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Sliding signal processing in telecommunication networks based on two-dimensional discrete Fourier transform

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Abstract. A method of vertical sliding processing of two-dimensional discrete signals in the spatial frequency domain is proposed — a method of fast vertically sliding two-dimensional discrete Fourier transform. The mathematical representation of the two-dimensional discrete Fourier transform in algebraic and matrix form is considered. An effective method of vertically sliding two-dimensional discrete Fourier transform is proposed. The algorithm developed in the framework of the proposed method allows calculating the coefficients (bins) of this transformation in real time.

1 Introduction

It is difficult to overestimate the role and place of digital spectral processing of discrete one-dimensional (1-D) and two-dimensional (2-D) signals in telecommunication networks (telephone, computer, television networks and radio networks). The importance and relevance of the development and improvement of the methods of digital spectral processing of 1-D and 2-D signals is increasing due to the creation of telecommunication universal multiservice networks that can equally effectively transmit any type of information: data, sound and video.

The classical method of spatial-frequency processing of two-dimensional discrete signals is the 1-D and 2-D discrete Fourier transform (DFT), which allows to obtain a 1-D and 2-D frequency spectrum [1-13]. At the same time, there are a number of applications [9-12] where it is necessary to find the values of the frequency spectrum not at all 1-D and 2-D frequencies, but at a subset of them. In this case, the application of the full version of 1-D and 2-D DFT, even on the basis of the fast Fourier transform (FFT), becomes ineffective, since most of the obtained 1-D and 2-D DFT coefficients are not used. The solution to this problem for 1-D signals is given in [9].

The article discusses the solution of this problem for 2-D signals, introduces the concept of moving spatial-frequency processing based on 2-D DFT. Specifically, the vertical spatial-frequency processing of 2-D signals is considered.

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2 Direct two-dimensional discrete Fourier transform

Suppose we are given a discrete two-dimensional signal $x(n_1, n_2)$ in the form of a two-dimensional sequence of finite length $0 \le n_1 \le (N_1 - 1)$ and $0 \le n_2 \le (N_2 - 1)$ or with a matrix of size $n_1 \times n_2$ in a rectangular reference zone (specifically on a plane).

The direct two-dimensional discrete Fourier transform (2-D DFT) of a two-dimensional signal $x(n_1, n_2)$ is a special case of a direct two-dimensional z-transform:

$$S_{N_1, N_2}(k_1, k_2) = X(z_1, z_2) \Big|_{z_1 = W_{N_1}^{k_1},\ z_2 = W_{N_2}^{k_2}} ; \tag{1}$$

and can be specified both in algebraic and in matrix form.

Algebraic form:

$$S_{N_1, N_2}(k_1, k_2) = \frac{1}{N_1 \cdot N_2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2)\ \exp\left[-j2\pi\left(\frac{k_1 n_1}{N_1} + \frac{k_2 n_2}{N_2}\right) \right] =$$

$$= \frac{1}{N_1 \cdot N_2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x(n_1, n_2)\ \cdot W_{N_1}^{k_1 n_1} \cdot W_{N_2}^{k_2 n_2} ; \tag{2}$$

where $k_1 = \overline{0, (N_1 - 1)}$, $k_2 = \overline{0, (N_2 - 1)}$ are spatial frequencies; $x(n_1, n_2)$ – two-dimensional signal; $n_1 = \overline{0, N_1 - 1}$, $n_2 = \overline{0, N_2 - 1}$, ; ;
$W^{k_2 n_2}_{N_2} = \exp(-j \frac{2\pi}{N_2}(k_2 n_2))$; $S_{N_1, N_2}(k_1, k_2)$ – coefficients (bins) of 2-D DFT (two-dimensional vector spatial-frequency spectrum).

Matrix form:

$$S_{N_1 \times N_2} = \frac{1}{N_1 \cdot N_2} F^{(2)}_{N_1 \times N_1} \cdot X_{N_1 \times N_2} \cdot F^{(1)}_{N_2 \times N_2} ; \tag{3}$$

where

$$X_{N_1 \times N_2} = \begin{array}{c|ccccc|c} & 0 & 1 & . \ . & (N_2 - 1) & n_2 \\ \hline 0 & x(0,0) & x(0,1) & . \ . & x(0, N_2 - 1) & \\ 1 & x(1,0) & x(1,1) & . \ . & x(1, N_2 - 1) & \\ . & . & . & . \ . & . & \\ . & . & . & . \ . & . & \\ (N_1 - 1) & x(N_1 - 1, 0) & x(N_1 - 1, 1) & . \ . & x(N_1 - 1, N_2 - 1) & \\ n_1 & & & & & \end{array} ; \tag{4}$$

$$F^{(1)}_{N_2 \times N_2} = \begin{array}{c|ccccc|c} & 0 & 1 & . \ . & (N_2 - 1) & k_2 \\ \hline 0 & W_{N_2}^{0 \cdot 0} & W_{N_2}^{0 \cdot 1} & . \ . & W_{N_2}^{0 \cdot (N_2 - 1)} & \\ 1 & W_{N_2}^{1 \cdot 0} & W_{N_2}^{1 \cdot 1} & . \ . & W_{N_2}^{1 \cdot (N_2 - 1)} & \\ . & . & . & . \ . & . & \\ . & . & . & . \ . & . & \\ (N_2 - 1) & W_{N_2}^{(N_2 - 1) \cdot 0} & W_{N_2}^{(N_2 - 1) \cdot 1} & . \ . & W_{N_2}^{(N_2 - 1) \cdot (N_2 - 1)} & \\ n_2 & & & & & \end{array} ; \tag{5}$$

$$F^{(2)}_{N_1\times N_1} = \begin{array}{c} \\ 0 \\ 1 \\ . \\ . \\ (N_1-1) \\ k_1 \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & 1 & . \; . & (N_1-1) & n_1 \end{array} \\ \left[\begin{array}{ccccc} W_{N_1}^{0\cdot 0} & W_{N_1}^{0\cdot 1} & . \; . & W_{N_1}^{0\cdot(N_1-1)} \\ W_{N_1}^{1\cdot 0} & W_{N_1}^{1\cdot 1} & . \; . & W_{N_1}^{1\cdot(N_1-1)} \\ . & . & . \; . & . \\ . & . & . \; . & . \\ W_{N_1}^{(N_1-1)\cdot 0} & W_{N_1}^{(N_1-1)\cdot 1} & . \; . & W_{N_1}^{(N_1-1)\cdot(N_1-1)} \end{array}\right] \\ \\ \end{array} . \qquad (6)$$

Hereinafter there is no loss of generality in omitting the multiplier $1/(N_1 \cdot N_2)$ in (3).

Due to the fact that for the product of the matrices (3) holds the associative property:

$$S_{N_1\times N_2} = F^{(2)}_{N_1\times N_1} \cdot \left[X_{N_1\times N_2} \cdot F^{(1)}_{N_2\times N_2} \right] = \left[F^{(2)}_{N_1\times N_1} \cdot X_{N_1\times N_2} \right] \cdot F^{(1)}_{N_2\times N_2} \qquad (7)$$

and the 2-D DFT core is separable, then, according to (7), you can get the 2-D DFT bins $S_{N_1\times N_2}$, $k_1 = \overline{0,(N_1-1)}$, $k_2 = \overline{0,(N_2-1)}$, in two ways, each consists of two stages.

It is easy to find out that for obtaining $S_{N_1,N_2}(k_1,k_2)$ $k_1 = \overline{0,(N_1-1)}$, $k_2 = \overline{0,(N_2-1)}$ it is necessary to perform $N_1 \cdot N_2$ one-dimensional 2-D DFTs, for calculating which the algorithms of the fast Fourier transform - FFT can be effectively applied.

2. Sliding spatial-frequency processing of discrete signals

Let us consider the spatial-frequency processing of two-dimensional discrete signals in a sliding spatial analysis window. In contrast to the one-dimensional case for the two-dimensional case, there are 4 possible ways of sliding of the spatial analysis window on the original two-dimensional discrete signal:

— 1st way consists of horizontal right shift (HS +) and horizontal left shift (HS–);
— 2nd way consists of vertical shift up (VS +) and vertical shift down (VS–);
— 3rd way consists of right diagonal shift up (RDS +) and right diagonal shift down (RDS–);
— 4th way consists of left diagonal shift up (LDS +) and left diagonal shift down (LDS–).

Fig. 1 shows a star diagram illustrating 4 types of sliding of a spatial analysis window using a discrete two-dimensional signal and examples of its shift.

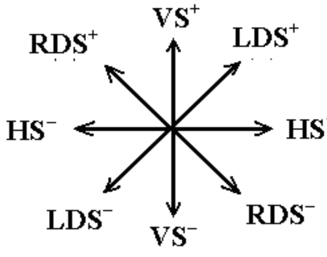

Fig. 1. The star chart illustrating 4 kinds of sliding of a spatial window.

Suppose we need to find the coefficient (bin) of a two-dimensional discrete transform $S_{N_1,N_2}(k_1 k_2)$ at the spatial frequency (k_1,k_2) from the samples of the input signal $x(n_1,n_2)$. In this case, the matrix equation (3) is converted into the form:

$$S(k_1,k_2) = [W_{N_1}^{k_1\cdot 0}, W_{N_1}^{k_1\cdot 1}, .. W_{N_1}^{k_1\cdot (N_1-1)}] \cdot \left\{ X_{N_1\times N_2} \cdot \begin{bmatrix} W_{N_2}^{0\cdot k_2} \\ W_{N_2}^{1\cdot k_2} \\ . \\ . \\ W_{N_2}^{(N_2-1)\cdot k_2} \end{bmatrix} \right\}. \quad (8)$$

At the first stage, according to (8), we multiply the basis function of frequency k_2 and duration N_2 by a matrix of a discrete two-dimensional signal $x(n_1,n_2)$. As a result, we obtain a columned matrix $S_{N_2}(n_1,k_2)$ of size N_2, having spent on this procedure $N_2 \cdot N_1$ complex multiplications and $(N_2-1)\cdot N_1$ complex additions. Further, at the second stage, we multiply the basis function of frequency k_1 and duration N_1 by a columned matrix of size obtained at the first stage, having spent on this procedure N_1 complex multiplications and (N_1-1) complex additions.

Thus, it is necessary to expend $N_2\cdot(N_1+1)$ complex multiplications and $(N_2-1)\cdot(N_1+1)$ complex additions to obtain one coefficient of a two-dimensional discrete transform $S_{N_1,N_2}(k_1 k_2)$ at spatial frequency (k_1,k_2). Considering that performing one complex multiplication requires four real multiplications and two real additions, and one complex addition of two real additions, it is necessary to spend $4\cdot N_1\cdot(N_2+1)$ real multiplications and $4\cdot N_1 N_2 + 2\cdot(N_1-1)$ real additions to obtain the value of one coefficient of a two-dimensional discrete transformation $S_{N_1,N_2}(k_1 k_2)$.

Note that this amount of computation needs to be performed at each shift of a two-dimensional spatial analysis window using a two-dimensional signal. At the same time, it is easy to see that for any kind of shift of a two-dimensional signal a large number of values $X_{N_1\times N_2}$ of the complex matrix in the spatial analysis window remains unchanged.

Note that the shift of the spatial window along a two-dimensional discrete signal can be considered as a shift of a two-dimensional discrete signal in the spatial analysis window in the opposite direction to the movement of the spatial window.

3. The algorithm of vertical sliding processing of 2-D signals based on 2-D DFT

Let us need to find one coefficient (bin) of a two-dimensional discrete transformation $S_{N_1,N_2}(k_1,k_2)$ at a spatial frequency (m_1,m_2) from the samples of the input signal $x(n_1,n_2)$. To obtain a bin, matrix equation (8) is converted to the form:

$$S_{N_1,N_2}(m_1,m_2) = [W_{N_1}^{m_1\cdot 0}, W_{N_1}^{m_1\cdot 1}, .. W_{N_1}^{m_1\cdot (N_1-1)}] \cdot \{X_{N_1\times N_2} \cdot [W_{N_2}^{m_2\cdot 0}, W_{N_{21}}^{m_2\cdot 1}, .. W_{N_2}^{m_2\cdot (N_1-1)}]^T\};(9)$$

where T is the transpose symbol.

The sequence of operations in the algorithm of vertical sliding processing of 2-D signals:

1. Find the column matrix $S_{N_2}(n_1,m_2)$ by size N_1, by multiplying the basis function duration N_2 and frequency m_2 on a matrix of a discrete two-dimensional signal $x(n_1,n_2)$.

2. Remember the column matrix $S_{N_2}(n_1,m_2)$ as a column matrix $S_{N_2}^{(n_1,m_2)}(0)$.

3. Calculate the 2-D coefficient of the discrete transform $S_{N_1,N_2}(m_1,m_2)\cdot$ by multiplying the column matrix $S_{N_2}(n_1,m_2)$ by a basis function of duration N_1 and frequency m_1. This stage ends the output of the algorithm to the operating mode.

4. Carry out the VS – shift of the discrete spatial window by one sample down the two-dimensional signal $x(n_1,n_2)$ and obtain the matrix of the discrete two-dimensional signal $x[(n_1-1),n_2]$.

5. Form a column matrix according to the ratio;

$$S_{N_2}^{(n_1,m_2)}(-1) = S_{N_2}^{((n_1+1),m_2)}(0);\ n_1 = \overline{0, N_1-2}$$

6. Calculate the coefficient value of a two-dimensional discrete transformation $S_{N_1,N_2}(m_1,m_2)$ by multiplying the column matrix $S_{N_2}^{(n_1,m_2)}(-1)$ by a basis function of duration N_1 and frequency m_1.

7. Go to the implementation of paragraph No. 4.

4. Conclusion

The effectiveness of the proposed algorithm for vertical sliding processing of two-dimensional discrete signals in the spatial frequency domain in comparison with the standard method for obtaining the coefficient of two-dimensional discrete transformation $S_{N_1,N_2}(m_1,m_2)$:

• the standard algorithm (8) of the sliding processing of two-dimensional discrete signals in the space-frequency domain requires to obtain a coefficient $S_{N_1,N_2}(m_1,m_2)$ of two-dimensional discrete conversion of $4\cdot N_1\cdot(N_2+1)$ real multiplications $4\cdot N_1\cdot N_2+2\cdot(N_1-1)$ of real additions;

• the developed algorithm for the sliding processing of two-dimensional discrete signals in the space-frequency region after reaching the operating mode requires $4\cdot(N_1+N_2)$ real multiplications and $4\cdot(N_1+N_2-1)$ real additions to obtain a two-dimensional discrete transform coefficient $S_{N_1,N_2}(m_1,m_2)$.

References

1. W.K. Pratt, *Digital image* processing (4th edn. Wiley-Interscience publication, 2007)
2. S.L. Marpl, *Digital Spectral Analysis with applications* (New Jersey, Prentice-hall, 1986)
3. L.P. Yaroslavsky, *Compression, restoration, resampling, "compressive sensing",* J. Optics 17(7), 073001 (2015)

4. M.N. Favorskaya, L.C. Jain, *Development of mathematical theory in computer vision*, (Springer International Publishing, Switzerland, 2015)
5. R.C. Gonzalez, R.E. Woods, *Digital image processing* (4th edn. Published by Pearson, 2018)
6. L. Rabiner, L., B. Gold, *Theory and application of digital signal processing* (New Jersey, Prentice-hall, 1975)
7. W. John, *Multidimensional signal, image, and video processing and coding*(Academic Press is imprint of Elsevier, 2006)
8. V.A. Ponomarev, O.V. Ponomareva, A.V. Ponomarev, *Method for effective measurement of a sliding parametric fourier spectrum* Optoelectronics, Instrumentation and Data Process. **50(2)**, p.p. 1–7 (2014)
9. O. Ponomareva, A. Ponomarev, V. Ponomarev, *Evolution of forward and inverse discrete fourier transform*, IEEE East-West Design & Test Symp., pp. 313–318 (2018)
10. O. Ponomareva, A. Ponomarev, N. Ponomareva*, Window - presume parametric discrete Fourier transform*, IEEE East-West Design & Test Symp., pp. 364–368 (2018)
11. O.V. Ponomareva, N. Ponomareva*, Increased accuracy and extended functionality of digital filters based on frequency sampling*, Devices and methodsof measuremants, v. **2(7),** p.p. 114-119, (2013)
12. Favorskaya, M.N., Jain, L.C., Savchina, E.I.: *Perceptually tuned watermarking using non-subsampled shearlet transform,* Computer Vision in Control Systems-3: Aerial and Satellite Image Processing, ISRL, v. **135**, pp. 41–69. (Springer International Publishing AG 2018)
13. Petrovsky, N.A., Rybenkov, E.V., Petrovsky, A.A.: *Two-dimensional non-separable quaternionic paraunitary filter banks*, IEEE Int. conf. on Signal Processing: Algorighms, Architectures, Arrangements, and Applications, Poznan, Poland, pp. 120–125 (2018)