

Identification of Nonparametric Nonlinear Systems

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Abstract. Presently, a modelling and identification of nonlinear systems is proposed. This study is developed based on spectral approach. The proposed nonlinear system is nonparametric and can be described by Hammerstein models. These systems consist of nonlinear element followed by a linear block. This latter (the linear subsystem) is not necessarily parametric and the nonlinear function can be nonparametric smooth nonlinearity. This identification problem of Hammerstein models is studied in the presence of possibly infinite-order linear dynamics. The determination of linear and nonlinear block can be done using a unique stage.

1 Introduction

The Hammerstein model is a series connection of a nonlinear function and a linear dynamical element (Fig. 1). Nonlinear system identification has been an active research area, especially over the last two decade [1]-[4]. Parameters determination of black-box nonlinear system is a very wide research area [1]-[4]. These models can describe several industrial systems, e.g. [1]-[2] and [5]. The diversity of nonlinear models and structures has led to a large variety of identification problems and identification methods.

In this paper, the problem of identifying Hammerstein systems in continuous time is addressed. Presently, the linear block is not necessarily parametric and can be of finite or infinite order. The system nonlinearity is nonparametric but of smooth shape. Furthermore, it is interesting to emphasize that, all the internal signals ($x(t)$, $w(t)$, and $\xi(t)$) are not accessible to measurement.

In view of these difficulties, it is not surprising that few parameter determination methods are available that deal with nonparametric Hammerstein models.

Unlike most of previous work, the parameter determination problem is developed with in the continuous-time context. The proposed approach is based on the Fourier expansion using a simple periodic or sine input signal. Here, the determination of linear block as well as the system nonlinearity is done at the same time (in one stage), unlike most of previous papers [1]-[4].

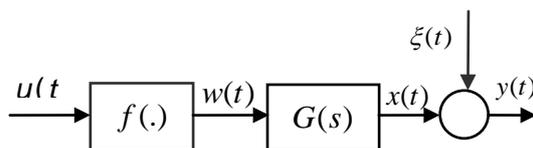


Fig. 1. Nonparametric Hammerstein system

Unlike many of previous works e.g. [4], the model structure of the linear block is entirely unknown. Furthermore, the system nonlinearity is of arbitrary-shape and can be noninvertible. In most previous works devoted to Hammerstein system identification, the nonlinear element is supposed to be smooth continuous function. Furthermore, the smoothness assumption implies that the system nonlinearity can be developed within any interval by a polynomial decomposition [6]-[10].

Among the most used identification methods, one can find the frequency approaches [11]-[14]. Several technics can be used, e.g. Fourier decomposition, geometric analysis, etc.

The proposed approach is allowed to concern a wide range of nonlinearity function. The identification problem amounts to obtain an accurate estimate of the complex frequency gain $G(j\omega)$ of linear element, for a set of frequencies $(\omega_1 \dots \omega_m)$, and the nonlinearity parameters. To the author's knowledge, the present identification solution will be performed in one-stage and using simple sine inputs (or periodic).

The paper is organized as follows: The identification problem is formally described in Section 2; relevant mathematical tools are described in Section 3; Section 4 is devoted to the determination of nonlinear element as well as the estimation of system nonlinearity.

2 Identification Problem Statement

Standard Hammerstein models consist of a nonlinear block $f(\cdot)$ followed in series by a linear dynamic

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element $G(s)$ (Fig. 1). This model is analytically described by the following equations:

$$w(t) = f(u(t)) \quad (1)$$

$$x(t) = g(t) * w(t) = g(t) * f(u(t)) \quad (2)$$

$$y(t) = x(t) + \xi(t) = g(t) * f(u(t)) + \xi(t) \quad (3)$$

where $g(t) = L^{-1}(G(s))$ is the inverse Laplace transform of linear transfer function $G(s)$, the notation $*$ refers to the convolution product; $w(t)$ is the internal signal (Fig. 1); $x(t)$ is the undisturbed system output; the extra-input $\xi(t)$ accounts for the noise signal. Note that, only the input $u(t)$ and output $y(t)$ signals are accessible to measurements.

The equation error $\xi(t)$ is a zero-mean stationary sequence of independent random variables; it is supposed to be ergodic (so that arithmetic averages can be substituted to probabilistic means whenever this is necessary).

As, the inner signals, $w(t)$, $x(t)$ and $\xi(t)$ are not accessible to measurements, the problem identification method must only rely on the external signals $u(t)$ and $y(t)$.

The linear block transfer function $G(s)$ is nonparametric and can be of unknown structure; it is only supposed to be stable and with nonzero static gain (i.e. $G(0) \neq 0$). The fact that $G(0) \neq 0$ implies that, without reducing generality, one can assume $G(0) = 1$. This statement will be justified in the next section.

In this work, the nonlinear function is not necessarily parametric and can be noninvertible. This latter is assumed to be smooth continuous which implies that, the nonlinear element $f(\cdot)$ can be approximated within any interval by a polynomial function of known degree m .

3 Mathematical preliminaries

In the proposed identification problem solution, the Hammerstein system is excited by periodic input signals, a simple sine signal can also be used. To make easy this method, let choose the following input signal:

$$u(t) = U \sin(\omega t) \quad (4)$$

The system is repeatedly excited by (4) for a set of frequencies $\omega \in \{\omega_1, \dots, \omega_n\}$. In this respect, note that the inner signals $w(t)$ and $x(t)$ are also periodic of the same period of input i.e. $T = 2\pi / \omega$. Then, this

immediately implies that, the inner signal $w(t)$ can be developed in Fourier expansion:

$$w(t) = \sum_{i=0}^{\infty} W_i \cos(i\omega t + \alpha_i) \quad (5)$$

Using the same thing, the undisturbed output $x(t)$ can be expressed as follows:

$$x(t) = \sum_{i=0}^{\infty} X_i \cos(i\omega t + \beta_i) \quad (6)$$

On the other hand, in Section 2 it was shown that, $G(0) = 1$. This result comes from the fact that, this problem identification does not have a unique solution (solution plurality). Specifically, if k is any nonzero real, so any model of the form $(f(v)/k, kG(s))$ is representative of the system.

The question is how to choose the scaling factor k ?

For convenience, let take the following choice:

$$k = |G(j\omega_l)| \quad (7)$$

where ω_l is any frequency $\omega_l \in \{\omega_1, \dots, \omega_n\}$.

4 Nonlinear systems Identification

In this section, an identification method is proposed to obtain an accurate estimate of the complex gain $G(s)$ (the phase $\varphi(\omega) = \arg(G(j\omega)) = \angle G(j\omega)$ and the modulus gain $|G(j\omega)|$) corresponding to the linear block for any frequency $\omega \in \{\omega_1, \dots, \omega_n\}$ as well as the system nonlinearity parameters.

On the other hand, it readily follows using (2) and (5) that:

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for any frequency $\omega \in \{\omega_1, \dots, \omega_n\}$ as well as the system nonlinearity parameters.

On the other hand, it readily follows using (2) and (5) that:

$$x(t) = \sum_{i=0}^{\infty} W_i |G(ji\omega)| \cos(i\omega t + \alpha_i + \varphi(i\omega)) \quad (8)$$

Then, it follows from (3) and (8) that:

$$y(t) = \sum_{i=0}^{\infty} W_i |G(ji\omega)| \cos(i\omega t + \alpha_i + \varphi(i\omega)) + \xi(t) \quad (9)$$

Therefore, one immediately gets using (6) and (8):

$$X_i = W_i |G(ji\omega)| \quad \text{for } i = 0, 1, \dots \quad (10a)$$

$$\beta_i = \alpha_i + \varphi(i\omega) \quad \text{for } i = 1, 2, \dots \quad (10b)$$

This result means that, if the spectrum (Fourier expansion) is available, then the amplitude $X_i = W_i |G(ji\omega)|$ and the argument $\beta_i = \alpha_i + \varphi(i\omega)$ can be given. Unfortunately, note that the inner signal $x(t)$ is not accessible. However, given that the undisturbed output $x(t)$ is periodic, with common period T of inupt $u(t)$, and $\xi(t)$ is a zero-mean ergodic white noise, the effect of the latter can be filtered considering the following trans-period averaging of the output:

$$\hat{x}(t) = \frac{1}{M} \sum_{k=1}^M y(t+kT) \quad \text{for } 0 \leq t < T \quad (11)$$

for some (large enough) integer M .

Finally, using the estimate of undisturbed output $x(t)$ and getting benefit from the plurality of solution (i.e. using the rescaling of nonlinear system (7)), this identification problem can be coped by exciting the system with different frequencies ($\omega, 2\omega, \dots$).

5 Conclusion

Presently, the problem of nonlinear system identification is dealt. The nonlinear system is described by Wiener model.

In this work, nonparametric identification solution is developed with continuous-time Hammerstein systems involving nonparametric smooth nonlinear element. The proposed method is built in one stage using periodic (or sine) input signal. Then, the frequency gain of the linear element is determined at a number of frequencies. The originality of the present study lies in the fact that the linear block and the system nonlinearity are all nonparametric. Accordingly, the linear block is not necessarily finite order and the nonlinearity element may be noninvertible. Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented.

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