

# Features of the construction of information transfer system which using two-dimensional signals

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**Abstract.** The article deals with analysis of the causes of intersymbol interference and interchannel interference. It is indicated that physically unrealizable orthogonal bases are used to describe systems and signals. The considered interference occurs due to the loss of orthogonality by the coordinate signals of the bases. Known methods for obtaining systems of orthogonal functions do not allow the formation of a coordinate basis corresponding to physically feasible systems and signals. It is proposed to use equidistant biased impulse characteristics of physically realizable linear systems as basic signals. An orthogonalization method based on determining the weight of orthogonality is described. It is shown that the resulting basis is quasi-orthogonal. It is determined that the conversion of the standard low-pass prototype filter into the filters of channel-forming equipment does not change the conditions of orthogonality. Structural schemes of a modulator and a demodulator of two-dimensional signals are proposed, based on the developed method of orthogonalization.

## 1 Target setting

The characteristic of modern digital information transmission systems (ITS) is that the levels of intersymbol interference (ISI) and inter-channel interference (ICI) is higher, than the noise level in the communication channel. The modern theory claims that the simultaneous decrease in the levels of ISI and ICI to any target value with the stipulated parameters of adjoined communication channels is difficult. It should be noted that even a slight increase in the noise immunity of systems leads to a significant complication of ITS.

Consider the causes of ISI and ICI and the relationship between them.

ISI is caused by the overlapping in time of the responses of linear devices of channel-forming equipment (CAE) to various elementary signals that carry information about the transmitted symbols, as a result of which the decoding of one symbol is influenced by several previous, and in subsequent channels with a large group delay time, subsequent symbols. ISI also appears as a result of multipath propagation of radio waves.

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The reason for the ICI is the penetration on the output of CAE of one signal channel of the signals of the next channels due to the overlap in frequency of the amplitude-frequency characteristics (AFC) of the filters of the channels of CAE.

Each signal, that carry information, is a linear combination of signals of the coordinate orthogonal basis. From this point of view, the ISI is a consequence of the waste of orthogonality by the signals by means of which message symbols are transmitted; and ICI arise as a result of the fact that the signals transmitted in adjacent communication channels, due to non-ideal frequency response of the channels, stops to be orthogonal.

Thus, the elimination of ISI and ICI is associated with the choice of an orthogonal signal basis.

The theory of special functions and the theory of linear integral transforms are used to analyze the general properties of systems of orthogonal functions and to obtain such systems.

The method of studying orthogonal series proposed by the theory of special functions is based on the study of the differential properties of the orthogonality weight of these functions. In accordance with this theory, the weight of orthogonality should be a non-negative function. The theory of special functions substantiates the Gram-Schmidt orthogonalization method, which allows one to obtain an ensemble of orthogonal functions from a system of linearly independent functions. The disadvantage of this theory is that it allows you to calculate orthogonal functions by a known weight, but does not answer the question of how to determine the orthogonality weight for already known linearly independent functions. In addition, the Gram-Schmidt orthogonalization method does not make it possible to take full advantage of many linearly independent functions, since the resulting orthogonal functions differ in form from the original ones. For example, for the most part, wavelet systems are systems of linearly independent non-orthogonal functions. Wavelet series converge quickly, because the basic functions are "similar" to the expandable function. The usage of the specified method of orthogonalization leads to a decrease in the rate of convergence of the series.

In the theory of linear integral transforms, it is proved that the own functions of these transformations are orthogonal. A special case of linear integral transforms is the Hilbert transforms with the reproducing kernel (GTRK). To describe the function space are used the own functions of the GTRK, reference theorems are proved. The most famous of them is V.A. Kotelnikov. Using the sample functions for the coordinate description of the signals is equivalent to expanding the signals in a series according to the equidistant offset impulse characteristics of an ideal linear system. However, ideal linear systems are not physically feasible. Their equidistant biased impulse responses are not orthogonal.

In addition, the signal system composed of all equidistant offset impulse characteristics of the information transmission systems of neighboring communication channels is not orthogonal.

Therefore, ISI and ICI are associated with the physical feasibility of ITS.

So, there is a scientific problem: on the one hand, the physical feasibility of information transmission systems imposes restrictions on the choice of the system of orthogonal functions required to describe signals; on the other hand, the existing theory of orthogonal series and integral transformations does not allow the creation of the indicated system of orthogonal functions.

Thus, the solution to the problem of simultaneously reducing the levels of ISI and ICI, and, consequently, increasing the noise immunity and efficiency of ITS is associated with the development of new methods for describing signals and information transmission systems.

In addition, these new methods should not lead to a significant complication of ITS.



necessary to introduce  $k = N(N + 1)/2$  the functions  $l_1(t), l_2(t), \dots, l_k(t)$ , that make up this weight function. In addition, a numerical experiment shows that depending on which functions  $l_i(t)$  are selected, different values can be obtained

$$I = \int_{t_1}^{t_2} h^2(t) dt, \tag{3}$$

called weight energy.

Usually a system consisting of an infinite number of functions. In this case, the weight has the form of a row.

Lemma 2 [1]. The weight optimal under the condition of minimum energy is a quadratic form of orthogonalizable functions.

$$h(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \lambda_{mn} \varphi_n(t) \varphi_m(t). \tag{4}$$

In order to determine the Lagrange multipliers, it suffices to substitute expression (4) into equations (1).

The energy of weight (3) is equal to the sum

$$I = \sum_{n=-\infty}^{\infty} \lambda_{nn}.$$

A feature of this method is the fact that the weight obtained is an alternating function (the classical theory of special functions claims that the weight of orthogonality should be a non-negative function).

Consider the features of orthogonalization of physically realized functions.

In practice, they approximate the ideal characteristics of CAE.

At the first stage, they switch to normalized prototype low-pass filters (NPF) of Chebyshev, Butterworth, Bessel and elliptic filters of order  $N$ . The transfer functions of NPFs are written as

$$K_{\text{Oix}}(s) = K_{\text{Oix}} \frac{1}{\prod_{j=1}^N (s - p_j)},$$

( $K_{\text{Oix}}$  — filter gain) have simple poles  $p_j$  and, therefore, impulse characteristics of the form

$$\varphi_0(t) = l(t) \sum_{k=1}^{N/2} A_k e^{\sigma_k t} \sin(\omega_k t + \vartheta_k), \tag{5}$$

when  $N$  is even number;

$$\varphi_0(t) = l(t) A_0 e^{\sigma_0 t} + \sum_{k=1}^{N-1} A_k e^{\sigma_k t} \sin(\omega_k t + \vartheta_k), \tag{6}$$

if  $N$  is an odd number,

where  $l(t)$  is the Heaviside function  $A_0, A_k, \vartheta_k$  are some known constant values  $\sigma_k$  and  $\omega_k$ , and are the real and imaginary parts of the  $k$ th pole of the transfer function CAE:  $\sigma_k + j\omega_k = p_k$ .

Let us introduce the system of functions obtained by shifting the impulse response of the LPF by the time interval  $\alpha$

$$\varphi_m(t) = 1(t - m\alpha)\varphi_0(t - m\alpha), \quad (7)$$

It is shown in [1] that the conditions of orthogonality of functions (7) can be observed only for the first  $N$  functions  $\varphi_m(t)$ , that is, a system composed of equidistantly shifted impulse characteristics of a CAE can only be a quasi-orthogonal system. An increase in the order of the filter reduces the error of the conditions of orthogonality. As an example, in [1], systems of functions composed of biased impulse responses of normalized Butterworth filters were considered.

At the second stage, the calculation of CAE is carried out by converting the characteristics of the NPF into a filter that forms a communication channel.

The NPF can be characterized by the duration of the transition process  $t_{ux} \cong 5\tau_k$ , where  $\tau_k$  is the maximum time constant corresponding to the minimum attenuation  $\sigma_k$ , which, in turn, corresponds to the  $k$ th pole  $p_k$ .

$$\alpha = \frac{t_{ux}}{N} = \frac{5\tau_k}{N} = \frac{5}{\sigma_k N}.$$

The impulse response  $g(t)$  of the NPF is recorded in the form (5, 6).

The conversion of the transfer function of the NPF  $K_{NPF}(p)$  to the transfer function of the low-pass filter with a cutoff frequency  $K_{LPF}(p)$  of 3 dB is carried out by formal replacement  $p$  with  $\frac{p}{\omega_c}$ . In [1], it was shown that the conversion of the NPF and the low-pass filter with a cutoff frequency  $\omega_c$  of 3 dB does not change the orthogonality conditions for equidistant biased impulse response.

The transfer function of the NPF  $K_{NPF}(p)$  to the transfer function of a band-pass filter (BF) with a passband  $\Delta\omega$  of 3 dB and a center frequency  $\omega_0$  is converted by formal replacement  $p$  with  $\frac{p^2 + \omega_0^2}{\Delta\omega p}$ . Notice, that  $\Delta\omega = 2\omega_c$ .

In this case, the order of the BF  $N_{i\hat{O}} = 2N$ , and the root  $p_k$  of the characteristic equation of the NPF corresponds to two roots  $\hat{p}_{k1}$  and  $\hat{p}_{k2}$  the characteristic equation of the BF.

Calculations show that the conversion of NPF to BF with a bandwidth  $\Delta\omega$  determined by the level of 3 dB does not change the conditions of orthogonality of equidistant biased impulse responses. In this case, compliance with the conditions

$$\omega_0 \hat{\alpha} = 2\pi l, \quad \hat{\alpha} = \frac{\hat{t}_{ux}}{N},$$

where  $\hat{\alpha}$  is the shift interval of the pulse characteristics of the BF,  $\hat{t}_{ux}$  is the duration of the transition process of the BF [1].

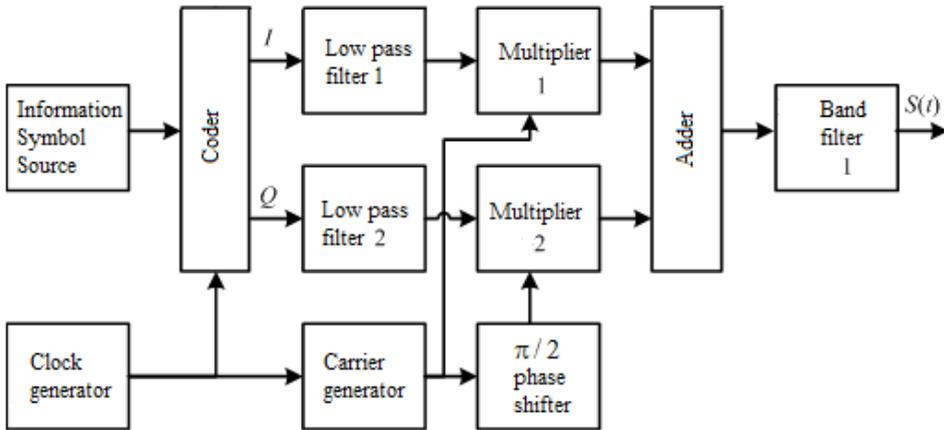
Consider the features of building information transmission systems, that implement a method of reducing intersymbol and interchannel interference.

### 3 Two-dimensional signal modulator

Two-dimensional signals include: phase and amplitude-phase modulation, quadrature amplitude modulation and quadrature modulation signals in the form of weight responses. The block diagram of the modulator of such signals is shown in Figure 1. The output signal of the modulator  $S(t)$  is the sum of two quadrature components  $s_I(t)$  and  $s_Q(t)$ . When transmitting the  $j$ -th information symbol at the outputs  $I$  and  $Q$  of the encoder via the communication channel, we have  $u_I(t) = a_j u(t - jT)$  and  $u_Q(t) = b_j u(t - jT)$ , where the coefficients  $a_j$  and  $b_j$  determine the coordinate of the  $j$ -th information symbol on the plane,  $u(t)$  is a rectangular pulse whose duration is equal to the oscillation period of the clock frequency  $T$ . Low-pass filters (LPF) 1 and 2 are identical and have a pulse characteristic  $g(t)$ . The signals taken from the outputs of the LPF 1 and the LPF 2 are described by convolutions:

$$y_I(t) = a_j u(t - jT) \oplus g(t - jT), \quad (8)$$

$$y_Q(t) = b_j u(t - jT) \oplus g(t - jT). \quad (9)$$



**Fig. 1.** The structural diagram of the modulator of two-dimensional signals.

The first inputs of multipliers 1 and 2 receive signals (8) and (9), and the second, respectively

$$v_I(t) = \cos \omega_0 t,$$

$$v_Q(t) = \sin \omega_0 t,$$

where  $\omega_0$  is the frequency of the carrier oscillation.

The signal at the output of the adder is

$$s_j(t) = (a_j \cos \omega_0 t + b_j \sin \omega_0 t)(u(t - jT) \oplus g(t - jT)).$$

Since the level of samples of intersymbol interference significantly depends on the instantaneous values of the transient signals occurring in the LPF 1 and LPF 2, as well as in the filters of the demodulator, for simplicity we can assume that the signals at the outputs  $I$  and  $Q$  of the encoder have the form

$$u_{I_j}(t) = a_j \delta(t - jT), \quad (10)$$

$$u_{Q_j}(t) = b_j \delta(t - jT), \quad (11)$$

where  $\delta(t)$  is the Dirac function.

In view of (10) and (11), expressions (8) and (9) will take the form

$$y_{I_j}(t) = a_j g(t - jT),$$

$$y_{Q_j}(t) = b_j g(t - jT),$$

and the signal at the output of the adder is written as

$$s_j(t) = (a_j \cos \omega_0 t + b_j \sin \omega_0 t) g(t - jT). \quad (12)$$

In the case when a sequence of information symbols is transmitted over the communication channel, the signal at the output of the adder takes the form

$$S(t) = \sum_{j=0}^M (a_j \cos \omega_0 t + b_j \sin \omega_0 t) g(t - jT). \quad (13)$$

## 4 Demodulation of two-dimensional signals

Let the functions  $g(t - kT)$  satisfy the conditions:

$$\left\{ \begin{array}{l} J_1 = \int_{nT}^{nT+\alpha} g(t - nT) g(t - kT) \cos^2 \omega_0 t \rho(t) dt = \begin{cases} 1, k = n, \\ 0, k \neq n, \end{cases} \\ J_2 = \int_{nT}^{nT+\alpha} g(t - nT) g(t - kT) \cos \omega_0 t \sin \omega_0 t \rho(t) dt = 0, \\ J_3 = \int_{nT}^{nT+\alpha} g(t - nT) g(t - kT) \sin^2 \omega_0 t \rho(t) dt = \begin{cases} 1, k = n, \\ 0, k \neq n, \end{cases} \end{array} \right. \quad (14)$$

where  $\alpha$  is the time interval for deciding, which of the information symbols was transmitted,  $\rho(t)$  is the weight function.

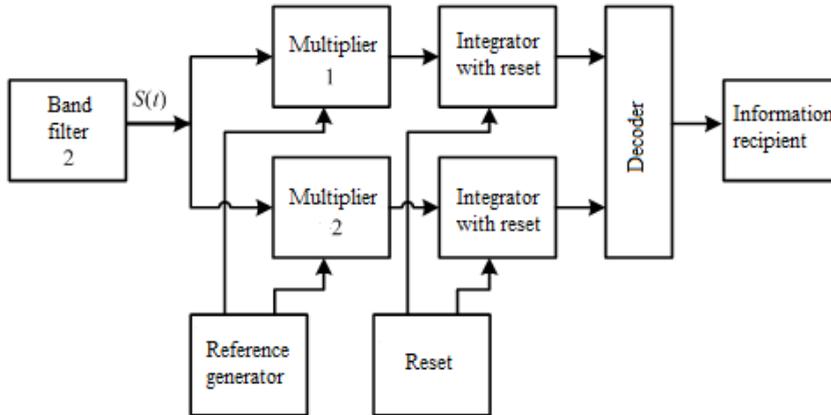
The existence condition is the consistency of the system of equations (14).

Under conditions (14), the structural diagram of the demodulator of two-dimensional signals can be represented by the circuit shown in Figure 2.

The first inputs of the multipliers receive a signal  $S(t)$ , that is recorded in accordance with expression (13). The second inputs of the multipliers 1 and 2 are fed, respectively, reference oscillations

$$z_I(t) = \sum_{k=1}^{\infty} g(t - kT) \cos \omega_0 t \rho(t), \quad (15)$$

$$z_Q(t) = \sum_{k=1}^{\infty} g(t - kT) \sin \omega_0 t \rho(t). \quad (16)$$



**Fig. 2.** The structural diagram of the demodulator of two-dimensional signals.

From the output of the upper integrator we get a signal

$$\int_{jT}^{jT+\alpha} S(t)z_I(t)dt =$$

$$= \int_{jT}^{jT+\alpha} \sum_{n=0}^M (a_n \cos \omega_0 t + b_n \sin \omega_0 t) g(t-nT) \sum_{k=1}^{\infty} g(t-kT) \cos \omega_0 t \rho(t) dt = a_j,$$

and from the output of the lower integrator —

$$\int_{jT}^{jT+\alpha} S(t)z_Q(t)dt =$$

$$= \int_{jT}^{jT+\alpha} \sum_{n=0}^M (a_n \cos \omega_0 t + b_n \sin \omega_0 t) g(t-nT) \sum_{k=1}^{\infty} g(t-kT) \sin \omega_0 t \rho(t) dt = b_n.$$

The integration time of the input signals by each integrator is  $\alpha$ .

## 5 Conclusion

Structural diagrams of the ITS modulator and demodulator with a lowered level of ISI and ICI can be constructed according to the classical schemes of the SPI modulator and demodulator ITS, which uses two-dimensional signals to transmit information. The difference of the proposed system from known systems is in the method of processing the received signals. So, the reference signal generator of the correlator of the proposed receiver does not produce a copy of the received signal, but a copy of the received signal, multiplied with the weight function. Thus, the proposed method for processing received signals does not lead to a significant complication of the information transmission system.

## References

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