On the wave solutions to the TRLW equation

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Abstract. In this study, a nonlinear model is investigated, namely; the time regularized long wave equation. Various solitary wave solutions are constructed such as the non-topological, compound topological-non-topological bell-type, singular and compound singular soliton solutions. Under the choice of suitable parameters values, the 2D and 3D graphs to all the obtained solutions are plotted. The reported results in this study may be helpful in explaining the physical meanings of some important nonlinear models arising in the field of nonlinear science.

1Introduction

For the past two decades, the investigation of the solutions to the various kind of nonlinear evolution equations (NLEEs) has attracted the attention of many researchers. Nonlinear evolution equations are often used to model various nonlinear complex physical aspects that arise in the various fields on nonlinear physical sciences, especially in physics plasma, biology and fluid mechanics and so on. Various analytical techniques have been used to explore search of several NLEEs [1–11].

However, in the present study, the extended sinh-Gordon equation expansion method [12-15] (ShGEEM) will be used to investigate the wave solutions to the time regularized long wave (TRLW) equation [16]. The time regularized long wave is given by

$$u_t + u_x + \beta u u_x + u_{xtt} = 0, \tag{1.1}$$

where$u(x,t)$ is the unknown function of x; the spatial coordinate and the time t, respectively, and β is a nonzero constant.

3Application

In this section, the application of the extended sinh-Gordon equation expansion method to the TRLW is presented.

Consider the travelling wave transformation

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$$u = \Psi(\eta),\ \eta = \mu(x - vt). \tag{2.1}$$

Inserting Eq. (2.1) into Eq. (1.1), yields

$$2\mu^2 v^2 \Psi'' + 2(1 - v)\Psi + \beta\Psi^2 = 0. \tag{2.2}$$

By the extended ShGEEM [12], the solutions of any given nonlinear partial differential equation are assumed to be of the forms

$$\Psi(w) = \sum_{j=1}^{m}\left[B_j \sinh(\theta) + A_j \cosh(w)\right]\cosh^{j-1}(w) + A_0, \tag{2.3}$$

$$\Psi(\eta) = \sum_{j=1}^{m}\left[\pm iB_j \operatorname{sech}(\eta) \pm A_j \tanh(\eta)\right]\tanh^{j-1}(\eta) + A_0, \tag{2.4}$$

$$\Psi(\eta) = \sum_{j=1}^{m}\left[\pm B_j cs\operatorname{ch}(\eta) \pm A_j \coth(\eta)\right]\coth^{j-1}(\eta) + A_0, \tag{2.5}$$

where $i = \sqrt{-1}$ and $w' = \sinh(w)$ [5]

Balancing the terms Ψ'' and Ψ^2, yields $m = 2$.

With m = 2, Eqs. (2.3), (2.4) and (2.5) take the forms

$$\Psi(w) = B_1 \sinh(w) + A_1 \cosh(w) + B_2 \cosh(w)\sinh(w) + \mathrm{A}_2 \cosh^2(w) + A_0 \tag{2.6}$$

Inserting Eq. (2.6) and its second derivative into Eq. (2.2), gives a polynomial in powers of hyperbolic functions. Summing each coefficients of the hyperbolic functions of the same degree and equating each summation to zero, gives a group of algebraic equations. The group of algebraic equations is then simplified to secure the values of the parameters involved into different cases. For each case, inserting the obtained values of the parameters into each of Eqs. (2.4) and (2.5) with $m =$ 2, yields the solutions to Eq. (1.1).

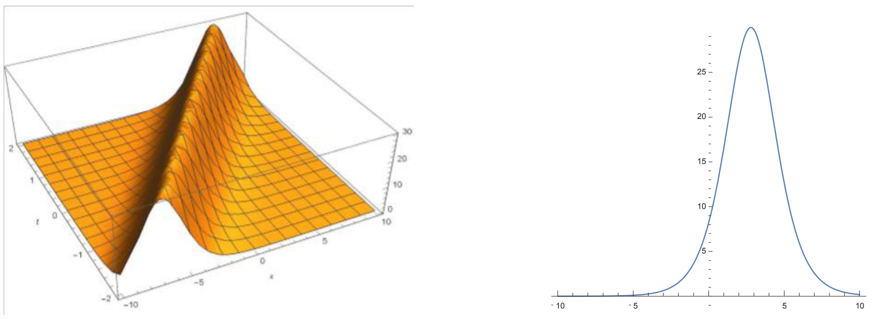
Case-1: When

$$A_0 = \frac{6(-1+v)}{\beta}, A_1 = 0, B_1 = 0, A_2 = \frac{6-6v}{\beta}, B_2 = -\frac{6\sqrt{(-1+v)^2}}{\beta}, \mu = -\frac{\sqrt{-1+v}}{v},$$

we get

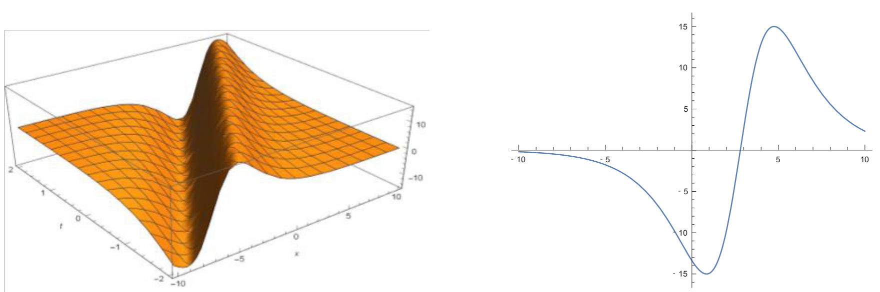
$$u_1(x,t) = \frac{1}{\beta}(6(-1+\nu)\,\mathrm{sech}\left[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}\right]^2 + 6\mathbb{i}(\nu-1)\mathrm{sech}\left[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}\right]\tanh[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}])\quad(2.7)$$

$$u_2(x,t) = -\frac{6\sqrt{(-1+\nu)^2}\coth\left[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}\right]\mathrm{csch}\,[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}]+6(-1+\nu)\,\mathrm{csch}\,[\frac{\sqrt{-1+\nu}(x-t\nu)}{\nu}]^2}{\beta}\quad(2.8)$$



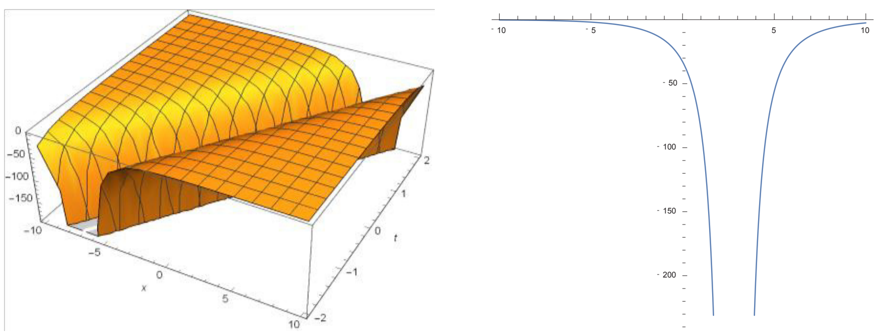
(a) (b)

Figure 1: The 3D and 2D graphs of the real part of Eq. (2.7).



(a) (b)

Figure 2: The 3D and 2D graphs of the imaginary part of Eq. (2.7).



(a) (b)

Figure 3: The 3D and 2D graphs of Eq. (2.8).

Case-2: When

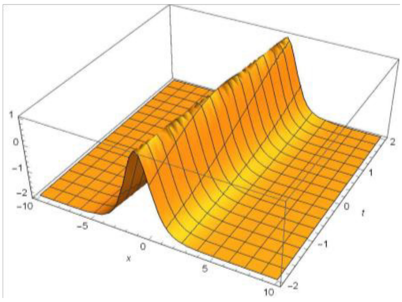
$$A_0 = \frac{1-\nu}{\beta}, A_1 = 0, B_1 = 0, A_2 = \frac{3(-1+\nu)}{\beta}, B_2 = 0, \mu = \frac{\sqrt{-(\nu-1)}}{2\nu},$$

we get

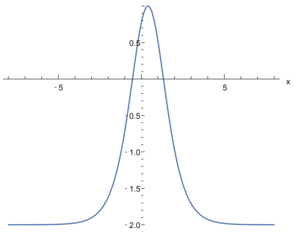
$$u_3(x,t) = \frac{(-1+\nu)(-2+\cosh[\frac{\sqrt{1-\nu}(x-t\nu)}{\nu}])sech^2[\frac{\sqrt{1-\nu}(x-t\nu)}{\nu}]}{\beta} \quad (2.9)$$

And

$$u_4(x,t) = \frac{(-1+\nu)(2+\text{Cosh}[\frac{\sqrt{1-\nu}(x-t\nu)}{\nu}])csch^2[\frac{\sqrt{1-\nu}(x-t\nu)}{\nu}]}{\beta} \quad (2.10)$$

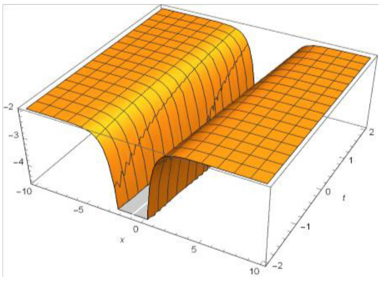


(a)

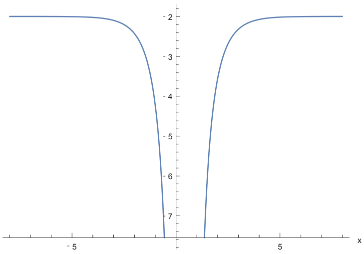


(b)

Figure 3: The 3D and 2D graphs of Eq. (2.9).



(a)



(b)

Figure 4: The 3D and 2D graphs of Eq. (2.10).

3 Conclusion

In this study, the time regularized long wave equation is investigated by using the extended sinh-Gordon equation expansion method with the aid of Wolfram Mathematica 11. Various soliton solutions such as the non-topological, compound topological, non-topological bell-type, singular and compound singular solitons are successfully extracted. Using suitable values of parameters, the 2D and 3D graphics to all the reported solutions are plotted.

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