The criterion for the existence of the weight of the orthogonality of equidistant elementary signals

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Abstract. To reduce the level of intersymbol interference and interchannel interference, it is proposed to form the transmitted signals using the basis functions obtained by shifting the impulse responses of linear systems by multiple time intervals. An algorithm for calculating the weight of the orthogonality of basis functions is proposed. A criterion for the existence of the indicated weight of orthogonality is formulated.

1 Introduction

A feature of modern digital information transmission systems (ITS) is that the levels of intersymbol interference (ISI) and inter-channel interference (ICI) exceed the noise level in the communication channel. In [1], it is shown that the solution to the problem of simultaneously reducing the ISI and ICI levels, and, consequently, increasing the noise immunity and efficiency of the ITS, is associated with the development of new methods for describing signals.

As elementary signals, it is proposed to accept signals that effectively use the bandwidth of the communication channel. Such signals should have the same spectral density moduli. Equidistant signals obtained by shifting the pulse characteristics of a linear system approximating a communication channel by multiple time intervals have a similar property.In the monograph [1], a method of orthogonalization of functions based on determining the weight of orthogonality is proposed. A feature of this method is the fact, that the weight obtained is an alternating function. To determine the weight, it is necessary to solve a system of linear equations with respect to weight coefficients, the number of which is equal to the number of orthogonalizable functions, which with a large number of equations leads to significant computational costs.

The number of equations can be reduced by using the fact that the receiver decides which of the message symbols was transmitted based on a comparison of the output signals of the correlators, each of which is tuned to one of the received signals. So, if various message symbols are transmitted by elementary orthogonal signals, then there is no need to fulfill the requirement of completeness of the system of orthogonal functions. It is enough that each individual signal is orthogonal to all other signals.

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Obviously, knowing the weight of the orthogonality of one of the signals to all other signals of the system, one can determine the weight of the orthogonality of the other signals by shifting it by the interval of the shift of signals.

Since the weight of orthogonality in the general case is an infinite number of known linearly independent functions, to reduce the computational cost, you can limit the error in calculating the weight coefficients to a value that is sufficient for practice.

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2 The algorithm for calculating the weight of the orthogonality of elementary signals

The algorithm for calculating the orthogonality weight of functions $\varphi_1(t),\ \varphi_2(t),\ \ldots,\ \varphi_N(t)$,described in [1] is based on solving a system of equations

$$\int_{t_1}^{t_2} \varphi_i(t)\varphi_j(t)h(t)\mathrm{d}t = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \tag{1}$$

where h (t) is the desired weight of orthogonality, $(t_1,\ t_2)$ is the interval of fulfillment of conditions (1).

Weight $h(t)$ is suggested to look in the form

$$h(t) = \sum_{i=1}^{N} b_i l_i(t)\,, \tag{2}$$

where $l_1(t), l_2(t), \ldots l_N(t)$ are the known functions, b_i are the desired coefficients, N is the number of orthogonalizable functions.

An algorithm that reduces computational costs is as follows. At the first stage, the weight $h_1(t)$ is calculated for the first N equations in accordance with (1).

So, at the first stage, as a weight estimate, you can take

$$h_1(t) = b_1 l_1(t) + b_2 l_2(t)\,.$$

Then the uranium system for estimating the first two weight coefficients has the form

$$\begin{cases} \int_{t_1}^{t_2} \varphi_0^2(t)\left[b_1 l_1(t) + b_2 l_2(t)\right]\mathrm{d}t = 1, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_1(t)\left[b_1 l_1(t) + b_2 l_2(t)\right]\mathrm{d}t = 0. \end{cases}$$

In the second and subsequent stages, the functions are calculated

$$h_2(t) = h_1(t) + \sum_{i=1}^{N+1} b_i^{/} l_i(t)\,,\quad h_3(t) = h_1(t) + h_2(t) + \sum_{i=1}^{N+2} b_i^{//} l_i(t)\,,$$

and so on.

Will accept

$$h_2(t) = h_1(t) + b_1^{/} l_1(t) + b_2^{/} l_2(t) + b_3^{/} l_3(t),$$

and determine the unknown coefficients from the system of equations

$$\begin{cases} \int_{t_1}^{t_2} \varphi_0^2(t)\left[h_1(t) + b_1^{/} l_1(t) + b_2^{/} l_2(t) + b_3^{/} l_3\right]\mathrm{d}t = 1, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_1(t)\left[h_1(t) + b_1^{/} l_1(t) + b_2^{/} l_2(t) + b_3^{/} l_3\right]\mathrm{d}t = 0, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_2(t)\left[h_1(t) + b_1^{/} l_1(t) + b_2^{/} l_2(t) + b_3^{/} l_3\right]\mathrm{d}t = 0. \end{cases}$$

Further, by accepting

$$h_3(t) = h_1(t) + h_2(t) + b_1^{//} l_1(t) + b_2^{//} l_2(t) + b_3^{//} l_3(t) + b_4^{//} l_4(t)$$

we can continue to calculate the coefficients by solving the system of equations

$$\begin{cases} \int_{t_1}^{t_2} \varphi_0^2(t)\left[h_1(t) + h_2(t) + b_1^{//} l_1(t) + b_2^{//} l_2(t) + b_3^{//} l_3(t) + b_4^{//} l_4(t)\right]\mathrm{d}t = 1, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_1(t)\left[h_1(t) + h_2(t) + b_1^{//} l_1(t) + b_2^{//} l_2(t) + b_3^{//} l_3(t) + b_4^{//} l_4(t)\right]\mathrm{d}t = 0, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_2(t)\left[h_1(t) + h_2(t) + b_1^{//} l_1(t) + b_2^{//} l_2(t) + b_3^{//} l_3(t) + b_4^{//} l_4(t)\right]\mathrm{d}t = 0, \\ \int_{t_1}^{t_2} \varphi_0(t)\varphi_3(t)\left[h_1(t) + h_2(t) + b_1^{//} l_1(t) + b_2^{//} l_2(t) + b_3^{//} l_3(t) + b_4^{//} l_4(t)\right]\mathrm{d}t = 0. \end{cases}$$

After each function calculation, the behavior of the coefficients is determined. If, after M calculations, the inequality

$$\left|b_i\right| > \left|b_i^{/}\right| > \left|b_i^{//}\right| > \ldots > \left|b_i^{//\ldots/}\right| < \varepsilon,$$

then with an error ε sufficient for practical purposes, we can take the value of the *i*th coefficient equal to

$$B_i = b_i + b_i^{/} + b_i^{//} + \ldots + b_i^{//\ldots/}.$$

In the third stage, the weight is taken equal

$$h(t) = \sum_{i=N+M}^{2N+M} B_i l_i(t). \tag{3}$$

3 The criterion for the existence of the weight of orthogonality

The obtained algorithm allows us to formulate a criterion for the existence of an orthogonality weight. If after M calculations at each second stage the inequality

$$\left|b_i\right| > \left|b_i^{/}\right| > \left|b_i^{//}\right| > \ldots > \left|b_i^{//\ldots/}\right| < \varepsilon,$$

then the orthogonality weight given in the form of a weighted sum of (2) functions $l_1(t), l_2(t), \ldots, l_N(t)\ldots$, exists and is equal to (3), where $B_i = b_i + b_i^{/} + b_i^{//} + \ldots + b_i^{//\ldots/}$.

4 Results of a numerical experiment

Let it be required to determine a function $h(t)$ for which the conditions are satisfied

$$\int_{-\infty}^{\infty} \frac{\sin t}{t} \cdot \frac{\sin(t - \pi n)}{t - \pi n} h(t)\mathrm{d}t = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases} \tag{4}$$

We will look for weight $h(t)$ in the form

$$h(t) = B_0 \frac{\sin t}{t} \cdot \frac{\sin t}{t} + \frac{\sin t}{t} \sum_{i=1}^{M} B_i \left(\frac{\sin(t - \pi i)}{t - \pi i} + \frac{\sin(t + \pi i)}{t + \pi i} \right). \tag{5}$$

In accordance with the considered algorithm, we solve the system of equations

$$\begin{cases} \displaystyle\int_{-\infty}^{\infty} \left(\frac{\sin t}{t} \right)^2 h_1(t)\mathrm{d}t = 1, \\ \displaystyle\int_{-\infty}^{\infty} \frac{\sin t}{t} \cdot \frac{\sin(t - \pi)}{t - \pi} h_1(t)\mathrm{d}t = 0, \end{cases}$$

considering that

$$h_1(t) = b_0 \frac{\sin t}{t} \cdot \frac{\sin t}{t} + b_1 \frac{\sin t}{t} \left(\frac{\sin(t - \pi)}{t - \pi} + \frac{\sin(t + \pi)}{t + \pi} \right).$$

We get

$$b_0 = 0{,}563036, \; b_1 = -0{,}563036.$$

Next we accept

$$h_2(t) = h_1(t) + b_0^{/} \frac{\sin t}{t} \cdot \frac{\sin t}{t} + b_1^{/} \frac{\sin t}{t} \left(\frac{\sin(t - \pi)}{t - \pi} + \frac{\sin(t + \pi)}{t + \pi} \right) +$$

$$+ b_2^{/} \frac{\sin t}{t} \left(\frac{\sin(t - 2\pi)}{t - 2\pi} + \frac{\sin(t + 2\pi)}{t + 2\pi} \right),$$

and solve the system of equations

$$\begin{cases} \displaystyle\int_{-\infty}^{\infty} \left(\frac{\sin t}{t} \right)^2 h_2(t)\mathrm{d}t = 1, \\ \displaystyle\int_{-\infty}^{\infty} \frac{\sin t}{t} \cdot \frac{\sin(t - \pi)}{t - \pi} h_2(t)\mathrm{d}t = 0, \\ \displaystyle\int_{-\infty}^{\infty} \frac{\sin t}{t} \cdot \frac{\sin(t - 2\pi)}{t - 2\pi} h_2(t)\mathrm{d}t = 0. \end{cases}$$

We have

$$b_0^{/} = 0{,}026410,\ b_1^{/} = -0{,}026410,\ b_3^{/} = 0{,}589446 .$$

Accept

$$h_2(t) = h_1(t) + b_0^{/}\frac{\sin t}{t}\cdot\frac{\sin t}{t} + b_1^{/}\frac{\sin t}{t}\left(\frac{\sin(t-\pi)}{t-\pi} + \frac{\sin(t+\pi}{t+\pi}\right) +$$

$$+ b_2^{/}\frac{\sin t}{t}\left(\frac{\sin(t-2\pi)}{t-2\pi} + \frac{\sin(t+2\pi}{t+2\pi}\right).$$

Similarly, we obtain

$$b_0^{//} = 0{,}012550,\ b_1^{//} = -0{,}012550,\ b_3^{//} = 0{,}012550,\ b_4^{//} = -0{,}601996 .$$

The calculations show, that a weight of the form (5) for which conditions (4) are satisfied exists. To the nearest hundredths, you can take

$$B_0 = 0{,}601996,\ B_1 = -0{,}601996 .$$

Given the regular behavior of the coefficients, we can assume that

$$B_n = (-1)^n 0{,}601996 \text{, where n=0,1,2,...}$$

Then

$$h(t) = B_0\left(\frac{\sin t}{t}\right)^2 + B_0\frac{\sin t}{t}\sum_{i=1}^{\infty}(-1)^i\left(\frac{\sin(t-\pi i)}{t-\pi i} + \frac{\sin(t+\pi i)}{t+\pi i}\right).$$

Having completed the summation of the series, we obtain

$$h(t) = B_0\frac{\sin 2t}{2t} .$$

The unknown coefficient is determined from the condition

$$\int_{-\infty}^{\infty}\left(\frac{\sin t}{t}\right)^2 h(t)\mathrm{d}t = 1 .$$

We have

$$\int_{-\infty}^{\infty}\left(\frac{\sin t}{t}\right)^2 B_0\frac{\sin 2t}{2t}\mathrm{d}t = 1 ,$$

Where do we get

$$B_0 = \frac{2}{\pi} .$$

We verify the orthogonality condition

$$\int_{-\infty}^{\infty}\frac{\sin t}{t}\cdot\frac{\sin(t-\pi n)}{t-\pi n}\frac{\sin 2t}{\pi t}\mathrm{d}t = \frac{\sin\pi n}{\pi n} .$$

Thus, condition (4) is satisfied. Consider the following example. Let it be required to determine a function, a function *h(t)* for which the conditions are satisfied

$$\int_{-\infty}^{\infty} \frac{\sin t}{t} \cdot \frac{\sin(t-\frac{\pi n}{2})}{t-\frac{\pi n}{2}} h(t)\mathrm{d}t = \begin{cases} 1, & n=0, \\ 0, & n \neq 0. \end{cases} \quad (6)$$

We look for weight *h(t)* in the form

$$h(t) = B_0 \frac{\sin t}{t} \cdot \frac{\sin t}{t} + \frac{\sin t}{t} \sum_{i=1}^{M} B_i \left(\frac{\sin(t-\frac{\pi i}{2})}{t-\frac{\pi i}{2}} + \frac{\sin(t+\frac{\pi i}{2})}{t+\frac{\pi i}{2}} \right). \quad (7)$$

Performing calculations similar to the previous calculations, we obtain

$$b_0 = 37{,}931943\,,\ b_1 = -27{,}910527\,;$$

$$b_0^{/} = 3436{,}120732\,,\ b_1^{/} = -2689{,}467325\,,\ b_3^{/} = 1139{,}131657\,;$$

$$b_0^{//} = 9{,}432245 \cdot 10^5\,,\ b_1^{//} = -1{,}741538 \cdot 10^5\,,\ b_3^{//} = 4{,}278934 \cdot 10^5\,,\ b_4^{//} = -27985{,}28068\,.$$

Thus, the weight of the form (7) at which conditions (6) are satisfied does not exist.

5 The direction of further research

The presented algorithm and the criterion for the existence of the orthogonality weight do not give an answer to the question of how functions should be selected $l_1(t), l_2(t), \ldots, l_N(t) \ldots$. The criterion allows you to determine the convergence of the weight, if the functions $l_1(t), l_2(t), \ldots, l_N(t) \ldots$ are already selected.

The direction of further research is the development of recommendations for the selection of functions $l_1(t), l_2(t), \ldots, l_N(t) \ldots$.

References

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