Analysis of the Limited Resources Queuing System with Signals and Multiple Flows of Customers

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Abstract. In the paper, a wireless network is modelled in terms of multiserver queuing system with limited resources, random resource requirements and signals that trigger resource reallocation procedure. For the queuing system, we derive the set of balance equations and propose the solution method. Moreover, the formulas for main performance measures are obtained.

1 Introduction

Improvements and advanced techniques in the future evolution of mobile systems, often referred to as fifth generation (5G) networks [1-2], are due to the significant increase in the user traffic demands in recent years. Besides the implemented advances in modulation, MIMO techniques, the network solutions are also designed to improve the wireless network performance. Among them are network densification [3], direct in-band and out-of-band device-to-device communications [4].

In this connection, mobile operators and network device manufacturers need an effective tool to estimate performance gains from emerging technologies. Classic queuing models fails to cover a specific mechanism of the frequency resources allocation in the contemporary networks. The physical resources are usually represented by a time-frequency resource grid. In a simple words, resource blocks are scheduled in respect to the radiochannel quality and required bitrate that a user equipment communicate with a base station [5]. The maximum achievable bitrate for a user session i can be provided by the well-known Shannon's formula:

$$C_i^{\max} = \omega \log_2(1 + \mathrm{SINR}_i \, p_{\max}),$$

where ω is a spectral bandwidth, $p_{\max}$ is the maximum transmit power of the base station and SINR_i is a signal-to-interference and noise ratio for an i-th user session. Basically, SINR depends on the radio signal propagation conditions, including the distance to the base station and obstacles between the mobile terminal and the base station. The session admission policy can be represented by the following inequality:

$$\sum_i \frac{C_i}{C_i^{\max}} \le 1, \qquad (1)$$

where C_i is the required rate of i-th user session. Thus, the resource requirements of each user session depends

on a number of parameters, basically connected to the signal propagation conditions, and vary for each session. Here the ratio $\dfrac{C_i}{C_i^{\max}}$ may be interpreted as a share of system resources required by the i-th user session.

The described above resource allocation process can be modelled in terms of multiserver queuing systems, in which customers require a server and a random amount of limited resources. Queuing systems with random resource requirements, Poisson arrivals and exponentially distributed service times were studied in [6-7]. The generalization of the model for multiple types of customers is presented in [8].

However, in the works mentioned above, the required volume of resources does not change during lifetime of user sessions. To model user mobility, the resource queuing system was extended in [9] by implementing signals that trigger resource reallocation process during service process.

In the paper, we propose further generalization of [9] for multiple flows of customers. The steady-state analysis of the system is provided and formulas for main performance characteristics are deduced. The proposed method of analysis is applied for only discrete resource requirements, the case of continuous resource requirements is left for further research.

The rest of the paper is organized as follows. Section II presents the detailed description of the queuing system, provides stationary probability distribution and formulas for main performance measures. Section III is devoted for numerical analysis, while section IV concludes the paper.

2 Mathematical model

In this section, we describe the queuing system with random resource requirements and multiple flows of

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customers and provide the analysis of its stationary characteristics and various performance measures.

2.1. Model description

Consider a multiserver queuing system with N servers and R units of discrete resources. For simplicity, we consider the system with only one type of resources. However, the results of the analysis can be easily extended to the case with vector of resources **R**.

Assume that there are L types of customers, customers of l-type arrive according to the Poisson process with intensity λ_l, the service times have exponential distribution with parameter μ_l. Upon arrival, a customer requests random amount of resources. If the system has enough free resources, then the customer is accepted and occupies the requested amount of resources. If there are not enough resources to meet the requirements, then the customer is lost. On the departure of a customer, it releases all the resources that were occupied by it. The resource requirements of l-type customers are defined by probability mass function $\{p_{l,r}\}, r \geq 0$.

Besides customer arrivals, each currently served l-type customer generates a Poisson flow of signals with intensity γ_l. On the arrival of a signal, the customer releases all the resources that were occupied by it, generates new resource requirement according to the same probability mass function $\{p_{l,r}\}$, $r \geq 0$ and tries to occupy new amount of resources. If the system has enough free resources, then the customer occupies new amount of resources. Otherwise, the customer is lost.

The behaviour of the system can be described by a Markov process $\tilde{X}(t) = (\xi(t), \theta(t), \eta(t))$, where $\xi(t)$ is the number of customers at time t, $\theta(t) = (\theta_1(t), \ldots, \theta_{\xi(t)}(t))$ is the vector of customers' types and $\eta(t) = (\eta_1(t), \ldots, \eta_{\xi(t)}(t))$ is the number of resources occupied by each customer.

The number of states of $\tilde{X}(t)$ is extremely high, so we use the simplification approach described in [7]. The main idea of the simplification is that instead of keeping track of the amount of occupied resources by each customer, we keep track of only total amount of occupied resources by all customers of specific type.

The simplified system is described by process $X(t) = (\xi(t), \delta(t))$, where $\xi(t) = (\xi_1(t), \ldots, \xi_L(t))$ is the number of customers of each type and $\delta(t) = (\delta_1(t), \ldots, \delta_L(t))$ is the total amount of occupied resources by customers of each type.

The state space S of $X(t)$ can be decomposed to the following disjoint sets:

$$S = \bigcup_{n_1 + \ldots + n_L \leq N} S_{n_1, \ldots, n_L}, \qquad (2)$$

$$S_{n_1, \ldots, n_L} = \left\{ ((n_1, \ldots, n_L), (r_1, \ldots, r_L)) \mid p_{l, r_l}^{(n_l)} > 0, \sum_{l=1}^{L} r_l \leq R, 1 \leq l \leq L \right\} (3)$$

In process $X(t)$, we do not know the exact number of resources that should be released on a departure and on a signal arrival. To cope with the problem, we use approximation based on Bayes' formula. I.e. if the system is in state $((n_1, \ldots, n_L), (r_1, \ldots, r_L))$ and a customer of type l departs from the system, then the customer will release j resources with probability $\dfrac{p_{l,j} p_{l, r_l - j}^{(n_l - 1)}}{p_{l, r_l}^{(n_l)}}$. Here $p_{l,r}^{(n)}$ is n-fold convolution of the distribution $\{p_{l,r}\}, r \geq 0$ and can interpreted as the probability that n customers of l-type occupy totally r resources.

The same Bayesian approach is used to determine state transitions on a signal arrival. If the system is in state $((n_1, \ldots, n_L), (r_1, \ldots, r_L))$, then on arrival of a signal to l-type customer, it releases j resource units with probability $\dfrac{p_{l,j} p_{l, r_l - j}^{(n_l - 1)}}{p_{l, r_l}^{(n_l)}}$ and requires new amount of resources according to the initial distribution $\{p_{l,r}\}$, $r \geq 0$. Thus, the probability that the system state will change to the state $((n_1, \ldots, n_l, \ldots, n_L), (r_1, \ldots, s, \ldots, r_L))$ is

$\sum_{j=0}^{r_l} \dfrac{p_{l,j} p_{l, r_l - j}^{(n_l - 1)}}{p_{l, r_l}^{(n_l)}} p_{l, s - r_l + j}$ in case of successful resource reallocation and the system state will change to the state $((n_1, \ldots, n_l - 1, \ldots, n_L), (r_1, \ldots, u, \ldots, r_L))$ with probability

$\dfrac{p_{l, r_l - u} p_{l,u}^{(n_l - 1)}}{p_{l, r_l}^{(n_l)}} \left(1 - \sum_{i=0}^{R-u} p_{l,i} \right)$ in case of loss of a customer.

2.2. Stationary probability distribution and performance measures

Denote stationary probabilities
$$q_{n_1, \ldots, n_L}(r_1, \ldots, r_L) = \lim_{t \to \infty} P\{\xi_l(t) = n_l, \delta_l(t) = r_l, 1 \leq l \leq L\}. \quad (4)$$
Then the stationary probabilities (4) obey the balance equations:

$$q_{0, \ldots, 0}(0, \ldots, 0) \sum_{l=1}^{L} \lambda_l \sum_{j=0}^{R} p_{l,j} = \sum_{l=1}^{L} \mu_l \sum_{r=0}^{R} q_{0, \ldots, 1, \ldots, 0}(0, \ldots, r, \ldots, 0) +$$

$$+ \sum_{l=1}^{L} \gamma_l \left(1 - \sum_{r=0}^{R} p_{l,r} \right) \sum_{r=0}^{R} q_{0, \ldots, 1, \ldots, 0}(0, \ldots, r, \ldots, 0) ; \quad (5)$$

$$q_{n_1, \ldots, n_L}(r_1, \ldots, r_L) \left(\sum_{l=1}^{L} \lambda_l \sum_{j=0}^{R - r_l} p_{l,j} + \sum_{l=1}^{L} n_l (\mu_l + \gamma_l) \right) =$$

$$\sum_{l=1}^{L}\lambda_l\sum_{j=0}^{\eta}q_{n_1,..,n_l-1,..,n_L}(r_1,...,j,...,r_L)p_{l,\eta-j}+$$

$$+\sum_{l=1}^{L}(n_l+1)\left(\mu_l+\gamma_l\left(1-\sum_{j=0}^{R-\eta}p_{l,j}\right)\right)\cdot$$

$$\cdot\sum_{j=0}^{\eta}q_{n_1,..,n_l+1,..,n_L}(r_1,...,r_l+j,...,r_L)\frac{p_{l,j}p_{l,\eta}^{(n_l)}}{p_{l,\eta+j}^{(n_l+1)}}+$$

$$+\sum_{l=1}^{L}\gamma_l\sum_{j=0}^{\eta}q_{n_1,..,n_l,..,n_L}(r_1,...,j,...,r_L)\sum_{s=0}^{\min(j,\eta)}\frac{p_{l,s}p_{l,j-s}^{(n_l-1)}}{p_{l,j}^{(n_l)}}p_{l,\eta-j+s},$$

$$\left((n_1,...,n_L),(r_1,...,.,r_L)\right)\in S_{n_1,..,n_L},\sum_{l=1}^{L}n_l<N; \quad (6)$$

$$q_{n_1,..,n_L}(r_1,...,r_L)\sum_{l=1}^{L}n_l(\mu_l+\gamma_l)=$$

$$\sum_{l=1}^{L}\lambda_l\sum_{j=0}^{\eta}q_{n_1,..,n_l-1,..,n_L}(r_1,...,j,...,r_L)p_{l,\eta-j}+$$

$$+\sum_{l=1}^{L}\gamma_l\sum_{j=0}^{\eta}q_{n_1,..,n_l,..,n_L}(r_1,...,j,...,r_L)\sum_{s=0}^{\min(j,\eta)}\frac{p_{l,s}p_{l,j-s}^{(n_l-1)}}{p_{l,j}^{(n_l)}}p_{l,\eta-j+s},$$

$$\left((n_1,...,n_L),(r_1,...,.,r_L)\right)\in S_{n_1,..,n_L},\sum_{l=1}^{L}n_l=N. \quad (7)$$

The system of balance equations (5)-(7) along with normalizing condition gives stationary probabilities (4).

With the stationary probability distribution, we can evaluate performance measures of the system. The probability $p_{B,l}$ that a *l*-type customer will be blocked on arrival is

$$p_{B,l}=1-\sum_{n_1+...+n_L<N}q_{n_1,..,n_L}(r_1,...,r_L)\sum_{j=0}^{R-\eta}p_{l,j}, \quad (8)$$

The probability $p_{T,l}$ that an accepted *l*-type customer will be blocked on signal arrival is calculated by formula

$$p_{T,l}=\sum_{0<n_1+...+n_L\le N}q_{n_1,..,n_L}(r_1,...,r_L)\sum_{j=0}^{R-r_l}\frac{p_{l,j}p_{l,r_l-j}^{(n_l-1)}}{p_{l,r_l}^{(n_l)}}\left(1-\sum_{s=0}^{R-r_l+j}p_{l,s}\right)(9)$$

Finally, the average amount of occupied resources *b* is

$$b=\sum_{n_1+...+n_L\le N}q_{n_1,..,n_L}(r_1,...,r_L)\sum_{l=1}^{L}r_l. \quad (10)$$

3 Numerical analysis

In this section, we provide some numerical results of performance measures analysis.

For simplicity, we assume only $L=2$ types of customers, with the same serving intensity $\mu_1=\mu_2=1$. Arrival intensity of 1-type customers is assumed to be $\lambda_1=2.5$, while arrival intensity of 2-type customers is $\lambda_1=5$. Signal arrival intensities are equal for both types of customers $\gamma_1=\gamma_2=3$.

The distribution of resource requirements is assumed to be geometric:

$$p_{l,r}=(1-\alpha_l)\alpha_l^r, r\ge 1, \quad (11)$$

with $\alpha_1=0.1$ and $\alpha_2=0.2$. Note that according to the properties of geometric distribution, the average resource requirement of 1-type customers is 10 and the average resource requirement of 2-type customers is 5.

Figures 1 and 2 show the behavior of and $p_{T,l}$ depending on the total offered load $\rho=\rho_1+\rho_2$.
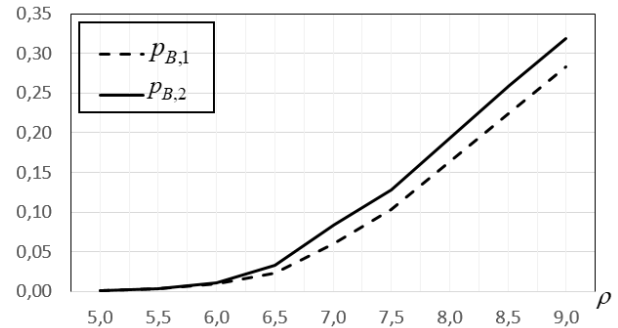

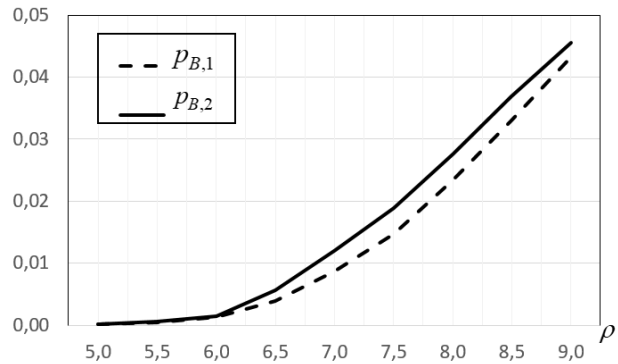**Fig. 1**. The probability that a customer will be blocked on arrival as a function of system load

Fig. 2. The probability that an accepted customer will be blocked on signal arrival as a function of system load.

The figures show that 1-type customers have lower blocking probabilities for both arriving and accepted customers. The reason of the effect is lower average resource requirements of 1-type customers. The effect is nearly negligible for small values of system load ρ and increases with the increase of ρ.

4 Conclusion

In the paper, we analysed a multiserver queuing system with discrete limited resources, multiple arrival flows and signals that trigger resource reallocation. The described queuing system may be applied for the analysis of contemporary wireless networks. We derived the system of balance equations that can be solved numerically. Formulas for the main performance measures were obtained. Finally, we illustrated our results by numerical example.

In our further research we plan to reduce the state space of the system by aggregating the arrival flows. As a result, the solution complexity of the balance equations will significantly decrease.

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