0$.

Theorem 3.2. Let $(X,\mu,\nu,*,\diamond)$ be a metrically convex if2ms. If a self-mapping T on $(X,\mu,\nu,*,\diamond)$ is (δ,λ)-uniform locally contractive, then T is a contractive mapping with the fuzzy contractive constant λ.

Proof. Let $x,y,z\in X$. Since $(X,\mu,\nu,*,\diamond)$ is metrically convex, there are points $x=x_0,x_1,x_2,\dots,x_{n-1},x_n=y$ and $t_0,t_1,\dots,t_n>0$ such that

$\mu(x,y,z,t)=\mu(x_0,x_1,z,t_0)*\mu(x_1,x_2,z,t_1)*\dots*\mu(x_{n-1},x_n,z,t_n)$ and

$\nu(x,y,z,t) = \nu(x_0,x_1,z,t_0) \diamond \nu(x_1,x_2,z,t_1) \diamond \ldots \diamond \nu(x_{n-1},x_n,z,t_n)$
where $t = t_0 + t_1 + \cdots + t_n$, $\mu(x_{i-1},x_i,z,t) > 1-\delta$ and $\nu(x_{i-1},x_i,z,t) < \delta$ for $i = 1,2,\ldots,n$. Also,
$\mu(Tx,Ty,z,t) = \mu(Tx_0,Tx_1,z,t_0) * \mu(Tx_1,Tx_2,z,t_1) * \ldots * \mu(Tx_{n-1},Tx_n,z,t_n)$ and
$\nu(Tx,Ty,z,t) = \nu(Tx_0,Tx_1,z,t_0) \diamond \nu(Tx_1,Tx_2,z,t_1) \diamond \ldots \diamond \nu(Tx_{n-1},Tx_n,z,t_n)$.
As T is (δ,λ)-uniform locally contractive, we have $\mu(Tx_{i-1},Tx_i,z,t_{i-1}) \geq \mu\left(x_{i-1},x_i,z,\frac{t_{i-1}}{\lambda}\right)$ and $\nu(Tx_{i-1},Tx_i,z,t_{i-1}) \leq \nu\left(x_{i-1},x_i,z,\frac{t_{i-1}}{\lambda}\right)$ for $i = 1,2,\ldots,n$. Hence, we have
$\mu(Tx,Ty,z,t) = \mu(Tx_0,Tx_1,z,t_0) * \mu(Tx_1,Tx_2,z,t_1) * \ldots * \mu(Tx_{n-1},Tx_n,z,t_n)$
$\geq \mu\left(x_0,x_1,z,\frac{t_0}{\lambda}\right) * \mu\left(x_1,x_2,z,\frac{t_1}{\lambda}\right) * \ldots * \mu\left(x_{n-1},x_n,z,\frac{t_n}{\lambda}\right) = \mu\left(x,y,z,\frac{t}{\lambda}\right)$ and
$\nu(Tx,Ty,z,t) = \nu(Tx_0,Tx_1,z,t_0) \diamond \nu(Tx_1,Tx_2,z,t_1) \diamond \ldots \diamond \nu(Tx_{n-1},Tx_n,z,t_n)$
$\leq \nu\left(x_0,x_1,z,\frac{t_0}{\lambda}\right) \diamond \nu\left(x_1,x_2,z,\frac{t_1}{\lambda}\right) \diamond \ldots \diamond \nu\left(x_{n-1},x_n,z,\frac{t_n}{\lambda}\right) = \nu\left(x,y,z,\frac{t}{\lambda}\right)$.
So T is a contractive mapping.

Definition 3.6. Let $(X,\mu,\nu,*,\diamond)$ be an if2ms and $\delta > 0$. A finite sequence $x = x_0,x_1,\ldots,x_{n-1},x_n = y$ is called a $\delta-$chain from x to y if $\mu(x_{i-1},x_i,z,t) > 1-\delta$ and $\nu(x_{i-1},x_i,z,t) < \delta$ for all $z \in X$, $t > 0$ and $i = 1,2,\ldots,n$. An if2ms $(X,\mu,\nu,*,\diamond)$ is called δ-chainable if for every $x,y \in X$ there is a δ-chain from x to y.

Theorem 3.3. Let $(X,\mu,\nu,*,\diamond)$ be a complete and δ-chainable if2ms. If a self mapping T on $(X,\mu,\nu,*,\diamond)$ is a (δ,λ)-uniform locally contractive, then T has a unique fixed point in X.

Proof Let $x \in X$ and $Tx \neq x$ (otherwise x is a fixed point of T). Since $(X,\mu,\nu,*,\diamond)$ is δ-chainable, there is a $\delta-$chain $x = x_0,x_1,\ldots,x_{n-1},x_n = Tx$ from x to Tx. From here, we have $\mu(x_{i-1},x_i,z,t) > 1-\delta$ and $\nu(x_{i-1},x_i,z,t) < \delta$ for all $z \in X$, $t > 0$ and $i = 1,2,\ldots,n$. Applying induction method, we obtain

$$\mu(T^m x_{i-1},T^m x_i,z,t) \geq \mu\left(x_{i-1},x_i,z,\frac{t}{\lambda^m}\right) \quad (3.1)$$
$$\nu(T^m x_{i-1},T^m x_i,z,t) \leq \nu\left(x_{i-1},x_i,z,\frac{t}{\lambda^m}\right) \quad (3.2)$$

for all $t > 0, z \in X, m \in \mathbb{N}$, $i = 1,2,\ldots,n$. Now, we have

$$1 \geq \mu(T^m x,T^{m+1}x,z,t) = \mu(T^m x_0,T^m x_n,z,t)$$
$$0 \leq \nu(T^m x,T^{m+1}x,z,t) = \nu(T^m x_0,T^m x_n,z,t)$$

for all $t > 0, z \in X, m \in \mathbb{N}, i = 1,2,\ldots,n$. Here, using the Lemma 2.1., we get that $\{T^m x\}$ is a Cauchy sequence in X. As $(X,\mu,\nu,*,\diamond)$ is complete, there is point $y \in X$ such that $\lim_{n\to\infty} T^m x = y$. Since T is continuous, we have $\lim_{n\to\infty} T^{m+1}x = Ty$. Hence $Ty = y$ and y is a fixed point of T. To show uniqueness, assume $Tw = w$ for some $w \in X$ $(w \neq y)$. Since $(X,\mu,\nu,*,\diamond)$ is δ-chainable, there is a $\delta-$chain $y = w_0,w_1,\ldots,w_{n-1},w_k = w$ from y to w. Now, for any $h \in \mathbb{N}$, we have

$$1 \geq \mu(y,w,z,t) = \mu(T^h y,T^h w,z,t) = \mu(T^h w_0,T^h w_k,z,t)$$
$$0 \leq \nu(y,w,z,t) = \nu(T^h y,T^h w,z,t) = \nu(T^h w_0,T^h w_k,z,t)$$

for all $t > 0$ $z \in X$, $i = 1,2,\ldots,k$. Using Lemma 2.1., equations (3.1) and (3.2), we have $\mu(y,w,z,t) = 1$ and $\nu(y,w,z,t) = 0$ for all $t > 0, z \in X$. So, we obtain that $y = w$.

Definition 3.7. [13] Let $(X,\mu,\nu,*,\diamond)$ be an if2ms and A,B be two self-mappings on $(X,\mu,\nu,*,\diamond)$. A pair (A,B) is said to be weak compatible if $Ax = Bx$ for some $x \in X$ implies $ABx = BAx$.

Theorem 3.4. Let $(X,\mu,\nu,*,\diamond)$ be a complete, δ-chainable if2ms and the self-mappings A,B,S,T on $(X,\mu,\nu,*,\diamond)$ satisfy the following conditions:
(i) $AX \subseteq TX$ and $BX \subseteq SX$.
(ii) $\exists k \in (0,1)$ such that
$\mu(Ax,By,z,kt) \geq \mu(Sx,Ty,z,t) * \mu(Ax,Sx,z,t) * \mu(By,Ty,z,t) * \mu(Ax,Ty,z,t)$ and
$\nu(Ax,By,z,kt) \leq \nu(Sx,Ty,z,t) \diamond \nu(Ax,Sx,z,t) \diamond \nu(By,Ty,z,t) \diamond \nu(Ax,Ty,z,t)$

for all $x, y, z \in X$ and $t > 0$.
(iii) The pairs (A, S) and (B, T) are weak compatible.
(iv) A and S are continuous.
Then A, B, S and T have a unique common fixed point in X.
Corollary 3.1. Let $(X, \mu, \nu, *, \diamond)$ be a complete, δ-chainable if2ms and the self-mappings A, B, S, T on $(X, \mu, \nu, *, \diamond)$ satisfy the following conditions:
(i) $AX \subseteq TX$ and $BX \subseteq SX$.
(ii) $\exists k \in (0,1)$ such that $\mu(Ax, By, z, kt) \geq \mu(Sx, Ty, z, t)$ and $\nu(Ax, By, z, kt) \leq \nu(Sx, Ty, z, t)$ for all $x, y, z \in X$ and $t > 0$.
Then A, B, S and T have a unique common fixed point in X.
If we take $S, T = I$ in the above corollary, we obtain the following result:
Corollary 3.2. Let $(X, \mu, \nu, *, \diamond)$ be a complete, δ-chainable if2ms and the self-mappings A, B on $(X, \mu, \nu, *, \diamond)$ satisfy the following conditions: $\exists k \in (0,1)$ such that

$$\mu(Ax, By, z, kt) \geq \mu(x, y, z, t)$$
$$\nu(Ax, By, z, kt) \leq \nu(x, y, z, t)$$

for all $x, y, z \in X$ and $t > 0$. Then A and B have a unique common fixed point in X.
If we take $A = B$ in the above corollary, we get the following result:
Corollary 3.3. Let $(X, \mu, \nu, *, \diamond)$ be a complete, δ-chainable if2ms and the self-mapping A on $(X, \mu, \nu, *, \diamond)$ satisfies the following conditions: $\exists k \in (0,1)$ such that

$$\mu(Ax, Ay, z, kt) \geq \mu(x, y, z, t)$$
$$\nu(Ax, Ay, z, kt) \leq \nu(x, y, z, t)$$

for all $x, y, z \in X$ and $t > 0$. Then A has a unique fixed point in X. Hence, we obtain the result stated in Theorem 3.1.

4 Conclusion

In this paper, we established some theorems related to the fixed point theory using some contractive mappings in a complete δ-chainable intuitionistic fuzzy 2-metric space. We obtained some generalizations of previous fixed point theorems in this spaces[4] and stated the relations of these theorems. Further, we investigated some theorems by using weak compatible mappings. In the future scope, we hope that some applications of these results can be founded for the real life problems.

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