

Modified method of identification of mutual fractional-order inductance

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Abstract. The paper presents a method for identifying the parameters M_γ , γ of a fractional-order transformer, which parameters $L_{\beta 1}$, β_1 , $L_{\beta 2}$, β_2 have been previously determined. This method is based on the measurement of the phase resonance frequency in a few systems containing: the investigated fractional-order transformer and two standard capacitors. The measurements need to be performed only for one series opposite-aiding connection of the fractional-order transformer. The dependencies allowing the determination of the fractional-order mutual inductance parameters have been given

1 Introduction

There are many works devoted to the analysis of systems with fractional-order elements L_β , C_α , their realization and parameter identification, e.g. [1-3].

For several years, there has been a rapid growth of interest in fractional differential-integral calculus application in describing fractional-order magnetically-coupled coils systems [4-6]. The work [4] describes the concept and properties of such fractional-order coupled inductances. In [5], the electromagnetic Maxwell equations of the fractional-order mutual inductance are analyzed. The wireless power transmission system has been modeled as a fractional-order coupled coils system in [6]. The existence of fractional-order coupled coils (fractional-order transformer) implies the need to determine the parameters of the fractional-order elements. In [7], a method has been proposed for parameters identification of the fractional-order coils with an iron core, which is based on the approximation of the transient response to the unit-step voltage using the least squares method.

The paper is an extension and continuation of [8], where the new method for the identification of all the parameters $L_{\beta 1}$, β_1 , $L_{\beta 2}$, β_2 , M_γ , γ , of the fractional-order coupled inductances, has been proposed. The paper presents a proposal for a modified method of the identification of the fractional-order parameters M_γ , γ of the mutual inductance, based on the phase resonance phenomenon in the series circuit of the class $RL_\beta C_\alpha$, compared to [8], without the need of the input impedance measurement in the combination of series and opposite-aiding connection of the transformer system.

2 Modification proposal

The equivalent circuit of the system for the parameters γ , M_γ determination of the fractional-order mutual inductance, is shown in Fig. 1.

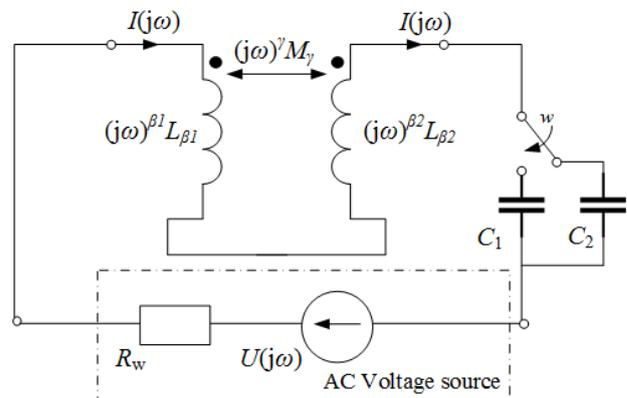


Fig. 1. The equivalent circuit for the fractional-order parameters identification by resonance method.

The circuit from Fig. 1 is supplied by the sinusoidal voltage source of adjustable frequency.

The circuit impedance, seen from the source terminals, is given by a formula:

$$Z(j\omega) = R + (j\omega)^{\beta_1} L_{\beta_1} + (j\omega)^{\beta_2} L_{\beta_2} - 2(j\omega)^\gamma M_\gamma - j \frac{1}{\omega C} \quad (1)$$

where: R - the equivalent resistance of the series connection of the coil resistances.

Transforming, the real and imaginary part of the impedance is:

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$$\begin{aligned} \operatorname{Re}\{Z(j\omega)\} = & R + \omega^{\beta_1} L_{\beta_1} \cos\left(\frac{\beta_1 \pi}{2}\right) + \omega^{\beta_2} L_{\beta_2} \cos\left(\frac{\beta_2 \pi}{2}\right) - \\ & - 2\omega^\gamma M_\gamma \cos\left(\frac{\gamma \pi}{2}\right) \end{aligned} \quad (2)$$

and:

$$\begin{aligned} \operatorname{Im}\{Z(j\omega)\} = & \omega^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \\ & - 2\omega^\gamma M_\gamma \sin\left(\frac{\gamma \pi}{2}\right) - \frac{1}{\omega C} \end{aligned} \quad (3)$$

Parameters L_{β_1} , β_1 , L_{β_2} , β_2 were determined previously, according to the procedure described in [8]. The circuit from Fig. 1 should be brought into phase resonance state, which will occur when the phase shift between the voltage measured on the series connection of the magnetically-coupled coil system as well as the capacitor C_1 and the current, will be equal to zero.

The measurement described above should be repeated twice, for two values of classic capacitances C_1, C_2 and the detection of the phase resonance frequency values, ω_1 , ω_2 respectively.

Next, using the general phase resonance condition $\operatorname{Im}\{Z(j\omega)\} = 0$ and transforming, we get the relation:

$$\left(\frac{\omega_1}{\omega_2}\right)^\gamma = \frac{\omega_1^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega_1^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \frac{1}{\omega_1 C_1}}{\omega_2^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega_2^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \frac{1}{\omega_2 C_2}} \quad (4)$$

Then the value of the parameter γ can be determined as:

$$\gamma = \log_{\left(\frac{\omega_1}{\omega_2}\right)} \left(\frac{\omega_1^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega_1^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \frac{1}{\omega_1 C_1}}{\omega_2^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega_2^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \frac{1}{\omega_2 C_2}} \right) \quad (5)$$

However, the parameter M_γ can be determined by substituting the obtained value of the coefficient γ with the formula (3) for one of the performed measurements, for example for:

$$M_\gamma = \frac{\omega_1^{\beta_1} L_{\beta_1} \sin\left(\frac{\beta_1 \pi}{2}\right) + \omega_1^{\beta_2} L_{\beta_2} \sin\left(\frac{\beta_2 \pi}{2}\right) - \frac{1}{\omega_1 C_1}}{2\omega_1^\gamma \sin\left(\frac{\gamma \pi}{2}\right)} \quad (6)$$

The described algorithm has been illustrated with a simulation example.

3 Example

The circuit from Fig. 1 has been supplied from a source with an adjustable frequency value, for which the input voltage value has been assumed $U(j\omega) = 1 \text{ V}$.

Parameters of the primary and secondary side of the transformer have been determined according to the procedure diagram [8] and were respectively: $\beta_1 = 0.503$, $L_{\beta_1} = 8.813 \text{ mH}\cdot\text{s}^{(1-\beta_1)}$, $\beta_2 = 0.502$, $L_{\beta_2} = 3.113 \text{ mH}\cdot\text{s}^{(1-\beta_2)}$. For two capacitors with known capacitances $C_1 = 10 \text{ mF}$, $C_2 = 3,53 \text{ mF}$ in the investigated circuit, as in Fig. 1, two values of resonance frequencies $f_1 = 100 \text{ Hz}$, $f_2 = 200 \text{ Hz}$ have been recorded. From the dependencies (4) and (5), the searched values of the fractional-order parameters of the mutual inductance have been determined:

$$\gamma = 0.503 \quad (7)$$

and:

$$M_\gamma = 1.554 \text{ mH}\cdot\text{s}^{(1-\gamma)} \quad (8)$$

4 Summary

The paper proposes a modified method, compared to the method presented in [8], for identifying M_γ , γ parameters of a fractional-order transformer. This method is based on the measurement of the phase resonance frequency in a circuit containing the analyzed transformer and two switchable standard capacitors. The dependencies allowing the determination of the fractional-order mutual inductance parameters have been given, on the basis of the described measurements.

The advantage of the modified method for determining the fractional-order parameters is the fact, that only one series opposite-aiding connection of the fractional-order coupled coils is enough to determine the searched parameters. The need to measure the input impedance of the circuit from Fig. 1 is also avoided.

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