Some New Characterizations Of Parallel Translation Surface According To Bishop Frame With Timelike M_1 In Minkowski 3-Space

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Abstract. In this paper we consider parallel translation surfaces, which are generated by spacelike curves, according to Bishop frame with timelike M_1 in Minkowski 3- space. Then, we obtain some characterizations of these surface.

1 Introduction

Creation of paralel surfaces is useful in design and manufacture. A surface M^r is parallel to M if points of M^r are at a constant distance along the normal from the surface M. So, there are infinite numbers of parallel surfaces, [8].

Kızıltuğ and Yaylı studied timelike curves on timelike parallel surfaces in Minkowski 3-spaces E_1^3. Then, they gave some characterization for its image curve in, [3].

Unluturk and Ekici obtain paralel surfaces of timelike ruled surfaces which are developable are timelike ruled Weingarten surfaces. Then, some properties of that kind paralel surfaces are obtained in Minkowski 3-space in [7].

Safiulina gave a complete classification of the existence and geometry of such two-dimensional Riemannian submanifolds (surfaces). Moreover, it is shown that in E_s^n with s>0 do exist not totally geodesic minimal semiparallel space-like surfaces, [6].

In [1], Calvaruso and Van der Veken completely classified surfaces with parallel second fundamental form in all non-symmetric homogeneous Lorentzian three manifolds. In [2], Fukui and Hasegawa investigate singularities of all parallel surfaces to a given regular surface.

2 Fundamental Properties of the Method

Let $\alpha: I \to E_1^3$ be a non geodesic spacelike curve on the E_1^3 parameterized by arc length. Let {T,N,B} be the Frenet frame. Then, for spacelike curves we have the following Frenet formulas:

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$$\frac{d}{ds}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{N}(s)\\ \mathbf{B}(s)\end{bmatrix}=\begin{bmatrix}0 & \kappa(s) & 0\\ -\varepsilon\kappa(s) & 0 & \tau(s)\\ 0 & \tau(s) & 0\end{bmatrix}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{N}(s)\\ \mathbf{B}(s)\end{bmatrix}, \quad (1)$$

where $\kappa\neq0$ and $\tau\neq0$ and

$g(T,T) = 1, g(N,N)=\varepsilon=\pm1, g(B,B)=1.$

The Bishop frame is an option when the spacelike curve has a vanishing second derivative. The Bishop frame is

$$\frac{d}{ds}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{M}_1(s)\\ \mathbf{M}_2(s)\end{bmatrix}=\begin{bmatrix}0 & k_1(s) & -k_2(s)\\ k_1(s) & 0 & 0\\ k_2(s) & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{M}_1(s)\\ \mathbf{M}_2(s)\end{bmatrix}, \quad (2)$$

where

$g(T,T) = -g(M_1, M_1)=g(M_2, M_2)=1.$

For timelike curves, we have the following Frenet formulas:

$$\frac{d}{ds}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{N}(s)\\ \mathbf{B}(s)\end{bmatrix}=\begin{bmatrix}0 & \kappa(s) & 0\\ \kappa(s) & 0 & \tau(s)\\ 0 & -\tau(s) & 0\end{bmatrix}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{N}(s)\\ \mathbf{B}(s)\end{bmatrix} \quad (3)$$

Then, the Bishop frame is expressed as

$$\frac{d}{ds}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{M}_1(s)\\ \mathbf{M}_2(s)\end{bmatrix}=\begin{bmatrix}0 & k_1(s) & k_2(s)\\ k_1(s) & 0 & 0\\ k_2(s) & 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{T}(s)\\ \mathbf{M}_1(s)\\ \mathbf{M}_2(s)\end{bmatrix} \quad (4)$$

Here, the set $\{T, M_1, M_2\}$ is called Bishop trihedra, k_1 and k_2 as Bishop curvatures, $\tau(s)=\theta'(s)$ and $\kappa(s)=\sqrt{(|k_2^2-k_1^2|)}$.

3 Parallel Translation Surfaces in E_1^3

Let $\alpha(u)$ and $\beta(v)$ be spacelike curves and $\theta(u,v)$ be a translation surface which is parametrized as

$$\theta(u,v)=\alpha(u)+\beta(v). \quad (5)$$

Then, parallel surfaces of the translation surface is

$$\phi(u,v)=\theta(u,v)+aU(u,v) \quad (6)$$

where a is a constant and $U(u,v)$ is unit normal vector field of the translation surface $\theta(u,v)$. So, it parameterized as

$$\phi(u,v)=\alpha(u)+\beta(v)+\frac{a}{\sin\alpha}(T_\alpha\times T_\beta) \quad (7)$$

where α is the constant angle between unit tangent vector field of curves T_α and T_β.

Theorem 1. Let $\alpha(u)$ and $\beta(v)$ be spacelike curves where M_1^α and M_1^β are timelike vector fields in Minkowski three space. Then if ϕ(u,v) is minimal,

$$E(-a^2(k_1^{\beta 2}-k_2^{\beta 2})^2+k_1^{\beta}<M_1^{\beta},U>-k_2^{\beta}<M_2^{\beta},U>+\frac{ak_1^{\beta}{}'}{\sin\alpha}<W,U>-\frac{ak_2^{\beta}{}'}{\sin\alpha}<X,U>)$$
$$-2F(\frac{ak_1^{\beta}}{\sin\alpha}(k_1^{\beta}<Q,U>-k_2^{\beta}<P,U>)+\frac{ak_2^{\alpha}}{\sin\alpha}(k_1^{\beta}<R,U>-k_2^{\beta}<S,U>) \quad (8)$$
$$+G(-a^2(k_1^{\alpha 2}-k_2^{\alpha 2})^2+k_1^{\alpha}<M_1^{\alpha},U>-k_2^{\alpha}<M_2^{\alpha},U>$$
$$+\frac{ak_1^{\alpha}{}'}{\sin\alpha}<W,U>-\frac{ak_2^{\alpha}{}'}{\sin\alpha}<X,U>)=0.$$

Proof. If we take derivatives of the parallel surface of the translation surface according to u and v, then from bishop equations, we have

$$\phi_u(u,v)=T_{\alpha}+\frac{ak_1^{\alpha}}{\sin\alpha}(M_1^{\alpha}\times T_{\beta})-\frac{ak_2^{\alpha}}{\sin\alpha}(M_2^{\alpha}\times T_{\beta})\ . \quad (9)$$

Similarly,

$$\phi_v(u,v)=T_{\beta}+\frac{ak_1^{\beta}}{\sin\alpha}(T_{\alpha}\times M_1^{\beta})-\frac{ak_2^{\beta}}{\sin\alpha}(T_{\alpha}\times M_2^{\beta}) \quad (10)$$

The compenents of the first fundamental form of ϕ(u,v) are

$$E=1-ak_1^{\alpha}<U,M_1^{\alpha}>+ak_2^{\alpha}<U,M_2^{\alpha}>+\frac{1}{\sin^2\alpha}(ak_1^{\alpha 2}(\left\|M_1^{\alpha}\right\|^2-<M_1^{\alpha},T_{\beta}>)$$
$$+ak_2^{\alpha 2}(\left\|M_2^{\alpha}\right\|^2-<M_2^{\alpha},T_{\beta}>)), \quad (11)$$

$$F=\cos\alpha+\frac{a^2k_1^{\alpha}k_1^{\beta}}{\sin^2\alpha}(<M_1^{\alpha},T_{\alpha}><M_1^{\beta},T_{\beta}>-\sin\alpha<M_1^{\alpha},M_1^{\beta}>)$$
$$-\frac{a^2k_1^{\alpha}k_2^{\beta}}{\sin^2\alpha}(<M_1^{\alpha},T_{\alpha}><M_2^{\beta},T_{\beta}>-\sin\alpha<M_1^{\alpha},M_2^{\beta}>)$$
$$-\frac{a^2k_2^{\alpha}k_1^{\beta}}{\sin^2\alpha}(<M_2^{\alpha},T_{\alpha}><M_1^{\beta},T_{\beta}>-\sin\alpha<M_2^{\alpha},M_1^{\beta}>) \quad (12)$$
$$-\frac{a^2k_2^{\alpha}k_2^{\beta}}{\sin^2\alpha}(<M_2^{\alpha},T_{\alpha}><M_2^{\beta},T_{\beta}>-\sin\alpha<M_2^{\alpha},M_2^{\beta}>)$$

$$G=1-ak_1^{\beta}<U,M_1^{\beta}>+ak_2^{\beta}<U,M_2^{\beta}>+\frac{1}{\sin^2\alpha}(ak_1^{\beta 2}(\left\|M_1^{\beta}\right\|^2-<M_1^{\beta},T_{\alpha}>) \quad (13)$$
$$+ak_2^{\beta 2}(\left\|M_2^{\beta}\right\|^2-<M_2^{\beta},T_{\alpha}>)).$$

The unit vector field of the parallel translation surface is

$$U=\frac{1}{\sqrt{EG}\sin\gamma}(\phi_u\times\phi_v) \quad (14)$$

where γ is the angle between vector fields ϕ_u and ϕ_v.

Then, components of the second fundamental form of the $\phi(u,v)$ are

$$h_{11} = -a^2(k_1^{\alpha 2} - k_2^{\alpha 2})^2 + k_1^{\alpha} < U, M_1^{\alpha} > - k_2^{\alpha} < U, M_2^{\alpha} >$$
$$+ \frac{ak_1^{\alpha}{}'}{\sin\alpha} < U, W > - \frac{ak_2^{\alpha}{}'}{\sin\alpha} < U, X >, \quad (15)$$

$$h_{12} = \frac{ak_1^{\alpha}}{\sin\alpha}(k_1^{\beta} < Q, U > - k_2^{\beta} < P, U >) + \frac{ak_2^{\alpha}}{\sin\alpha}(k_1^{\beta} < R, U > - k_2^{\beta} < S, U >), \quad (16)$$

$$h_{22} = -a^2(k_1^{\beta 2} - k_2^{\beta 2})^2 + k_1^{\beta} < U, M_1^{\beta} > - k_2^{\beta} < U, M_2^{\beta} >$$
$$+ \frac{ak_1^{\beta}{}'}{\sin\alpha} < U, W > - \frac{ak_2^{\beta}{}'}{\sin\alpha} < U, X >, \quad (17)$$

Where

$$M_1{}^{\alpha} \times T_{\beta} = W,\ M_2{}^{\alpha} \times T_{\beta} = X,\ T_{\alpha} \times M_1{}^{\beta} = Y,\ T_{\alpha} \times M_2{}^{\beta} = Z,\ M_1{}^{\alpha} \times M_1{}^{\beta} = Q,$$
$$M_1{}^{\alpha} \times M_2{}^{\beta} = P,\ M_2{}^{\alpha} \times M_1{}^{\beta} = R,\ M_2{}^{\alpha} \times M_2{}^{\beta} = S. \quad (18)$$

Then the proof is complete.

Corollary 1. Let α(u) be timelike and β(v) be spacelike curves where M_1^{β} is timelike vector field . Then if parallel surfaces of the translation surface $\tilde{\phi}(u,v)$ is maximal,

$$\tilde{E}(a^2(k_1^{\beta 2} - k_2^{\beta 2})^2 + k_1^{\beta} < M_1^{\beta}, \tilde{U} > - k_2^{\beta} < M_2^{\beta}, \tilde{U} > + \frac{ak_1^{\beta}{}'}{\cosh\alpha} < W, \tilde{U} > - \frac{ak_2^{\beta}{}'}{\cosh\alpha} < X, \tilde{U} >)$$
$$-2\tilde{F}(\frac{ak_1^{\alpha}}{\sin\alpha}(k_1^{\beta} < Q, \tilde{U} > - k_2^{\beta} < P, \tilde{U} >) + \frac{ak_2^{\alpha}}{\cosh\alpha}(k_1^{\beta} < R, \tilde{U} > - k_2^{\beta} < S, \tilde{U} >) \quad (19)$$
$$+\tilde{G}(a^2(k_1^{\alpha 2} - k_2^{\alpha 2})^2 + k_1^{\alpha} < M_1^{\alpha}, \tilde{U} > - k_2^{\alpha} < M_2^{\alpha}, \tilde{U} >$$
$$+ \frac{ak_1^{\alpha}{}'}{\cosh\alpha} < W, \tilde{U} > - \frac{ak_2^{\alpha}{}'}{\cosh\alpha} < X, \ddot{U} >) = 0$$

where $\tilde{E}$, $\tilde{F}$ and $\tilde{G}$ are components of the first fundamnetal form.

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