

Structure of the emission field of ensemble of ultra-wideband chaotic sources

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Abstract. The structure of the emission field of ensemble of independent ultra-wideband chaotic sources in collective emission mode is investigated analytically and numerically, including power density, directivity, and far zone border. The waves emitted by independent individual sources are incoherent; hence in the reception point the created incoherent fields are summed by power, and this summation gives no additional directivity to the ensemble emission pattern. If all the individual antennas are equal, emission pattern of the entire ensemble is the same as the emission pattern of any of the individual emitters.

1 Introduction

Ensembles of transmitters operating in collective emission mode are a popular way of increasing transmission power, which gives higher range or higher SNR in communications [1]. Classical collective emission systems are antenna arrays, in particular, phased antenna arrays [1, 2]. Since classical communication systems operate with narrowband signals, classical antenna arrays emit coherent signals, which gives directed emission patterns, even if only omnidirectional antennas are used [2].

Collective emission mode is also used with ultra-wideband (UWB) signals in the form of ultrashort pulses to increase emission power in radio location [3–6]. This effect is also usually implemented with antenna arrays. In this case, the signals in the reception point appear correlated, and the whole system demonstrates directed emission diagram [3–6].

Unlike antenna arrays, collective emission of direct chaotic transmitter ensembles is based on nonsynchronous (noncoherent) emission of ensemble elements [7, 8]. Each ensemble element has a transmitter with its own independent chaotic oscillator. Since the signals of chaotic oscillators are uncorrelated, the electromagnetic fields of partial emitters are also uncorrelated, both in space and time. As a result, emission characteristics of such an ensemble are essentially different of those of narrowband transmitter ensembles.

2 Analytical model of UWB chaotic source ensemble

Let us investigate emission characteristics of the ensemble of UWB chaotic sources. We will concentrate on energy emission parameters, such as the power field, spatial structure of

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the energy flow, and far zone border. This interest in energy characteristics is caused by the direct chaotic communication scheme, in which incoherent (energy) receiver is used. Also, energy characteristics are considered more appropriate for description of UWB systems [9].

The analysis is made in frequency domain. For simplicity, the scalar case is considered (polarization issues omitted).

The electric field of a single emitter, placed in the origin of spherical coordinates, in far zone looks like a diverging spherical wave [1]:

$$E(\theta, \alpha, r, \omega) = \frac{A\omega}{r\sqrt{4\pi}} S(\omega)D(\theta, \alpha, \omega) \exp[-j(kr + \varphi)], \quad (1)$$

where E is electric field complex amplitude; (θ, α, r) is receiver point coordinates; A is emitted signal strength; $S(\omega)$ is complex spectral density of the signal at the antenna input; $D(\theta, \alpha, \omega)$ is normalized directivity of the emitting antenna; r is distance between the emitter and the receiver; $\varphi(\omega)$ is random phase of the antenna excitation current; $k = \omega/c = 2\pi/\lambda$ is wave number; ω is circular frequency; c is light speed.

Each individual source has its own chaotic oscillator. All oscillators generate band-limited UWB signals of the same frequency band $F = [\omega_{lo}, \omega_{up}]$, where ω_{lo} and ω_{up} are lower and upper frequency boundaries, respectively. However, all oscillators are independent, so their chaotic signals are uncorrelated, as well as the corresponding emission fields.

The number of sources in ensemble is N . Antennas of the sources can be different, their directivity patterns are described with parameters $D_i(\theta, \alpha, \omega)$, $i = 1, \dots, N$.

3 Emission power of ensemble of UWB sources

Instantaneous field power density in reception point $M(\theta, \alpha, r)$ according to Parseval theorem [10] is equal to

$$P(\theta, \alpha, r) = \frac{1}{2Z_0} \int_0^\infty |E|^2 d\omega, \quad (2)$$

where Z_0 is free space wave resistance. Since the total field in reception point $M(\theta, \alpha, r)$ is equal to the sum of the fields of all the emitters [1], the field power density of the ensemble is equal to

$$P_\Sigma = \sum_{n=1}^N P_n(\theta_n, \alpha_n, r_n) = \int_0^\infty \frac{1}{2Z_0} \left| \sum_{n=1}^N E_n \right|^2 d\omega = \frac{1}{2Z_0} \sum_{n=1}^N \int_{\omega_{lo}}^{\omega_{up}} |E_n|^2 d\omega + \frac{1}{2Z_0} \sum_{i \neq j} \int_{\omega_{lo}}^{\omega_{up}} |E_i E_j| d\omega \approx \sum_{n=1}^N P_n. \quad (3)$$

Thus, the power density of the sum field is the sum of the power densities created in this point by the individual ensemble elements. If the ensemble is made of equal elements, its emission power is proportional to the number of its elements.

4 Energy emission pattern of UWB ensemble

In classical radio systems emission directivity is usually described by antenna directivity parameters $D(\theta, \alpha, \omega)$ (for amplitude) and $D^2(\theta, \alpha, \omega)$ (for power). Obviously, this is not very convenient in case of UWB signals which frequency F spans from ω_{lo} to ω_{up} . Here, an

integral (over frequency) parameter is necessary, which means energy (or power) emission characteristics.

To describe directional properties of an UWB source, we derive energy emission pattern (EEP) $H(\theta, \alpha)$ and define it as the energy flow as a function of the direction in space.

At first, let us derive EEP for a single UWB chaotic source, and then consider the whole ensemble.

4.1 Energy emission pattern of a single UWB chaotic source

Mathematically, $H(\theta, \alpha)$ can be defined as the ratio of field power density $P(\theta, \alpha, r)$ of an UWB chaotic emitter with $D(\theta, \alpha, \omega)$ antenna to the power density $P_i(r)$ of the field created in the same point by the same emitter with isotropic (omnidirectional) antenna:

$$H(\theta, \alpha) = \frac{P(\theta, \alpha, r)}{P_i(r)}. \quad (4)$$

Obviously, $P_i(r) = P_0/4\pi r^2$, where P_0 is emission power. Using Eqs. (1) and (2), P_0 can be calculated as

$$P_0 = \frac{A^2}{Z_0} \int_{\omega_b}^{\omega_{up}} |S(\omega)|^2 d\omega. \quad (5)$$

Similarly, instantaneous field power density $P(\theta, \alpha, r)$ is:

$$P(\theta, \alpha, r) = \frac{A^2}{4\pi r^2} \int_{\omega_b}^{\omega_{up}} |S(\omega)|^2 D^2(\theta, \alpha, \omega) d\omega. \quad (6)$$

Thus, EEP of a chaotic emitter is equal to

$$H(\theta, \alpha) = \frac{\int_{\omega_b}^{\omega_{up}} |S(\omega)|^2 F^2(\theta, \alpha, \omega) d\omega}{\int_{\omega_b}^{\omega_{up}} |S(\omega)|^2 d\omega}. \quad (7)$$

Note, that energy emission pattern of an UWB source is determined not only by the antenna directivity, but also by the emitted signal spectrum.

4.2 Energy emission pattern of an ensemble of UWB chaotic sources

Similar to definition (4), we define energy emission pattern $H_{\Sigma}(\theta, \alpha)$ of the ensemble as the energy flow of the ensemble as a function of the direction:

$$H_{\Sigma}(\theta, \alpha) = \frac{P_{\Sigma}(\theta, \alpha, r)}{P_{\Sigma, i}(\theta, \alpha, r)}, \quad (8)$$

where $P_{\Sigma}(\theta, \alpha, r)$ is the field power density of the ensemble, and $P_{\Sigma, i}(r)$ is the field power density of the same ensemble with all its antennas made omnidirectional. As is shown above, in far zone $P_{\Sigma}(\theta, \alpha, r) = \sum_{n=1}^N P_n(\theta_n, \alpha_n, r_n)$, since the signals of independent chaotic sources are uncorrelated, so the corresponding fields are summed by power. The same

refers to the ensemble with omnidirectional antennas, i.e., $P_{\Sigma,i}(\theta, \alpha, r) = \sum_{n=1}^N P_{i,n}(r_n)$, where $P_{i,n}$ is the field power density of n th source in the reception point by isotropic emission. Note that in far zone all r_n can be substituted by r . Consequently, we obtain

$$H_{\Sigma}(\theta, \alpha) = \frac{\sum_{n=1}^N P_{i,n} H_n(\theta, \alpha)}{\sum_{n=1}^N P_{i,n}} = \sum_{n=1}^N a_n H_n(\theta, \alpha), \tag{9}$$

where $a_n = P_{Tx,n} / \sum_{n=1}^N P_{Tx,n}$, and $P_{Tx,n}$ is n th source emission power. Thus, EEP of the ensemble is a linear combination of EEPs of its elements, whereas the weights are the values of the relative emission power of these elements.

This analytical result has a number of important consequences. For example, if all the antennas are equal and are equally oriented in space, i.e., have the same directivities $F_n(\theta, \alpha, \omega) = F_0(\theta, \alpha, \omega)$ and consequently the same EEP $H_n(\theta, \alpha) = H_0(\theta, \alpha)$, then $H_{\Sigma}(\theta, \alpha) = H_0(\theta, \alpha)$, i.e., EEP of the entire ensemble coincides with EEP of each individual element. This means that adding of many signals from the ensemble elements gives no additional directivity. This property of the ensemble of chaotic sources is essentially different of traditional ensembles of coherent emitters (narrowband, as well as UWB ultrashort pulses), which are characteristic of directional emission even if the individual emitters are omnidirectional [2–6].

These theoretical results are confirmed by numerical simulation. For example, in fig. 1 the diagrams of the signal power at the input of the receiver are shown for an ensemble (linear grid) of $N = 16$ independent UWB chaotic sources with circular directivity antennas. As can be seen, EEP of the ensemble repeats the circular form of EEP of the ensemble emitter antennas. As it was expected from relation (9), in numerical experiments, the form of EEP is independent of the distance d between the ensemble emitters, as well as of the ensemble topology. Small variations (within 0.4 dB) of the received signal power are due to variations of the emitted signal power, caused by chaos generation.

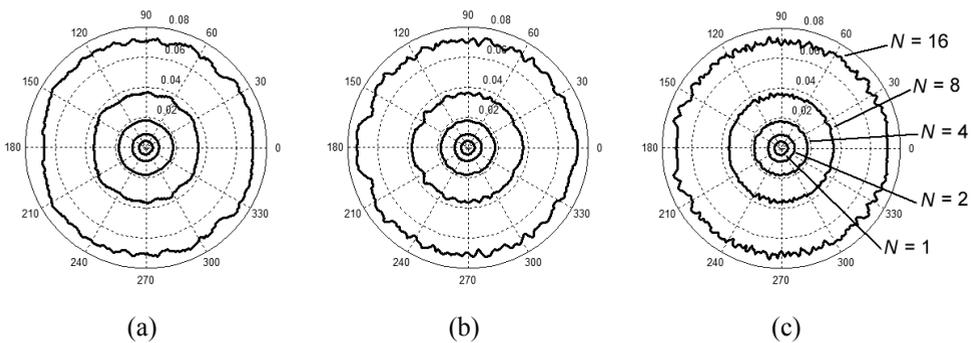


Fig. 1. The signal power $P_{rx}(\alpha, r)$ of the ensemble for $N=1, 2, 4, 8, 16$ UWB chaotic sources. $r = 200\lambda$. Distance of the emitters in the linear grid (a) $d = \lambda/2$, (b) $d = \lambda$, (c) $d = 2\lambda$ (λ is average wavelength of UWB chaotic oscillations).

5 Far zone border of UWB chaotic ensemble

Let us consider the far zone of ensemble of UWB chaotic sources in the case of isotropic (omnidirectional) antennas of individual emitters. As follows from the previous section, in this case the ensemble as whole has no directivity, its emission pattern is also isotropic, thus, in far zone, $r \gg L$, the ensemble wave front is an ideal sphere, where r is the distance

from the geometric center of the ensemble, and L is its maximum geometric size. As the receiver moves from far zone toward the center of the ensemble, and as it approaches the border, this sphere is being gradually distorted, because at the border the differences of the angles and wave paths from different ensemble elements can no longer be ignored.

Since the waves of individual sources that are summed in the reception point are incoherent, and the sum signal power is estimated, then the time delays and phase shifts are irrelevant. So, in order to estimate the far zone border r_{FZ} , we use geometric criterion, i.e., geometric distortion of the wave front. Let us measure the sphere distortion with the ratio of its most distorted diameters (let us denote them D_1 and D_2). For the ideal sphere this ratio is $D_1/D_2 = 1$. If the sphere is slightly squashed, then this ratio becomes $D_1/D_2 = (1+\delta)$, where δ is tolerance. For small δ , we obtain the following equation for the far zone border

$$r_{FZ} = \frac{L}{2} \sqrt{3 \left(1 + \frac{1}{\delta} \right)} \approx \frac{L}{2} \sqrt{\frac{3}{\delta}} \quad (10)$$

i.e., for $\delta = 20\%$ tolerance the border estimate is $r_{FZ} \approx 2.1L$, whereas for $\delta = 10\%$ $r_{FZ} \approx 2.9L$. Such tolerances seem quite reasonable from the practical viewpoint, they correspond to 0.4-0.8 dB distortion of the sphere front. So, a simple estimate for the far zone border r_{FZ} can be written as

$$r_{FZ} = (2-3)L. \quad (11)$$

Note that this estimate differs drastically from the estimate for narrowband signals $r = 2L^2/\lambda$ [1], in particular, it doesn't even include the signal wavelength. The main reason is that here we derive power characteristics, integral over the signal spectrum, and phase relations here are irrelevant.

In fig. 2 the power field of an ensemble with isotropic antennas is depicted.

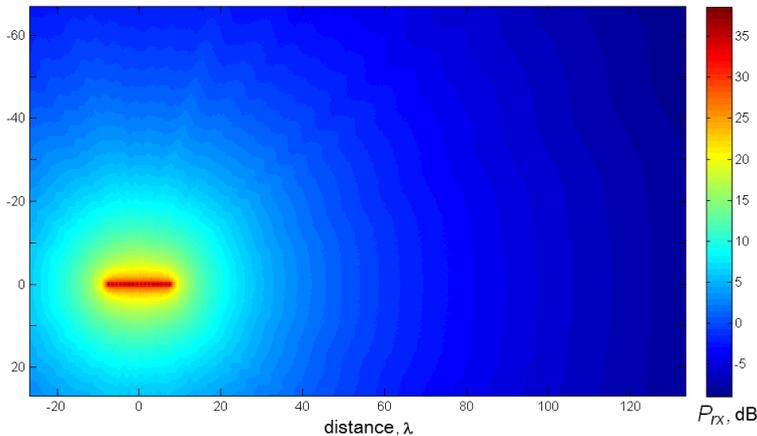


Fig. 2. Ensemble power field $P_{rx}(\alpha, r)$. Establishment of the spherical wave front.

For a linear grid (a line) of UWB chaotic sources, the received signal power $P_{rx}(\alpha, r)$ was calculated in the space around the ensemble. The ensemble had the following parameters: grid step $d = 0.8\lambda$, number of sources $N = 16$, overall ensemble dimension $L = 12\lambda$ (here, λ is the average wavelength of the UWB chaotic oscillations). All antennas are equal, omnidirectional and equally oriented in azimuth plane.

As can be seen in fig. 2, in near zone ($r < 20\lambda$) the ensemble energy emission pattern $P_{rx}(\alpha, r)$ is determined by the ensemble topology, but as the distance r is increased, the

pattern quickly (already at $r \approx 25\lambda$) becomes omnidirectional. Thus, this numerical experiment confirms that beginning from the distance $r \sim 2L$, information on the ensemble topology is lost, and the ensemble wave front becomes practically spherical (in azimuth plane). Small fluctuations of power values $P_{rx}(\alpha, r)$ are associated with variations of the mean power of chaotic signal sum. With a fair accuracy we can assert that the far zone border of the ensemble is at the distance of the order of $r_{FZ} = 2L$.

5 Conclusions

The spatial structure of the emission power field is fully investigated for the ensemble of ultrawideband direct chaotic emitters. The field power density in reception point is shown to be the sum of the power densities of the fields, formed in this point by partial chaotic sources. Relations for the energy emission pattern of an individual UWB source and of an ensemble of UWB noncoherent sources are obtained analytically. Energy directivity pattern of UWB ensemble is a linear combination of energy patterns of the ensemble elements. If all the chaotic sources have the same equally oriented antennas, then the ensemble emission pattern coincides with that of a partial source. For instance, in the case of omnidirectional antennas, emission power pattern of the ensemble is also omnidirectional. This feature of the ensemble of UWB chaotic transmitters makes it extremely different from ensemble of coherent emitters, in which emission power field is directed even if ensemble elements emit omni-directionally.

The far zone border is estimated for ensemble of UWB chaotic emitters. Since phase and delay relations of the fields in reception point are irrelevant when uncorrelated signal power sum is estimated, the far zone (power) border is essentially (by orders of magnitude) lower than in the case of narrowband emitters.

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