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Dynamic Reliability Evaluation of Linear Consecutive k-out-of-n: F System With Multi-State Components

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Abstract. In engineering applications, analyzing a technical system vary according to the operating principles of the system. In some situations, the status of the system is a function of stresses which act on the system and cause degradation. In order to efficiently analysis the reliability of a system which operates under stress, assigning the various states to the components depending on their operating performance is very important. In this paper, we have investigated the linear consecutive k-out-of-n: F system and assigned multiple states to its components. Due to the reason, the operating performance of the components can easily be controlled. Apart from that the reliability of the system depending on the states of its components can be calculated at any time interval. In the numerical example, the states of the components and the reliability calculation of the system at specific time intervals are shown clearly.

1 Introduction

A linear consecutive k-out-of-n:F (Lin/Con/k/n:F) system consists of n linearly arranged components such that the system fails if and only if at least k consecutive components fail. The first report for the Lin/Con/k/n:F system was presented by [1].

For reliability analysis and engineering applications, depending on the operating environment of the system, it may be exposed to some stresses and the system may respond to these strength. In this process, which is called the stress-strength model in the literature, the reliability of the system is the probability that the system is strong enough to counteract the stresses applied on it. The stress-strength model remains important since it is a viable model in engineering application, and many studies have been done in the literature on the analysis of systems and components using this model. [2, 3, 4, 5].

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Theoretically, in the stress-strength model, the stress and strength concepts are considered as random variables that do not change with time. However, in engineering applications, the status of the stress and strength variables may improve or deteriorate over time. As a result, the status of these variables may change over time. Therefore, it will be more meaningful to consider the stress-strength model depending on time. Regarding this issue, [6] defined the random lifetime, T, of the system as

$$T = \inf\{t: t \geq 0, Y(t) \leq X(t)\}, \tag{1}$$

where $X(t)$ and $Y(t)$ are the stress and strength random variables at time t, respectively. From (1) and for $(0, t_0]$, the reliability of the system is $R(t_0) = P(T > t_0)$.

Depending on the location of the system and external factors, the system may be exposed to more than one stress. Although the system is not completely degraded as a result of the stresses, there may be various degradation levels in system performance. Such a situation corresponds to a multi-state system [5]. In general, two different system structures are studied, namely binary system and multi-state system. In a binary system, the definition domains of the states of the system and its components are $\{0,1\}$. In a multi-state system, the definition domains of the states of the system and its components are $\{0,1,2,\ldots,M\}$. In engineering applications, the multi-state system has a more flexible structure than the binary system when the binary system and the multi-state system are compared. In literature, much attention has been paid to multi-state system modeling [7, 8, 9, 10, 11].

The failure rate $\lambda(t)$ which is corresponds to the absolutely continuous cumulative distribution function $F(t)$, is represented by

$$\lambda(t) = \frac{f(t)}{\bar{F}(t)}, \tag{2}$$

and defines the probability that an operating component will fail in the next sufficiently small unit interval of time. The failure rate plays an exceptional role in reliability engineering that mostly deals with positive random variables [12].

Clearly, as $F(t)$ is absolutely continuous, the probability distribution function $f(t) = \frac{d}{dt}F(t)$ exists almost everywhere. Then, from (2), we have main exponential formula

$$F(t) = 1 - \exp\left(-\int_0^t \lambda(u)du\right), \tag{3}$$

where $\lambda(t)$ is the corresponding failure rate. Equation (3) shows that the failure rate $\lambda(t)$ uniquely defines the absolutely continuous cdf $F(t)$.

In this paper, we aim to propose a new method for computing the dynamic reliability of the Lin/Con/k/n: F system depending on the states of its components. A method is presented for the case in which the system consists of n independent components whose deteriorating strengths are independent and identically distributed and these components are subjected to two random stresses.

2 Proposed Method

Suppose that the Lin/Con/*k*/*n*:F system consists of *n* independent components whose deteriorating strengths $Y_i(t) \geq 0$, $i = 1, 2, \ldots, n$ are stochastically decreasing in time and independent and identically with continuous cumulative distribution function $G_{i(t)}(x) = P\{Y(t) \leq x\}$. Also assume that, these components are subjected to two types of stresses X_l, $l = 1, 2$ and stresses remain fixed over time with cdf $F_l(x) = P\{X_l \leq x\}$. Let $EY_i(t)$ and EX_l are the finite mean of $Y_i(t)$ and X_l, respectively.

Using the finite mean of $Y_i(t)$ and X_l the operation states of the components, depending on the number of stresses, can be defined as follows

$$S_i(t):\begin{cases}3, & EX_1+EX_2<EY_i(t)\\ 2, & EX_2<EY_i(t)\\ 1, & EX_1<EY_i(t)\\ 0, & EY_i(t)<EX_1\end{cases},$$

where $S_i(t)$ is the state of the component i at time t and $EX_1 < EX_2$. The states 0 and 3 represent completely failed and perfectly working states, respectively and the others degraded states.

There is no doubt that the operation time in the possible states of the system will be reduced as time progresses, due to reduction in the operation performance of the components which work under stresses. This case causes a problem such that analyzing how to reduce of the operation performance of the system in the course of time. Hence, it is necessary considering as a dynamic structure of the system and also making examination at a certain time interval for the reliability of the system. Therefore, calculation of efficient operation performance of the system at a certain time periods is important.

Let, $Z_{i,j}(t)\geq 0$ denote the performance rate of the component i in state j at time t and $p_{i,j}(t)$ be the operating performance probability of the component i in state j at time t, $1\leq i\leq n$, $1\leq j\leq 3$. Then,

$$p_{i,j}(t)=P\{Z_{i,j}(t)>t+h|Z_{i,j}(t)>t\}=\frac{P\{Z_{i,j}(t)>t+h\}}{P\{Z_{i,j}(t)>t\}}=\frac{1-H_i(t+h)}{1-H_i(t)} \tag{4}$$

where $H_i(t)=P\{Z_{i,j}(t)\leq t\}$, $i=1,2,\dots,n$. Now using (3) in (4), we can obtain the dynamic operation performance probabilities as follows

$$p_{i,j}(t)=\exp\left(-\int_t^{t+h}\lambda_{i,j}(u)du\right). \tag{5}$$

where $\lambda_{i,j}(t)$ is the corresponding failure rate for the component i in state j at time t. The failure rate for the component i in state “3”, “2” and “1” at time t is as follows, respectively.

$$\lambda_{i,3}(t)=\frac{1}{(EY_i(t)-EX_1-EX_2)+(EY_i(t)-EX_2)+(EY_i(t)-EX_1)}\psi_i^{(3)},$$

where $\psi_i^{(3)}$ is an indicator function such that

$$\psi_i^{(3)}=\begin{cases}1, & \text{if event } EY_i(t)-EX_1-EX_2>0 \text{ occurs at time } t\\ 0, & \text{if event } EY_i(t)-EX_1-EX_2>0 \text{ does not occurs at time } t.\end{cases}$$

$$\lambda_{i,2}(t)=\frac{1}{(EY_i(t)-EX_2)+(EY_i(t)-EX_1)}\psi_i^{(2)},$$

where $\psi_i^{(2)}$ is an indicator function such that

$$\psi_i^{(2)}=\begin{cases}1, & \text{if event } EY_i(t)-EX_2>0 \text{ occurs at time } t\\ 0, & \text{if event } EY_i(t)-EX_2>0 \text{ does not occurs at time } t.\end{cases}$$

and

$$\lambda_{i,1}(t)=\frac{1}{(EY_i(t)-EX_1)}\psi_i^{(1)},$$

where $\psi_i^{(1)}$ is an indicator function such that

$$\psi_i^{(1)}=\begin{cases}1, & \text{if event } EY_i(t)-EX_1>0 \text{ occurs at time } t\\ 0, & \text{if event } EY_i(t)-EX_1>0 \text{ does not occurs at time } t.\end{cases}$$

3 Conclusions

The evaluation of the operating performance of the system which works under varies stresses is very important for investigation of the technical system. For this reason, the correct evaluation depending on number of stresses plays an important role for determining the reliability of the system. In case of more than one stress, the expected values of the stresses are arranged in descending order. At the beginning when the system starts working, the expected value of strength of each component is greater than sum of all stresses. As the process continuous, the expected value of the strength for each component reduces within the operating time until reaches the minimum expected stress. For each process, we have assigned a state. Finally using the states of the components, the reliability of the system is determined successively without assigning any state to the system.

References

1. Kontoleon, J. M. (1980). Reliability determination of a r-successive-out-of-n:F system. *IEEE Trans. Reliab.*, 29, 437.
2. Chandra, S., & Owen, D. B. (1975). On estimating the reliability of a component subject to several different stresses (strengths). *Naval Res. Log. Quart.*, 22, 31-40.
3. Kotz, S., Lumelskii, Y., & Pensky, M. (2003). *The Stress-Strength Model and its Generalizations. Theory and Applications*. Singapore: World Scientific.
4. Nadarajah, S., & Kotz, S. (2006). Reliability for some bivariate exponential distributions. *Mathematical Problems in Engineering*, 2006, 1-14.
5. Eryılmaz, S., & İşçioğlu, F. (2011). Reliability evaluation for a multi-state system under stress-strength setup. *Commun. Statist. Theor. Meth.*, 40, 547-558.
6. Basu, A. P., & Ebrahimi, N. (1983). On the reliability of stochastic systems. *Statist. Probab. Lett.*, 1, 265-267.
7. El-Neweihi, E., Proschan, F. & Sethuraman, J. (1978). Multi-state coherent system. *Journal of Applied Probability*, 15, 675-688.
8. Hudson, J. C., & Kapur, K. C. (1983). Reliability analysis for multistate systems with multistate components. *IIE Transactions*, 15, 127-135.
9. Ebrahimi N. (1984). Multistate reliability models. *Naval Research Logistics Quarterly*, 31, 671-680.
10. Brunelle R. D., & Kapur, K. C. (1999). Review and classification of reliability measures for multistate and continuum models. *IIE Transactions*, 31, 1171-1180.
11. Kuo, W., & Zuo, M. J. (2003). *Optimal Reliability Modeling: Principles and Applications*. New York: John Wiley & Sons.
12. Finkelstein, M., & Cha, J. H. (2013). *Stochastic modelling for reliability. shocks, burn-in and heterogeneous populations*. London: Springer