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Reproducing kernel functions for the generalized Kuramoto-Sivashinsky equation

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Abstract. Reproducing kernel functions are obtained for the solution of generalized Kuramoto–Sivashinsky (GKS) equation in this paper. These reproducing kernel functions are valuable in the reproducing kernel Hilbert space method. They will be useful for interested researchers.

1 Introduction

We consider the following problem[1]:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \alpha\frac{\partial^2 v}{\partial x^2} + \beta\frac{\partial^3 v}{\partial x^3} + \gamma\frac{\partial^4 v}{\partial x^4} = 0, \tag{1}$$

where α , β and γ are nonzero [2]. This equation has been obtained in the context of plasma instabilities, flame front propagation, and phase turbulence in reaction–diffusion system [3]. We obtain reproducing kernel functions to get approximate solutions of the following problem:

Example In this example, we consider the Kuramoto–Sivashinsky equation [5]

$$v_t + vv_x + v_{xx} + v_{xxxx} = 0,$$

with the initial condition [5]

$$v(x,0) = exp(-x^2),$$

and the boundary conditions

$$v(a,t) = 0, \quad v(b,t) = 0, \quad v_x(a,t) = 0, \quad v_x(b,t) = 0.$$

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2 Reproducing kernel functions

We define $V_2^5[0,1]$ as

$$V_2^5[0,1] = \left\{ \begin{array}{ll} v, v', v'', v''', v^{(4)} & \text{are absolutely continuous functions,} \\ v^{(5)} \in L^2[0,1], & v(a) = v'(a) = v(b) = v'(b) = 0. \end{array} \right\}$$

We give

$$\begin{aligned} \langle v, S_z \rangle_{V_2^5[a,b]} &= v(a)S_z(a) + v'(a)S_z'(a) + v''(a)S_z''(a) + v'''(a)S_z'''(a) \\ &+ v^{(4)}(a)S_z^{(4)}(a) + \int_a^b v^{(5)}(t)S_z^{(5)}(t)d(t). \end{aligned}$$

We obtain

$$\begin{aligned} \langle v, S_z \rangle_{V_2^5[a,b]} &= v(a)S_z(a) + v'(a)S_z'(a) + v''(a)S_z''(a) + u'''(a)S_z'''(a) \\ &+ v^{(4)}(a)S_z^{(4)}(a) + v^{(4)}(b)S_z^{(5)}(b) - v^{(4)}(a)S_z^{(5)}(a) \\ &- v^{(3)}(b)S_z^{(6)}(b) + v^{(3)}(a)S_z^{(6)}(a) \\ &+ v''(b)S_z^{(7)}(b) - v''(a)S_z^{(7)}(a) - v'(b)S_z^{(8)}(b) \\ &+ v'(a)S_z^{(8)}(a) + v(b)S_z^{(9)}(b) - v(a)S_z^{(9)}(a) \\ &- \int_a^b v(t)S_z^{(10)}(t)d(t). \end{aligned}$$

We have

$$\begin{aligned} S_z(a) &= 0 \\ S_z(b) &= 0 \\ S_z'(a) &= 0 \\ S_z'(b) &= 0 \end{aligned}$$

by boundary conditions. Therefore, we obtain

$$\begin{aligned} \langle v, S_z \rangle_{V_2^5[a,b]} &= v''(a)S_z''(a) + v'''(a)S_z'''(a) + v^{(4)}(a)S_z^{(4)}(a) \\ &+ v^{(4)}(b)S_z^{(5)}(b) - v^{(4)}(a)S_z^{(5)}(a) - v^{(3)}(b)S_z^{(6)}(b) + v^{(3)}(a)S_z^{(6)}(a) \\ &+ v''(b)S_z^{(7)}(b) - v''(a)S_z^{(7)}(a) - \int_a^b v(t)S_z^{(10)}(t)d(t). \end{aligned}$$

We have

$$\begin{aligned} S_z^{''}(a) - S_z^{(7)}(a) &= 0 \\ S_z^{'''}(a) + S_z^{(6)}(a) &= 0 \\ S_z^{(4)}(a) - S_z^{(5)}(a) &= 0 \\ S_z^{(5)}(b) &= 0 \\ S_z^{(6)}(b) &= 0 \\ S_z^{(7)}(b) &= 0 \end{aligned}$$

Therefore, we obtain

$$\langle v, S_z \rangle_{V_2^5[a,b]} = -\int_a^b v(t) S_z^{(10)}(t) d(t) = v(z).$$

Thus, we get

$$S_z^{(10)}(t) = -\delta(t - z).$$

When $t \neq z$

$$S_z^{(10)}(z) = 0.$$

Then, we acquire

$$S_z(t) = \begin{cases} \sum_{i=1}^{10} c_i(z) t^{i-1} & , \quad t \le z, \\ \sum_{i=1}^{10} d_i(z) t^{i-1} & , \quad t > z. \end{cases}$$

3 Conclusion

In this paper, we obtained very useful reproducing kernel function for solving the generalized Kuramoto–Sivashinsky equation. This reproducing kernel function is very useful to apply reproducing kernel Hilbert space method.

References

[1] A.H. Khater, R.S. Temsah, Numerical solutions of the generalized Kuramoto–Sivashinsky equation by Chebyshev spectral collocation methods, Comput. Math. Appl. 56 1465-1472 (2008).

[2] Y. Kuramoto, T. Tsuzuki, Persistent propagation of concentration waves in dissipative media far from thermal equilibrium, Prog. Theor. Phys. 55 356-369 (1976).

[3] J. Rademacher J, R. Wattenberg, Viscous shocks in the destabilized Kuramoto–Sivashinsky, J. Comput. Nonlinear Dynam. 1 336-347 (2006).

[4] R. Conte R, Exact solutions of nonlinear partial differential equations by singularity analysis, in: Lecture Notes in Physics, Springer, Berlin, 2003, pp. 1-83.

[5] M. Lakestani, M. Dehghan, Numerical solutions of the generalized Kuramoto-Sivashinsky equation using B-spline functions, Applied Mathematical Modelling 36 605-617 (2012).

[6] F. Genga, F. Shenb, Solving a Volterra integral equation with weakly singular kernel in the reproducing kernel space, Mathematical Sciences, Vol. 4, No. 2 159-170 (2010).