

Integro-differential equations of magnetostatics for calculation of nonlinear inductances in low-pass filters

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Abstract. In this work on the basis of the obtained integral relations, including integral equations, taking into account the influence of magnetic screens from a nonlinear magnetic on the inductance of multilayer coils in dependence of the current strength, the frequency and nonlinear properties of inductances of noise suppression radio reactors of electric transport are obtained. The radio reactors properties behavior in the circuit under nonstationary effects is studied.

1 Introduction

The electric vehicle electrical equipment operation is characterized by various transient processes in high-power consumers of electric energy, such as floating contact of current collector and contact network, switching of electric motor windings, switching on / off of climatic equipment, as well as operation of power switches in modern inverter drives [1]. All these processes are characterized by powerful current and voltage surges occurring during their course in electric circuits extended along the length, and, as a consequence, significant electromagnetic interference in a wide frequency range, which requires the development of special procedures to suppress them.

For noise suppression mainly are used the low-pass filters on the basis of the inductance coils, called in this industry sector as the noise suppression reactors (NSR) Fig. 1. To significantly reduce the dimensional specifications and weight of NSR, it seems advisable to use magnetic screens made of ferromagnetic materials, which allows the inductance value increasing at low frequencies and low currents by several tens or more times [2]. At that the suppression of parasitic high-frequency fluctuations in ensuring accomplishment of requirements for electromagnetic compatibility (EMC) it is necessary to provide in the conditions of flow along inductance coils of such reactors in operating modes of a direct current in hundreds of amperes, and in the widest frequency range. Therefore, it is important to esteem the inductance value reduction of such coil with magnetic screens from ferromagnetic under condition of high currents flowing, as well as under condition of the parasite signals frequency increasing. The present work is devoted to the development of a mathematical model for calculating the influence of these factors on the inductance of a multilayer coil with magnetic screens. For this purpose, the integro-differential equation

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(IDE) of magnetostatics, written with respect to the magnetic field \mathbf{H} . On principle, the using of such equation is more convenient than the using of differential equations of magnetostatics. IDE can be formulated with respect to magnetization or with respect to vector-potential.

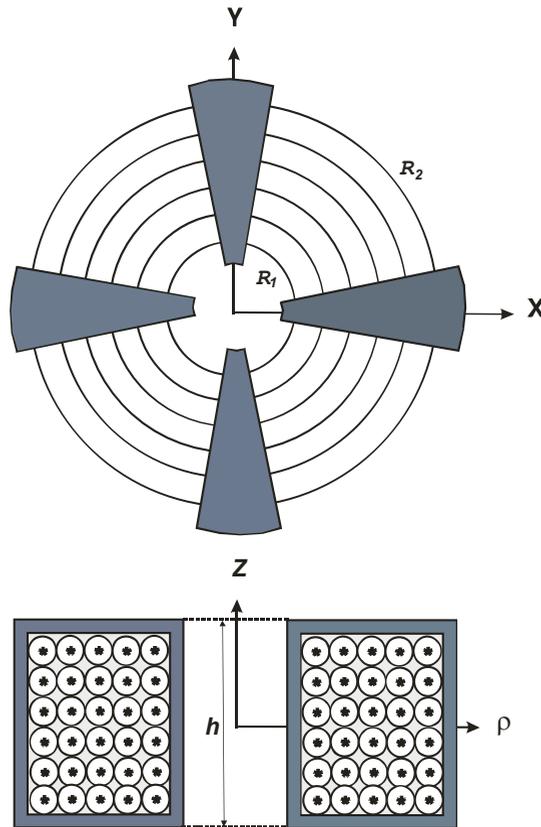


Fig. 1. Top view of the coil with screens in the form of brackets (top part of the picture) and the cross-section of the coil through the screen (low part).

2 Nonlinear inductance of coils with solenoidal current in the presence of a magnetic

In magnetostatics for a non-magnetic conductor with dimensions substantially smaller than the wavelength, the linear inductance is determined by the integral [3,4]

$$L_0 = \frac{\mu_0}{4\pi I^2} \iint_V \frac{\mathbf{J}(\mathbf{r})\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r d^3r'. \quad (1)$$

Here V - is the volume occupied by the currents, d^3r' - the volume element, I - is the total current flowing through any cross-section s of the conductor, which, due to solenoidality, is considered independent of the cross-section location. While the calculation of inductance, it is convenient to divide the current into current-tubes, taking into account the mutual inductances of the circuits. While the cross-section s decreasing, the current density is increasing. This is true for any closed solenoidal ($\nabla \cdot \mathbf{J} = 0$) loop of current. This current can be represented as the rotor of a certain vector, namely - as $\mathbf{J} = \nabla \times \mathbf{H}_0$. For a closed

metal conductor, the current density is constant across the cross-section, but when taking into account the skin effect, the inductance drops [4], since the current flows not along the full cross-section of the conductor radius r , but along the ring with thickness $\delta \ll r$. The frequency dependence of the inductance is conveniently to determine by the introduction of the surface current density, depending on the thickness of the skin layer or the surface impedance [3]. For a low profile non-magnetic coil, the inductance is approximately proportional to the square of the number of turns $L_0 \approx n^2 \tilde{L}_0$, where \tilde{L}_0 is the inductance of the average turn.

In the case of magnetic screens, it is necessary to solve integro-differential equations of magnetostatics. They can be formulated with respect to the field \mathbf{H} or with respect to the vector-potential \mathbf{A} [5]. It is necessary to note that in [6] such equations are formulated with respect to magnetization $\mathbf{M} = (\mu-1) \mathbf{H}$. By magnetic screens we will assume the soft magnetic material with high magnetic permeability. The best screen is the transformer steel. Since the magnetism of steel is mainly conditioned by the orientation mechanism of polarization of magnetic domains, the dependence of the magnetic permeability on the frequency is characterized by the Debye formula

$$\mu(\mathbf{r}, \omega, H(\mathbf{r}, \omega)) = 1 + \frac{\chi(\mathbf{r}, H(\mathbf{r}, 0))}{1 + (\omega\tau)^2} - j \frac{\omega\chi(\mathbf{r}, H(\mathbf{r}, 0))}{1 + (\omega\tau)^2}. \quad (2)$$

Here $H(\mathbf{r}, 0)$ is the magnetic field of the constant component of the current. The total DC field (low-frequency) is taken as the sum of the solenoidal part and the magnetization field. The first, according to the Bio-Savard-Laplace formula, has the form

$$\mathbf{H}_0(\mathbf{r}) = \int_{V_0} G(\mathbf{r} - \mathbf{r}') \nabla' \times \mathbf{J}(\mathbf{r}') d^3 r' = \nabla \times \int_{V_0} G(\mathbf{r} - \mathbf{r}') \mathbf{J}(\mathbf{r}') d^3 r'. \quad (3)$$

Here $G(\mathbf{r}) = (4\pi r)^{-1}$. In the volume of the magnet V_0 we set the solenoidal current distribution with density $\mathbf{J} = \nabla \times \mathbf{H}_0$. The full field subject IDE [5]

$$\mathbf{H}(\mathbf{r}) = \int_{V_0} G(\mathbf{r} - \mathbf{r}') \nabla' \times \mathbf{J}(\mathbf{r}') d^3 r' - \int_{V_M} G(\mathbf{r} - \mathbf{r}') \nabla' F(\mathbf{r}', H(\mathbf{r}')) d^3 r', \quad (4)$$

which is written with respect to the magnetic field \mathbf{H} . Here we introduced the function

$$F(\mathbf{r}, H(\mathbf{r})) = \nabla \cdot \mathbf{H}(\mathbf{r}) = -\left\{ \mu^{-1}(\mathbf{r}, H(\mathbf{r})) \left[\nabla \mu(\mathbf{r}, H(\mathbf{r})) + \mu_1(\mathbf{r}, H(\mathbf{r})) \nabla H(\mathbf{r}) \right] \cdot \mathbf{H}(\mathbf{r}) \right\},$$

the gradient is taken by the first argument \mathbf{r} , the value $\mu_1(\mathbf{r}, H(\mathbf{r})) = (\partial / \partial H) \mu(\mathbf{r}, H(\mathbf{r}))$ is calculated at a constant \mathbf{r} . This IDE is derived from the equation of magnetostatics $\nabla \times \mathbf{H} = \mathbf{J}$, taking into account the fact that magnetization in magnetic screens also acts as a source of the full field. For alternating currents and fields, the magnetization changes over time, and flow equations and the magnetization dynamics equation should be used. However, for soft magnetic screens, hysteresis can be neglected, and the magnetization becomes a one-to-one nonlinear function of the current or current field. Therefore, we can introduce a nonlinear inductance linking the induction \mathbf{B} (and flow) with the field \mathbf{H} . The main task then is to determine the current $I(t)$ in a nonlinear circuit, for which we approximate the inductance $L(I(t)) = k\mu(HI(t))L_0$ and magnetic permeability, which is determined by magnetization. Here k is the fill factor estimated with respect to the magnetostatic energies without the screen and with the screen. In the transition from the fields to the schematic description, replace the vector quantities with their averaged by volume RMS values, for example, $H = \sqrt{\langle \mathbf{H}^2(r) \rangle}$. The magnetic moment is approximated by a sigmoid function without hysteresis (see [5]). In particular, the following approximation

was used - $P_m(H) = B(H) - \mu_0 H = 2\pi^{-1} P_m(0) \text{arctch}(H/\bar{H})$, where \bar{H} is the saturation field, $P_m(0)$ is the magnetic moment of saturation. We have an approximation

$$\mu(H) = 1 + \chi(H) = 1 + 2\pi^{-1} P_m(0) / \left[(\mu_0 \bar{H}) \left(1 + (H/\bar{H})^2 \right) \right]. \quad (5)$$

The current density in the conductor is defined as the ratio of I to the wire cross-section s . Specifying heterogeneous or homogeneous current density, solving IDE for a given value μ , we find the field distribution. This allows to adjust consistently μ , using (5), calculate the inductance and to determine its relationship with the current (Fig. 2). Since the energy of the magnetic field is expressed both through current and inductance, and through magnetic permeability and magnetic field, we have

$$W_H = \mu_0 / 2 \int_{V_0 + V_M} \mu(H) H^2 dv = LI^2 / 2. \quad (6)$$

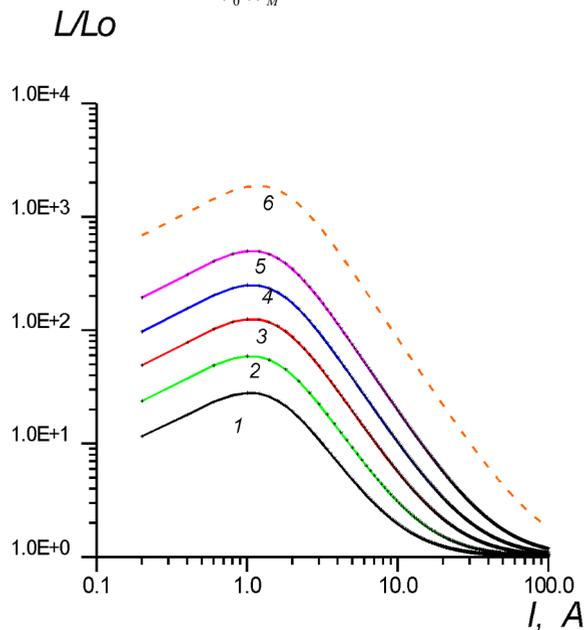


Fig. 2. The dependence of L/L_0 of the coil current I (A) at different angles of filling the volume by screen material: $\varphi=\pi/2$, $t=8$ (1), 16 (2), 32 (3) mm; $t=32$, $\varphi=\pi$ (4) $\varphi=2\pi$ (5); $\varphi=2\pi$, $t=100$ mm (6).

Substitution of the IDE solution in this equation for a given current and taking into account the connection of the magnetic permeability with the field makes it possible to find the connection of the nonlinear inductance and current. However, due to the nonlinearity, the process should be carried out iteratively, taking into account the magnetization curve $B(H)$ and determining $\mu(H)$ through its derivative. By determining the current density and the magnetic current density of polarization $\mathbf{J}_p^m = i\mu_0\omega(\mu-1)\mathbf{H}$, it is possible to calculate the radiation pattern of NSR. While calculating the radiation field in the ratio (4) it should be substituted the Green's function $G(\mathbf{r}) = (4\pi\epsilon)^{-1} \exp(-i\omega|\mathbf{r}|/c)$. The coil length of 30 m corresponds to half of the wavelength at a frequency of 500 MHz, so the delay should be taken into account. The method allows also to calculate the radiation field of NSR taking into account the nonlinearity. We are interested in the behavior of NSR in the AC circuit as a series circuit with the inclusion of resistance, which describes the inclusion of the motor in the contact circuit. So, the algorithm has the following form. We set the coil current, determine the field \mathbf{H}_0 , inductance L_0 and the field \mathbf{H} from the IDE for $\mu(0)$. Then we

recalculate value $\mu(H)$ for the received field and re-solve the IDE. After several iterations, we obtain the inductance value for the intended current. This inductance value can be used to calculate the current in the circuit. If a time-varying process is considered, the current value in the circuit changes, and for the different point in time the algorithm should be applied again.

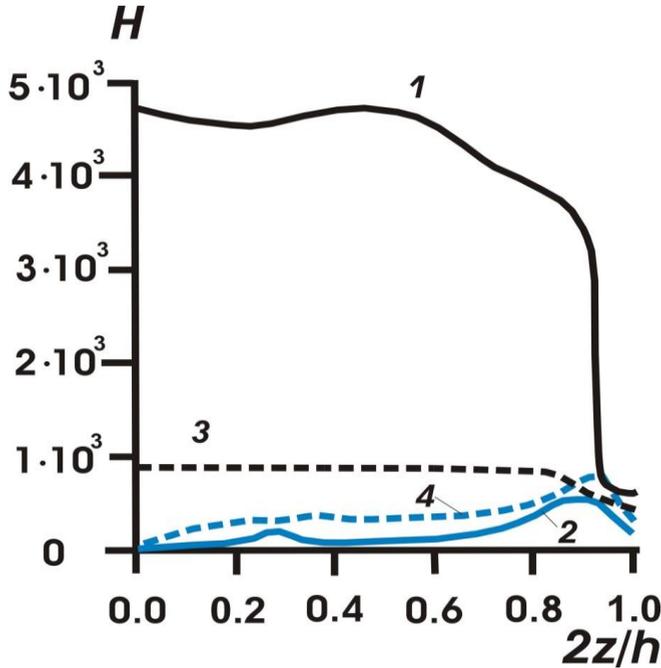


Fig. 3. The components of the magnetic field (A/m) in NSR at a current of 10 A in dependence on the coordinate along the axis. Curves 1 and 2: H_z and H_ρ with $\rho = 0$; curves 3 and 4: $-H_z$ and H_ρ with $\rho/R_2 = 1.05$.

In the case of nonlinear quasi-monochromatic processes it is possible to obtain related equations for harmonics $(2n-1)\omega$. In the equation for the first harmonic one should use the value $\mu(\omega, H(\omega)) = 1 + \chi(H(0)) / [1 + (\omega\tau)^2]$ in which the susceptibility $\chi(H)$ from (5) should be used. Formally to solve IDE (4) for a given current I and its density J for determination components of the magnetic field is necessary only inside the screen. In the Fig. 2 the distribution of the magnetic field in NSR with radii $R_1=10$, $R_2=15$ cm and height $h=10$ cm with five windings and four brackets in the form of sections with an angle of 10^0 is presented (the current was taken 10 A.) It was determined numerically by successive approximations for (4) taking into account (3) and approximation of μ . The dependence of the inductance of the coil from Fig. 1 at different angles of screen filling is shown in Fig. 3. It is seen a multiple inductance decrease at a current of the order of 10 A and more. Nonlinear responses in the form of current in the circuit of NSR and resistance are shown in Fig. 4 and Fig. 5. A nonlinear IDE equation for the loop current was numerically integrated when a sinusoidal voltage source was switched on at the initial moment.

3 Description of processes in a low-signal approximation

In the low-signal approximation, the frequency dependence of the magnetic permeability (2) provides the main limitation of the filtering properties of NSR. Another limitation is associated with resonances in NSR, which at high frequencies can no longer be considered

as a concentrated element. Basically, the resonances are caused due to the inter-turn capacity and are manifested at frequencies from 500 MHz and above. The calculation of the transfer coefficient taking into account the inter-turn capacitances and magnetic permeability is given in Fig. 6. There are also represented the results when two parallel capacitances are switched on at the inductance ends, providing improved filtration. In this approximation, iterations are not required and the IDE can be solved only once. However, the low-signal approximation for the considered NSR designs can be used for currents less than 5 A. At higher levels of the average current, its high-frequency oscillations are possible (Fig. 5) relatively large average level. In this case, the inductance is estimated according to Fig. 1 and it is also possible to use a spectral approach and consider filtering properties using a linear chain. Otherwise, it is necessary to numerically integrate nonlinear time IDEs or differential circuit equations.

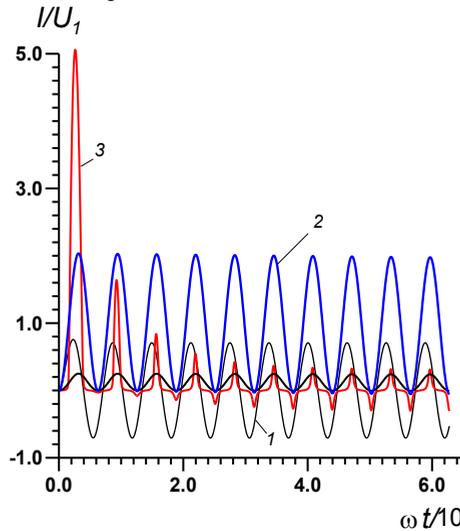


Fig. 4. Establishing of a transient process in a circuit with nonlinear normalized inductance $L_0=0.1$, (curves 2, 3) when a harmonic source is switched on. Curve 1 is linear case (zero harmonic $U_0=0$); 2 and 3 - values I/U_1 and $10I/U_1$ respectively for first harmonic of voltage $U_1=10^2$ and $U_1=10^3$.

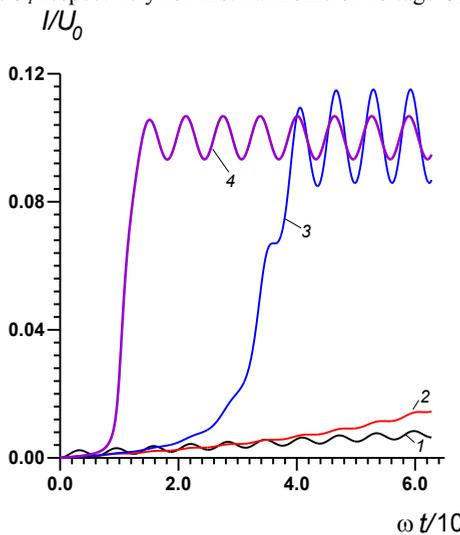


Fig. 5. Establishing of a transient processes for a nonlinear circuit with $U_1=10 : U_0=10$ (curve 1), $U_0=10^2$ (2), $U_0=3 \cdot 10^2$ (3), $U_0=10^3$ (4).

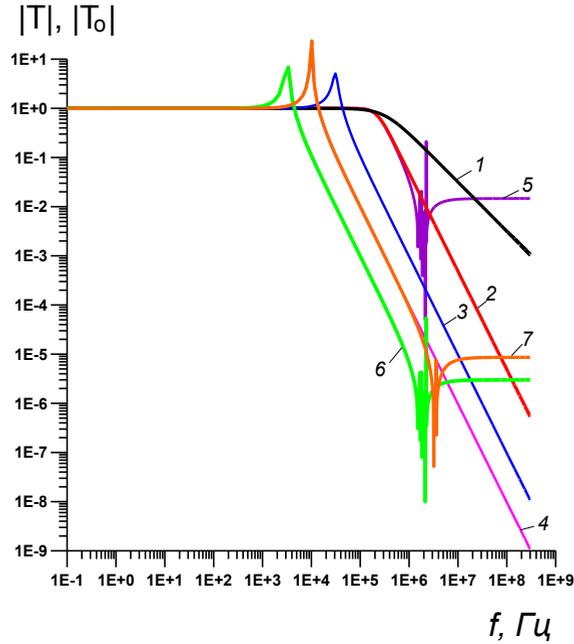


Fig. 6. Transmission coefficients T_0 (without shield, curve 1) and T (2-6) for $L=130 \mu\text{H}$ (1-6) and $13 \mu\text{H}$ (7) at capacitances respectively $1.92 \cdot 10^{-9}$ (2, 5), 10^{-7} (3), 10^{-6} (4), 10^{-5} (6) F. Curves 5 and 7 are constructed taking into account the inter-turn capacities.

4 Conclusions

In this paper the integro-differential equations of magnetostatics for a magnetic field are obtained in the case of a given external magnetic field created by a constant solenoid current in a conductor (coil) and in the presence of a magnetic body with a scalar inhomogeneous magnetic permeability, which can be a nonlinear function of the field. The equations are defined inside the body and on its surface. They are integro-differential in the sense that under the sign of the integral operator are unknown quantities and their derivatives.

Transient processes in a nonlinear circuit with NSR and resistance are calculated. The possibility of oscillatory modes with only positive or only negative current values depending on the positive or negative value of the voltage amplitude at the initial moment is shown.

The relations obtained in this paper and the resulting expression for the inductance value allows to carry out further calculations of this value for multilayer inductors with magnetic screens. On the basis of the derived relations it is possible to carry out the calculations for different materials of screens and different values of the direct current flowing through them, in a wide frequency range of parasitic signals, passing through multilayer coils. They are fundamentally suitable to analyse the effect of the magnetic permeability of the screen material on the saturation processes in the course of direct currents of different magnitude.

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