Optimization of signal processing algorithm for spacecraft identification

A.L. Polyakov¹,*, I.L. Afonin¹, and D.A. Polyakov²

¹Radio Electronics and Information Security Institute of Sevastopol State University
   st. Universitetskaya, 33, Sevastopol, Russia, 299053
²The military unit 17204, Gorodok St., Kolomna-1, Moscow Region, 140401, Russian Federation

Abstract. The article is devoted to the optimization of the signal processing algorithm of uncontrolled radiation (UCR) on-board equipment of spacecraft (SC) in order to identify them.

1 Introduction

The operation of the SC identification system, as noted in [3,4], depends on the possibility of receiving radio signals of the SC UCR and algorithms for processing these signals. Reception of signals of low power (about ~10⁻⁸ W), that is the UCR signals depends on the technical characteristics used by the RES and, as shown in the works provide no particular complexity under certain circumstances. Means of processing of such signals, for example, increase of reliability and efficiency of this processing are not sufficiently studied. This is due to the fact that the processes of instability of the master generators of onboard radio systems (MG ORES), which serve to identify the SC, determine the need to select the optimal algorithm for processing information about these processes with the assessment of the parameters of the selected model against the background of additive noise measurements. Of particular interest is the joint study of slowly changing m(t) and rapidly changing n(t) components of the identification process g(t).

Known signal processing algorithms take into account interfering effects in RES, as a rule, of Gaussian character. The existing techniques and methods of processing under non-Gaussian noise conditions lead to a significant complication of the algorithms for the necessary calculations significantly affecting the speed and quality of identification of the UCR. At present, the phase signal processing, carried out on the basis of the analysis of the intersection points of received zero level signals, is widely used. The rationale and various aspects of the use of such treatment are set out in work [9]. However, the extent of the influence of selective circuits of RES in the distribution of the zero level intersection points has not been investigated. In addition, there is a certain difficulty in constructing an algorithm for processing composite signals, which are signals of UCR. Therefore, the use of computer technology for processing such signals also makes it relevant to develop the principles of discrete algorithms construction that allow for acceptable losses in comparison with optimal processing, to build simpler signal processing algorithms.

* Corresponding author: al_polykov@inbox.ru
2 Main part

In this article the problem of algorithms development of identification – estimation of frequency instability processes of the SC master generators is considered.

The main features of the frequency instability processes of the master oscillators determine the requirements for the algorithms for estimating the parameters of the instability models. The analysis indicates the feasibility of applying recurrent processing procedures.

However, the features of the components of the instability process require the development of specific algorithms for their analysis.

The main difference between the term \( g(t) \) is its very slow change in the observation interval. Therefore, it is advisable to consider it constant at this interval and not to consider it in the instability models analysis.

The component \( m(t) \) of the process is a locally stationary process with a sufficiently long correlation time. As noted earlier, to determine its characteristics it is advisable to use methods of least squares [5] or dynamic filtering [6,9]. In order to reduce computational costs and improve efficiency, it is necessary to use their recurrent modifications with the involvement of a relatively small part of the accumulated information in the processing.

The process component \( n(t) \) of the process describes the fast fluctuations of phase changes in time, the most informative in terms of identifying the features of the master generators. The higher information content of \( n(t) \) compared to \( m(t) \) is due to the presence of unknown Doppler frequency shift components in the component \( m(t) \). The presence of the Doppler shift often leads to the impossibility of identification of the generators on component \( m(t) \).

We consider the component \( m(t) \) of the phase change process \( \varphi(t) \) in the framework of the model described by the equations of state and observation (1)

\[
\begin{align*}
\dot{x}(t) &= Fx(t) + Gg(t) \\
m(t) &= Hx(t) \\
\end{align*}
\]

(1)

\[
\begin{align*}
z(t) &= H_1m(t) + n(t) \\
z(t) &= \varphi(t) - g(t) \\
\end{align*}
\]

(2)

bearing in mind that the process \( n(t) \) is considered white noise so as its correlation time is significantly less than the correlation time of the component \( m(t) \). This model will be used to build a modification of the dynamic filtering algorithm.

In addition, the estimation of the model parameters of the components \( m(t) \) for the least squares method should be considered

\[
Z(t) = AM(t) + n(t)
\]

(3)

where \( A \) is the communication matrix;
\( M(t) \) – vector of the linear model parameters estimated by LSM;
\( n(t) \) – a rapidly changing component of the instability process,

\[
z(t) = \varphi(t) - q(t) = m(t) + n(t)
\]

The practice of studying stationary fast fluctuating processes, such as \( n(t) \), shows a high efficiency of application for their description of autoregressive models [1,2,8]. Therefore, in the future, the description and analysis of the components \( n(t) \) will be carried out in terms of autoregressive models:
where \( a_i \) - the coefficients of the autoregressive model;

\[
e_1(t) = \sum_{i=1}^{n} a_i n(t - i) + e_1(t - i);
\]

\( e_1(t) \) – generating noise,

\[
e_2(t) = e_1(t) - \sum_{i=1}^{n} a_i e_2(t - i);
\]

\( e_2(t) \) – white measurement noise

\( y(t) \) is the process of oscillation phase change caused by a weakly correlated instability component;

\( n(t) \) – component of the process.

We obtain the relations that determine the algorithms for estimating the components \( m(t) \) and \( n(t) \) for models (1)...(4).

Taking into account the peculiarities of the instability model, let’s investigate the possibility of constructing a recurrent LSM algorithm on a “sliding” window. To do this, first of all, consider the linear model of the component \( m(t) \) of the phase change process \( \phi(t) \), in the form

\[
Z(t) = AH + n(t)
\]

with the measurement vector, where \( z_1...z_N \) – the essence of measuring the phase of the signal at a time \( t_1...t_N \), matrix \( A = [A_1^T, ..., A_n^T] \),

where - i-th row of matrix A, vector of estimated parameters \( M = [M_1, ..., M_r]^T \) and vector of random measurement errors \( [n_1, ..., n_n]^T \),

where \( n_1, ..., n_n \) is the value of the component \( n(t) \) at a time \( t_1...t_n \). Value on the method of least squares can be derived based on the measurements \( zn+1...zn+\ell \) using matrix row \( A c(n+1) \) to \( (n+\ell) \).

Define

\[
z(n, n + \ell) = (z_{n+1}, ..., z_{n+\ell})^T
\]

\[
A(n, n + \ell) = [A_{n+1}^T, ..., A_{n+\ell}^T]
\]

a block consisting of \( \ell \)-rows matrix. We denote as the product of

\[
[A^T(n, n + \ell)A(n, n + \ell)]^{-1}
\]

Then the estimate of the vector \( M(e) \) at the time \( tm+1 \) when involved in the processing of measurements from \( tm \) to \( tm+\ell \).
\[ \tilde{M}(n, n + \ell) = \sum_{n=1}^{T} (n, n + \ell) A^T (n, n + \ell) z(n, n + \ell) \]  
(5)

The evolution of the estimate \( \tilde{M}(n, n + \ell) \) with the change \( n \) is described by the following system of recurrent relations [8]:

1. Operator for the new dimension selection

\[ \tilde{M}(n, n + \ell + 1) = \tilde{M}(n, n + \ell) + K^{(1)}(n, n + \ell) \left[ z_{n + \ell + 1} - A^T_{n + \ell + 1} \tilde{M}(n, n + \ell) \right] \]  
(6)

\[ K^{(1)}(n, n + \ell) = \sum_{n=1}^{T} (n, n + \ell) A_{n+\ell+1} \left[ J + A^T_{n+\ell+1} \sum_{n=1}^{T} (n, n + \ell) A_{n+\ell+1} \right]^{-1} \]  
(7)

\[ \sum_{n=1}^{T} (n, n + \ell + 1) = \left( J - K^{(1)}(n, n + \ell) A^T_{n+\ell+1} \right) \sum_{n=1}^{T} (n, n + \ell) \]  
(8)

where \( \ell \) is the unit matrix, or in expanded form

\[ \sum_{n=1}^{T} (n, n + \ell + 1) = \sum_{n=1}^{T} (n, n + \ell) - \sum_{n=1}^{T} (n, n + \ell) A_{n+\ell+1} \times \]  
(9)

\[ \left( A^T_{n+\ell+1} \sum_{n=1}^{T} (n, n + \ell) A_{n+\ell+1} + 1 \right)^{-1} A_{n+\ell+1} \sum_{n=1}^{T} (n, n + \ell) \]

2. Operator of forgetfulness

\[ \tilde{M}(n + 1, n + \ell + 1) = \tilde{M}(n, n + \ell + 1) - K^{(2)}(n, n + \ell + 1) \times \]  
(10)

\[ \left[ z_{n+1} - A^T_{n+1} \tilde{M}(n, n + \ell + 1) \right] \]

\[ K^{(2)}(n, n + \ell + 1) = \sum_{n=1}^{T} (n, n + \ell + 1) A_{n+1} \left[ J - A^T_{n+1} \sum_{n=1}^{T} (n, n + \ell + 1) A_{n+1} \right] \]  
(11)

\[ \sum_{n=1}^{T} (n+1, n + \ell + 1) = \left( J + K^{(2)}(n, n + \ell + 1) A^T_{n+\ell+1} \right) \sum_{n=1}^{T} (n, n + \ell + 1) \]  
(12)

or in expanded form

\[ \sum_{n=1}^{T} (n + 1, n + \ell + 1) = \sum_{n=1}^{T} (n, n + \ell + 1) + \sum_{n=1}^{T} (n, n + \ell + 1) \times \]  
(13)

\[ A_{n+1} \left[ J - A^T_{n+1} (n, n + \ell + 1) A_{n+1} \right]^{-1} A^T_{n+1} \sum_{n=1}^{T} (n, n + \ell + 1) \]
The vectors $K(1)$ and $K(2)$ of dimension $r+1$ are called respectively the transmission coefficient with the introduction of the new dimension and forgetting. You can use another convenient for implementation on a computer representation of the transmission coefficient $K(1)$

$$K^{(1)}(n,n+\ell) = \sum (n,n+\ell+1)A_{n+\ell+1}$$ (14)

and coefficient $K(2)$

$$K^{(2)}(n,n+\ell+1) = \sum (n+1,n+\ell+1)A_{n+1}$$ (15)

Then multiplying the matrix $A^T(n,n+\ell+1)$ and the vector $z(n,n+\ell+1)$ by blocks , we find out that

$$A^T(n+1,n+\ell+1)z(n+1,n+\ell+1) = A(n,n+\ell+1)z(n,n+\ell+1) - A_{n-1}z_{n+1}.$$ (16)

We consider two cases illustrating the application of these results.

Example 3.1. Suppose the results of the phase run-in measurements per second of the process $z(t) = \phi(t) - q(t), z_1, z_2, \ldots, z_N$ have the following structure:

$$z_i = m + n_i, \quad i = 1, \ldots, N,$$

where $m$ is an unknown parameter of phase foray per unit of time to be estimated; $n_1, \ldots, n_N, \ldots$ - process counts $n(t)$ at time $t_1 \ldots t_n$, treated as random measurement errors.

The least squares estimate is a sample mean

$$\hat{m}(n,n+\ell) = \frac{1}{\ell} \sum_{i=n+1}^{n+\ell} z_i.$$ (20)

The measurement of the estimate in time on a sliding window of size $\ell$ is described by the relations:

$$\hat{m}(n,n+\ell+1) = \hat{m}(n,n+\ell) + \frac{1}{\ell+1} [z_{n+\ell+1} - \hat{m}(n,n+\ell)];$$

$$\hat{m}(n+1,n+\ell+1) = \hat{m}(n,n+\ell) + \frac{1}{\ell+1} [z_{n+\ell+1} - \hat{m}(n,n+\ell)].$$

If the errors $n_1, \ldots, n_N, \ldots$, are uncorrelated random variables with zero mean and constant variance $\sigma^2$, then

$$\text{D}\hat{m}(n,n+\ell) = \frac{\sigma^2}{\ell}.$$

Example 3.2. Estimation of the slope change coefficient $m(t)$ on the “sliding window”.

Let the results of measurements of the foray phase per unit of time $z_1, z_2, \ldots, z_N$ can be represented as

$$z_i = m\phi_0 + n_i, \quad i = 1, \ldots, N,$$
where \( m \) – the estimated parameter (coefficient) of proportionality, which relates the phase value \( \varphi_0 \) oscillations with a frequency of \( f_0 \) and the current phase value, different from \( \varphi_0 \) due to instability;

\( n_1, \ldots n_N, \ldots \) - measurement errors, which are the counts of the process \( n(t) \). The LSM score in this case is as follows:

\[
\tilde{M}(n, n + \ell) = \frac{\sum_{i=n+1}^{n+\ell} \varphi_0 z_i}{\sum_{i=n+1}^{n+\ell} \varphi_0^2},
\]

the variance of the estimate \( \text{D} \tilde{m}(n, n + \ell) = \sigma^2 \left[ \sum_{i=n+1}^{n+\ell} \varphi_0^2 \right] \),

where \( \sigma^2 \) is the variance of the measurement noise.

\[
\tilde{M}(n, n + \ell + 1) = \tilde{M}(n, n + \ell) + K^{(1)}(n, n + \ell) [z_{n+\ell+1} - \varphi_0 \tilde{m}(n, n + \ell)];
\]

\[
K^{(1)}(n, n + \ell) = \varphi_0 \sum (n, n + \ell + 1)
\]

\[
\sum (n, n + \ell + 1) = \sum (n, n + \ell) \left[ 1 + \varphi_0^2 \sum (n, n + \ell) \right]^{-1}
\]

\[
\text{D} \tilde{m}(n, n + \ell) = \sigma^2 \sum (n, n + \ell)
\]

The action of the forgetting operator is described similarly:

\[
\tilde{\Theta}(n + 1, n + \ell + 1) = \tilde{\Theta}(n, n + \ell + 1) - K^{(2)}(n, n + \ell + 1) \quad x
\]

\[
x [z_{n+1} - \varphi_0 \tilde{\Theta}(n, n + \ell + 1)];
\]

\[
K^{(2)}(n, n + \ell + 1) = \varphi_0 \sum (n + 1, n + \ell + 1)
\]

\[
\sum (n + 1, n + \ell + 1) = \sum (n, n + \ell + 1) \left[ 1 - \varphi_0^2 \sum (n, n + \ell + 1) \right]^{-1}
\]
Thus, the use of recurrent procedures of the LSMs in the sliding window gives the opportunity to organize a relatively simple procedure for estimating the parameters of the components m(t) process φ(t). Such procedure will be used to process the measurement results of the UCR signal phase.

For a complete study and synthesis of the optimal algorithm for the identification of the SC UCR we analyze the evaluation procedure of the linear Kalman filter with finite memory components m(t) of the process φ(t) alternatively, modification of the LSM (6)...(13).

To do this, consider a dynamic system with discrete time, which is analogous to (1)...(2), and described by the equations of state

\[ x(k + 1) = \Phi(k + 1, k)x(k) + G(k + 1)\xi(k + 1); \]
\[ m(k + 1) = Hx(k + 1) \]

and observation equations

\[ z(k) = H_1m(k) + n(k); \]
\[ z(k) = \varphi(k) - q(k) = m(k) + n(k), \]

where \( \Phi(k + 1, k) \) is the transition matrix; k is a variable indicating the number of the filtration step.

It is well known that the optimal algorithm for estimating the state vector at the interval \([k+1, k+\ell] \) when changing \( \ell \) is described by a system of recurrent Kalman filtering equations

\[ x(k, k + \ell + 1) = \Phi(k + \ell + 1, k + 1)x(k, k + \ell), \]
\[ K^{(1)}(k, k + \ell + 1) = [z(k + \ell + 1) - H_1(k + \ell + 1)Hx(k + \ell + 1)k]x(k, k + \ell + 1) \]

where \( K^{(1)}(k, k + \ell + 1) \) is the matrix gain of the optimal filter of dimension \( m \times s \)

\[ K^{(1)}(k, k + \ell + 1) = R(k, k + \ell + 1)H_1^T(k + \ell + 1) - Q^{-1}(k + \ell + 1) = R(k, k + \ell + 1)H_1^T(k + \ell + 1) \times \]
\[ \left[H_1(k + \ell + 1)H_1R(k, k + \ell + 1)H_1^T(k + \ell + 1) + Q(k + \ell + 1) \right]^{-1} \]

where \( Q \) is the noise intensity matrix of measurements; R – covariance matrix of noise filtration;

\[ R(k, k + \ell + 1) = R(k, k + \ell + 1) - R(k, k + \ell + 1)H_1^T(k + \ell + 1)H^T \times \]
\[ \left[H_1(k + \ell + 1)H^TR(k, k + \ell + 1)H^TH_1(k + \ell + 1) + Q(k + \ell + 1) \right]^{-1} \times \]
\[ H^TH_1(k + \ell + 1)R(k, k + \ell + 1) \]

and the covariance matrix of extrapolation errors is
\[ R(k, k + l + 1) = \Phi(k + l + 1, k + 1)R(k, k + 1)\Phi^T(k + l + 1, k + 1) + 
+ G(k + l + 1, k + 1)Q_1(k + 1) \times 
\begin{align*}
G^T(k + l + 1, k + 1),
\end{align*}
\]

where \( Q_1 \) is the matrix of generating noise intensity.

The system of relations (21)...(22) is described by the operator of the introduction of a new measurement \( z(k+l+1) \). Let's add this system with the operator of forgetting outdated information of the form

\[ \tilde{x}(k+1, k + l + 1) = \tilde{x}(k, k + l + 1) - 
-K^2(k, k + l + 1)z(k + 1) - H_1(k + 1)H^T_1\tilde{x}(k, k + l + 1), \]

and to develop a matrix forgetfulness coefficient \( K(2) \) of dimension \( m \times s \), as proposed in work, we will monitor the evolution of the matrix \( C \), determined by the ratio

\[ \tilde{z}(k, k + 1) = C(k, k + 1)z(k, k + 1), \]

where \( z(k,k+1) \) is a block vector-column of dimension \( s \times 1 \) whose blocks are \( s \)-dimensional vectors of dimensions \( z(k+1),...,z(k+l) \)

\[ z(k, k + 1) = \left\{ z^T(k + 1),...,z^T(k + 1) \right\}^T, \]

so the dimension of the matrix \( C(k,k+1) \) is \( m \times s(l+1) \). The matrix \( C(k,k+1) \) \( m \) dimensional \( m \times s(l+1) \) has a block structure

\[ C(k, k + l + 1) = 
\begin{align*}
J - K^{(1)}(k, k + l)H(k + l + 1)H^T \Phi(k + l + 1, k + 1)C(k, k + l + 1)K^{(1)}(k, k + l),
\end{align*}
\]

and in the transition from \( C(k,k+\ell+1) \) to \( C(k+1,k+\ell+1) \), the leftmost block of dimension \( m \times s(l+1) \) equal to \( K(2)(k,k+\ell+1) \) is discarded, and the remaining block of dimension \( M \times s(\ell+1) \) is multiplied by the matrix on the left

\[ J + K^{(2)}(k, k + l + 1)H_1(k + 1)H \]

At the same time, as can be seen from the above analysis to describe the evolution of the correlation matrix of filtering errors \( R \) here, it is best to focus on the procedure (20)...(21).

### 3 Conclusion

The developed algorithm for processing information about the processes of instability of the ORES master generators can significantly simplify the evaluation of the parameters of the studied models. At the same time, due to the use of a “sliding window” in the assessment of the processed parameters, the quality and efficiency of identification of the SC UCR is increased. However, in real SC identification complexes it is necessary to take into account the effect of noise interference on the functioning of the RES. Therefore, it seems appropriate to analyze the effect of noise on the measurement of UCR RES signals, as well as to investigate the features of the joint evaluation of rapidly changing and slowly changing processes of measuring the phase of MG ORES.
References


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