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Comparison of unit-step responses of parametric filter and fractional-order filter

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Abstract. The paper presents transmission models of a parametric filter with non-periodic variable parameters and a fractional-order filter. The responses of these filters on a unit-step excitation have been examined as well as the dependence of filters time constants on their parameters. The obtained results have been illustrated by examples.

1 Introduction

Parametric systems are non-stationary deterministic systems with parameters variable in time. They are also shortly called LTV (linear time varying) systems. Many theoretical works e.g. [1,2] as well as practical applications have been devoted to these systems. In particular, LTV systems can be applied in signal processing especially in amplifiers [1], sampling systems, signal filtering and noise reduction [3] as well as current compensation in power networks [4], medicine devices [5] and many others. The parametric systems are described by the differential equations with time-varying parameters [6]. There are some methods of LTV system analysis based on transformation of some differential equations e.g. Riccati, Mathieu, Meissner or Hill eqs. [1]. Analytic solutions to the above mentioned equations exist only in some specific cases [1, 2] which depend strictly on the variability of system coefficients.

Fractional-order system are described by fractional-order differential eq. The most common fractional-order elements in electrical engineering are supercapacitors C_α, [7] and coils L_β with soft ferromagnetic cores [8]. Systems with such elements find many applications, e.g. in the construction of generators, energy storage systems for electric and hybrid vehicles, batteries and fuel cells [9]. There is a growing interest on the realization and analysis of analog and digital fractional-order filters [10], also using electronic active circuits (e.g. with MOS transistors) [11]. The aim of this paper is the comparison of the dynamic properties of a parametric filter with non-periodically variable parameters and fractional-order.

2 LTV filter model

The elementary low pass LTV filter (0) is described by the first order parametric differential equation:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + \omega(t)y(t) = cu(t) \qquad (1)$$

The solution to parametric equation (1) is given by [2]:

$$y(t) = y_0(t)\mathrm{e}^{-\int_0^t \omega(t)\mathrm{d}t} + \int_0^t \mathrm{e}^{-\int_0^t \omega(t)\mathrm{d}t}\,\mathrm{e}^{\int_0^t \omega(\tau)\mathrm{d}\tau}\,u(\tau)\mathrm{d}\tau \qquad (2)$$

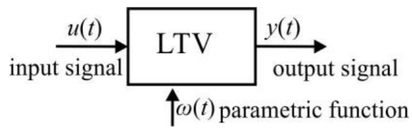

Fig. 1. Model of the first order LTV filter.

By the solution of the statement (2) results, one can express output signal of any type [2], e.g. the unit-step response:

$$y(t) = \left[\mathrm{e}^{\frac{C}{\gamma}\exp(-\gamma t) - \omega_0 k} \sum_{k=0}^{N} \frac{(-1)^k C^k}{k!\gamma^k} \frac{\mathrm{e}^{t(\omega_0 - \gamma k)} - 1}{\omega_0 - \gamma k} \right] \qquad (3)$$

The main advantages of LTV system application include improving the dynamic properties of systems and reducing the transient state.

3 Fractional-order filter model

The model of the corresponding fractional-order filter is shown in fig. 2. It consists of a supercapacitor, modeled as an ideal fractional-order capacitance C_α. The analyzed filter is described by the differential equation of fractional order, in the following form:

$$\frac{\mathrm{d}^\alpha y(t)}{\mathrm{d}t^\alpha} + \frac{1}{RC_\alpha} y(t) = \frac{1}{RC_\alpha} u(t) \qquad (4)$$

Using the two-parameter Mittag-Leffler function as a result of inverse Laplace transform of the impulse response [12], the convolution and Laplace transform linearity theorems, the solution can be written in time domain for any type of excitation, as:

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$$y(t) = (RC_\alpha)^{-1} \sum_{k=0}^{\infty} (-RC_\alpha)^{-k} \left(\Gamma(\alpha(k+1))\right)^{-1} \int_0^t u(t-\tau)\tau^{\alpha(k+1)-1} d\tau \qquad (5)$$

where: $\Gamma(\alpha(k+1))$ – special gamma Euler function.

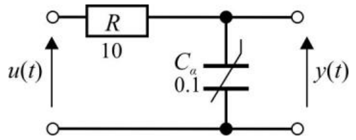

Fig. 2. Model of the fractional-order passive RC_α filter.

The unit-step system response takes the form:

$$y(t) = \frac{1}{RC_\alpha} t^\alpha \sum_{k=0}^{\infty} \left(-\frac{1}{RC_\alpha} t^\alpha\right)^k \frac{1}{\Gamma(\alpha(k+1)+1)} \qquad (6)$$

The introduced relation has been illustrated with a simulation example and compared to the dynamic properties of a similar parametric filter with non-periodically variable parameters.

4 Simulation studies

The first order low-pass LTV filter with a varying parameter $\omega(t)$ and fractional-order filter have been analyzed for the cut off angular frequency of a stationary prototype ω_0=1 rad/s.

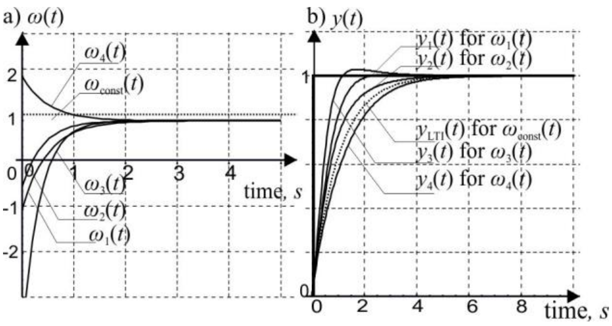

Fig. 3. LTV filter (a) parametric functions, (b) step responses.

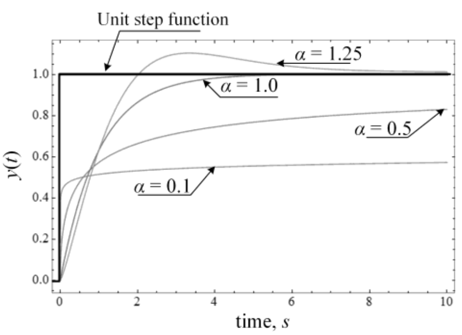

Fig. 4. Fractional-order filter step responses.

In the first diagram in fig 3 the cut off angular frequency variation is presented, while in the second one - the filter responses. The proper choice of the variability of parameters allows to shorten the transient state. Fig. 4 shows, that the value of parameter α can shape the signal response of fractional-order filter. For both simulation examples, the mean square error has been adopted as a quality criterion for assessing the dynamic properties of the filters. The results of the obtained errors are summarized in table 1. The calculation presented above, show that the dynamic properties of parametric filters are clearly better than of the fractional-order filters.

Table 1. Mean square error of the filters response for the a few considered examples of low-pass filters.

LTI	LTV		Fractional-order	
MSE, %	$\omega(t)$	*MSE, %*	α	*MSE, %*
5.0	$\omega_1(t)$	3.6	0.5	8.7
	$\omega_2(t)$	4.1	0.7	6.1
	$\omega_{const}(t)$	**5.0**	1.0	**5.0**
	$\omega_3(t)$	2.9	1.25	5.6
	$\omega_4(t)$	5.8	1.5	7.7
	$\omega_1(t)$	3.6	0.5	8.7

5 Summary

The paper aims to compare the dynamic properties of parametric filters with non-periodic variable parameters and fractional-order filters. The comparison was based on the filter response for unit step excitation. The simulations results show that the dynamic properties of parametric filters are clearly better than of fractional-order filters. Time-varying filter parameters and fractional-order filters as well, allow to design more flexible systems for which the properties can be designed more freely.

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