Double hybrid Kalman filtering for state estimation of dynamical systems

Jacek Michalski[1,*], *Piotr* Kozierski[2,1], and *Joanna* Zietkiewicz[1]

[1]Poznan University of Technology, Faculty of Electrical Engineering, Institute of Control, Robotics and Information Engineering, Division of Control and Robotics
[2]Poznan University of Technology, Faculty of Computing, Institute of Automation and Robotics, Division of Electronic Systems and Signal Processing

Abstract. In this paper authors present a new approaches to the hybrid Kalman filtering and modified hybrid Kalman filtering, with the changed order of methods inside (Unscented Kalman Filter and Extended Kalman Filter). For these algorithms, the modification based on double use of Hybrid Kalman Filters (Excented and Unscented) has been proposed. This new modification has been checked for Hybrid Kalman Particle Filters too, for the variable number of particles. Based on the obtained results, one can see that duplication of hybrid filters can improve the estimation quality.

1 Introduction

Very important branch of science, especially in the noisy environment of measurements, is state estimation. This paper develops the problem of state estimation of dynamical systems and refers to research from [1], where authors proposed modification of Hybrid Kalman Filter and Hybrid Kalman Particle Filter [2].

There are a lot of applications of state estimation [3, 4] and many different types of estimation methods [5-7]. The need and use of state estimation have been described in details in Introduction of [1].

In Section 2, the main goal of state estimation is presented. In Section 3, one can find description of the two proposed methods. Section 4 presents results of simulation. Conclusions are presented in the last section.

2 Examined algorithms of state estimation

Extended Kalman Filter (EKF) uses linearization of nonlinear plants by developing functions into Taylor series. Unscented Kalman Filter (UKF) does not use linearization, but unscented transformation instead. In Bootstrap Particle Filter (BPF), algorithm draws particles from the transition model and weights are calculated based on the measuerment model. In Hybrid Kalman Particle Filter (HKPF), the Probability Density Function (PDF) is used to draw the particles and weights are determined from the results of Hybrid Kalman Filter (HKF), which combine UKF and EKF algorithms.

New algorithms of state estimation are presented in [1], including modified hybrid Kalman filters (HKFmod and HKPFmod) with the changed order of filters (EKF-UKF instead of UKF-EKF). The basic methods like EKF, UKF, BPF were compared with HKF, HKPF and its modifications: HKFmod and HKPFmod.

All above mentioned methods are explained in more details in [1].

Based on these research, one can say that for proposed modification of hybrid Kalman filters the quality of state estimation can be improved. In the next section, another modification of HKFs will be presented.

3 Double hybrid Kalman filtering

New modification of the HKF algorithms is proposed in this section. It is based on duplication of UKF and EKF filtrations, which were used in hybrid methods. It combines double use of EKF and UKF in double HKF (dHKF) algorithm and double use of EKF and UKF, for each particle, in HKPF algorithm (dHKPF). Below, there are presented Doubled Modified Hybrid Kalman Filter (dHKFmod) and Doubled Modified Hybrid Kalman Particle Filter (dHKPFmod) algorithms. They are based on HKFmod and HKPFmod from [1], where for double methods based on traditional HKF and HKPF [2], the order of EKF and UKF algorithms inside were reversed.

<u>Algorithm 1: Doubled Modified Hybrid Kalman Filter</u>

$$[\boldsymbol{x}_{EKF1}{}^{(k|k)}, \mathbf{P}_{EKF1}{}^{(k|k)}] = \mathrm{EKF}[\boldsymbol{x}_{est}{}^{(k\text{-}1)}, \mathbf{P}^{(k\text{-}1)}, \boldsymbol{y}^{(k)}]$$

$$[\boldsymbol{x}_{UKF1}{}^{(k|k)}, \mathbf{P}_{UKF1}{}^{(k|k)}] = \mathrm{UKF}[\boldsymbol{x}_{EKF1}{}^{(k|k)}, \mathbf{P}_{EKF1}{}^{(k|k)}, \boldsymbol{y}^{(k)}]$$

$$[\boldsymbol{x}_{EKF2}{}^{(k|k)}, \mathbf{P}_{EKF2}{}^{(k|k)}] = \mathrm{EKF}[\boldsymbol{x}_{UKF1}{}^{(k|k)}, \mathbf{P}_{UKF1}{}^{(k|k)}, \boldsymbol{y}^{(k)}]$$

$$[\boldsymbol{x}_{UKF2}{}^{(k|k)}, \mathbf{P}_{UKF2}{}^{(k|k)}] = \mathrm{UKF}[\boldsymbol{x}_{EKF2}{}^{(k|k)}, \mathbf{P}_{EKF2}{}^{(k|k)}, \boldsymbol{y}^{(k)}]$$

$$\boldsymbol{x}_{\mathrm{est}}{}^{(k)} = \boldsymbol{x}_{UKF2}{}^{(k|k)},\ \mathbf{P}^{(k)} = \mathbf{P}_{\mathrm{UKF2}}{}^{(k|k)}$$

* Corresponding author: jacek.michalski95@wp.pl

Algorithm 2: Doubled Modified Hybrid Kalman Particle Filter

1. Initialization. Draw N_p particles from initial PDF $\boldsymbol{x}^{i,(0)} \sim p(x^{(0)})$.
2. For each particle, obtain $\boldsymbol{x}_{est}{}^{i,(k)}$ and $\mathbf{P}^{i,(k)}$ ($i = 1,\dots,N_p$) according to Algorithm 1.
3. Prediction. Draw N_p particles values $\boldsymbol{x}^{i,(k)} \sim g(\boldsymbol{x}^{(k)}|\boldsymbol{x}^{i,(k\text{-}1)}, \boldsymbol{y}^{(k)}) = N(\boldsymbol{x}_{est}{}^{i,(k)}, \mathbf{P}^{i,(k)})$.
4. Update. Calculate particles weights from $q^{i,(k)} = p(\boldsymbol{y}^{(k)}|\boldsymbol{x}^{i,(k)})\, p(\boldsymbol{x}^{i,(k)}|\boldsymbol{x}^{i,(k\text{-}1)})\, g^{-1}(\boldsymbol{x}^{i,(k)}|\boldsymbol{x}^{i,(k\text{-}1)}, \boldsymbol{y}^{(k)})$.
5. Normalization. Scale the weights in such a way that their sum be equal to one.
6. Resampling – re-drawing of particles (systematic resampling was used [8]).
7. End of the iteration. Calculate the estimate, update time step $k = k + 1$, go to Step 2.

4 Simulation results

For simulations the plant based on the power system model, was used. The choice was dedicated by the fact that in power system each additional node is associated with two additional state variables (voltage magnitude and phase angle), and, in the proposed network, only one state is related to each node. The plant contains 4 state variables (is composed of 4 nodes). This system was proposed and described as the 4-th version of system presented in [9].

$$
\begin{aligned}
x_i^{(k+1)} &= x_i^{(k)} + v_i^{(k)},\ i = 1,\dots,4,\\
y_{ij}^{(k)} &= \textstyle\sum_{j=1,\dots,Nx} [x_i^{(k)} x_j^{(k)} \mu_{ij} \sin(x_i^{(k)} - x_j^{(k)} - \mu_{ij})],\ i = j,\\
y_{ij}^{(k)} &= [x_i^{(k)}]^2 - x_i^{(k)} x_j^{(k)} \mu_{ij} \cos(x_i^{(k)} - x_j^{(k)} - \mu_{ij}),\ i \neq j,\\
\mu_{41} &= \mu_{14} = 0;\ \text{for other } i,j\ \mu_{ij} = 1.
\end{aligned}
$$

For each method, simulations with M = 1000 time steps were performed. Each simulation was repeated minimum 1000 times in order to decrease standard deviations (according to 68-95-99.7 rule [10]). Estimation quality was evaluated by *aRMSE* (average RMSE – mean of *RMSE* values through all state variables) quality index [6]. The simulation results are presented in Fig. 1.

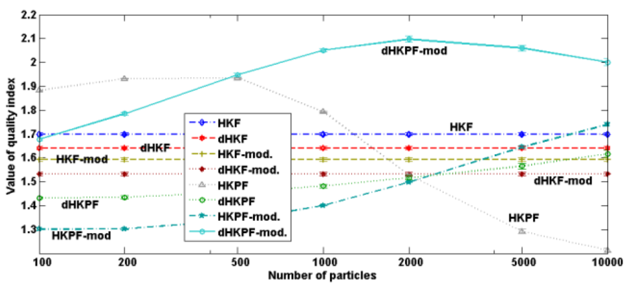

Fig. 1. Values of *aRMSE* quality index.

5 Conclusions

Based on the simulation results, one can see that duplication of hybrid Kalman filters can reduce the index value, but calculation time always increases for this modification, because EKF and UKF methods are triggered two times. HKF and HKFmod with duplicated filtration work better than single ones. Changed versions of HKPF and HKPFmod in this paper work better for applicable number of particles. The influence of changed order of filters in hybrid Kalman filters was shown in [1]. Also, there was confirmed that as the number of particles increases, particle filter quality gets better.

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