

Features of mode selection in a combined Fabry–Perot cavity with distributed feedback of counter-propagating waves

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Abstract. An analysis is made of the possibility of isolating relatively high-Q modes or groups of such modes in low-Q combined Fabry-Perot cavities with distributed feedback of counter-propagating waves in order to ensure resonant interaction of the electromagnetic field with the polarization oscillations of an amplifying or absorbing medium filling the cavity. We considered two-level active media with homogeneous or inhomogeneous broadening of a spectral line, which can be both smaller or larger than the photon bandgap of a cavity. Particular attention is paid to the change in the well-known spectrum of polariton modes which takes place due to the transition from an absorbing to an amplifying medium under conditions that allow the realization of spontaneous or laser generation of the so-called superradiant modes.

1 Introduction

Distributed feedback (DFB) of counter-propagating waves, due to various periodic structures (photonic crystals) and accompanying the reflection of waves from the mirrors of Fabry-Perot cavities (FP), is widely used in various physical research and modern technologies, for example, semiconductor ones [1-3]. For example, it allows one to control the quality factor of modes in relatively low-Q cavities, which are required for the unique lasing regimes available in the presence of highly amplifying active media (both classical, such as electron beams, and quantum, such as semiconductor heterostructures) [4, 5]. In this regard, the properties of the combined Fabry-Perot cavity with distributed feedback (FP-DFB) are not yet sufficiently studied, and in this paper we consider the features of the mode spectra of such a cavity both in the absence of an active medium (the so-called cold modes) and in the presence of an absorbing or amplifying medium with homogeneous or inhomogeneous broadening of the spectral line (the so-called hot modes).

2 Dispersion of the cold and hot modes

We confine ourselves to the analysis of a one-dimensional (1D) laser model with a two-level active medium uniformly filling a low-Q combined Fabry–Perot cavity, along which

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(along the z axis) there is distributed feedback of counter-propagating waves due to the Bragg reflection of waves on periodic modulation (with amplitude $\bar{\beta}$) of the dielectric permittivity $\varepsilon = \bar{\varepsilon} \text{Re}[1 + 4\bar{\beta} \exp(2ik_0z)]$ of the active medium or corrugation waveguide layers. Here $k_0 = \omega_0 \sqrt{\bar{\varepsilon}} / c$ is a wave number corresponding the Bragg resonance at a frequency ω_0 , $c / \sqrt{\bar{\varepsilon}}$ is the light velocity in a matrix of the active medium. Characteristic mode spectra, i.e. dependences of the mode growth rates Γ on frequency Ω , are found by solving the linearized Maxwell – Bloch equations [4, 5] for a given population inversion of the working levels of active centers, $n_p = \text{const}$:

$$|\beta|^2 + \kappa^2 = \left(\Omega + \frac{n_p}{\Omega + \Phi + i(\Delta + \Gamma_2)} \right)^2, \quad (1)$$

$$\frac{R\beta^* + R\beta}{1 + R^2} \pm \sqrt{\kappa^2 + |\beta|^2} + \kappa \frac{1 - R^2}{1 + R^2} \cdot \frac{1 + e^{2i\kappa L}}{1 - e^{2i\kappa L}} = 0. \quad (2)$$

The shape of these spectra is largely dictated by the so-called cooperative frequency $\nu_c = \sqrt{2\pi d^2 \omega_0 \Delta N / \bar{\varepsilon} \hbar}$, which is determined by the dipole moment d of the transition at frequencies close to ω_0 , $\bar{\varepsilon}$ is the dielectric constant of the active medium matrix averaged over the cavity and frequencies, and ΔN is the concentration of active centers in which the population inversion is created at the working levels, $n_p \leq 1$. The spatial scale of the field coherence is determined by a cooperative length, $B_c = c / (\nu_c \sqrt{\bar{\varepsilon}}) \equiv \lambda / (2\pi\sqrt{I})$, associated with the cooperative frequency. Accordingly, all temporal quantities are normalized to ν_c , and spatial ones (including the cavity length, L) are normalized to B_c , λ is the wavelength in the medium. The quantities $\kappa = (k - k_0)c / \nu_c \sqrt{\bar{\varepsilon}}$ и $\Omega = (\omega - \omega_0) / \nu_c$ are the dimensionless shifts of a wave number and a frequency from the Bragg wave number k_0 and the Bragg frequency ω_0 , respectively, $\Phi = (\omega_0 - \omega_{21}) / \nu_c$ is the detuning between the Bragg resonance frequency ω_0 and the center of a spectral line of an active medium, ω_{21} , normalized on the cooperative frequency ν_c . Here and below the following dimensionless parameters are used: $I = \nu_c^2 / \omega_{21}^2 \ll 1$, $\beta = \bar{\beta} / \sqrt{I}$ is the dimensionless amplitude of the modulation of permittivity which describes the coupling the counter-propagating waves (and is equal to the ratio of halfwidth of bandgap to the cooperative frequency), $b = \beta L$ is the DFB parameter (we assume $\beta = \beta^*$ for simplicity), R is reflection factor in the Fabry-Perot cavity (by amplitude of field). $\Gamma_2 = (\nu_c T_2)^{-1}$ is a dimensionless rate of the polarization relaxation determined by a homogeneous broadening inversely proportional to a polarization lifetime T_2 , $2\Delta = 2(\nu_c T_2^*)^{-1}$ is the inhomogeneous broadening of spectral line of the working transition of an active medium.

High-frequency oscillations of the electromagnetic field in an empty cavity are characterized by the photon lifetime T_E , which determines the Q-factor of cold modes (see an example in Fig. 1, triangles). Since all modes have the same Q-factor in a pure Fabry-Perot cavity (Fig. 1a), a distributed feedback of counter-propagating waves may be used for the selection of various laser modes (Fig. 1b). A set of modes with different Q-factors may be created in such combined cavities. In the presence of an inverted active medium these

cold modes turn into hot modes with different growth rates that allow one to obtain generation regimes corresponding to various dynamic classes of lasers. Referring for a detailed description of such mode selection to the review [5], we will formulate shortly the obtained qualitative conclusions. A single-mode monochromatic generation of the highest-quality modes with $T_E \gg T_2$ is typical of the traditional class B lasers. Modes with the photon lifetime comparable to the phase relaxation time of the polarization of active centers, $T_E \sim T_2$, are usually self-modulated and have a broadened spectrum (laser class C). For the lowest quality modes, superradiant generation with a pronounced pulsed character of radiation is possible (yet unrealized class D laser).

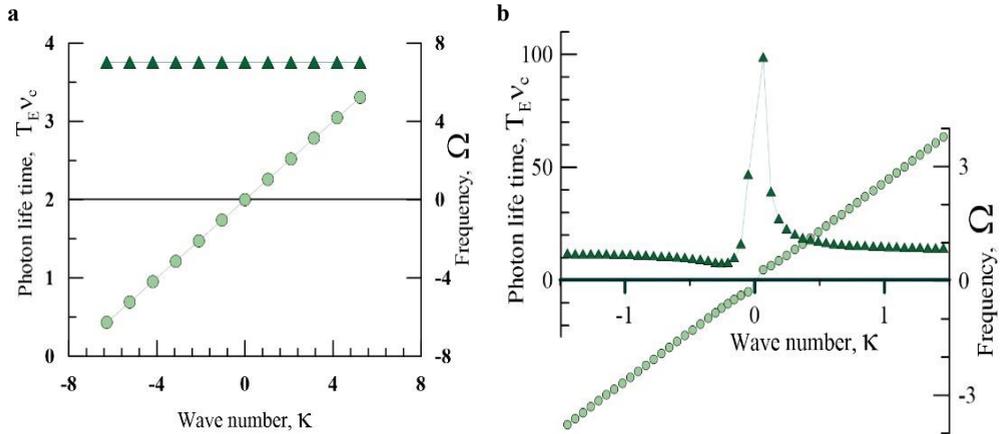


Fig. 1. Dependences of the photon lifetimes, $T_E \nu_c$, (triangles) and frequencies, $\Omega = (\omega - \omega_0) / \nu_c$, (curcles) on a normalized wave number, $\kappa = (k - k_0)c / \nu_c \sqrt{\epsilon}$. (a) Cold modes of a pure Fabry-Perot cavity with reflection factor $R = 0.45$ and length $L = 3$. (b) Cold modes of a combined FP-DFB cavity with reflection factor $R = 0.2$, parameter DFB $b = 5$, and length $L = 20$.

In the sufficiently low-Q cavities, in which the lifetime of the oscillations of the high-frequency dipole moments of the active centers, T_2 , exceeds the lifetime of the photons, T_E , the polarization oscillations of the active medium influence a lot the spectra and structure of the modes. In this case, the coupled waves of the electromagnetic field and oscillations of the dipoles of the active medium arise (polarization waves, or polaritons) [4–7] which have a dispersion different from the dispersion of electromagnetic waves if the inhomogeneous broadening of the spectral line of a medium does not exceed the cooperative frequency (cf. Figs. 2a and 2b, as the typical examples). Unlike the previously studied polariton modes for an absorbing medium with negative inversion, in this paper special attention is paid to polaritons in an inverted active medium (for example, heterolasers in which a polariton is formed by an exciton and a photon), where the dispersion becomes qualitatively different. Along with mode growth rates, it substantially depends on not only both indicated relaxation times T_E and T_2 , but also on the ratio of the cooperative frequency in a medium with positive inversion and the following three frequencies: the intermode interval, π / L , the bandgap of the FP-DFB cavity, 2β , and the inhomogeneous broadening, 2Δ .

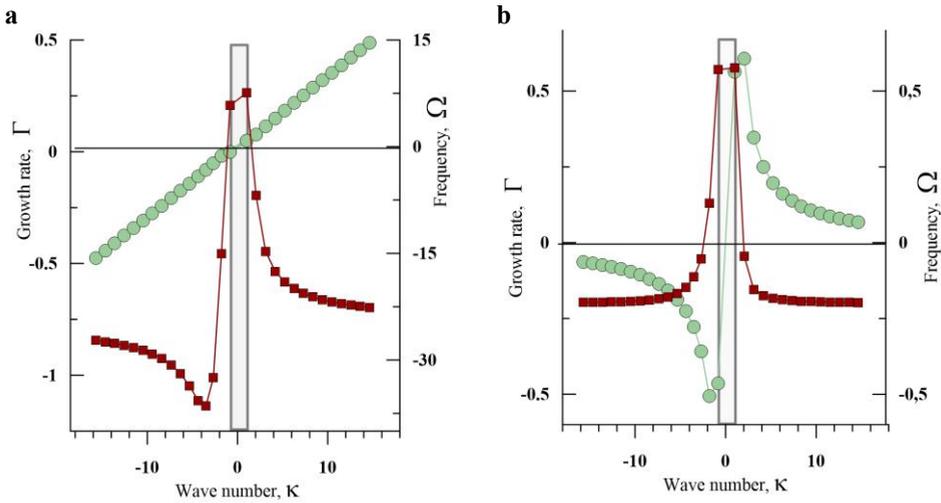


Fig. 2. Dependences of the normalized growth rates, Γ , (squares) and frequency shifts Ω , (circles) of hot modes of a combined cavity with length $L = 3$, reflection factor $R = 0.1$ and DFB parameter $b = 2$ on normalized wave number, κ . (a) Electromagnetic waves in the case of an inverted active medium with strong inhomogeneous broadening of a spectral line, $\Delta = 1$, and low relaxation rate of a polarization, $\Gamma_2 = 0.02$, when two modes with $T_E \nu_c \approx 3.7$ became unstable (the polariton modes decay and not shown). (b) Polariton modes in the case of an inverted active medium with weak inhomogeneous broadening of a spectral line, $\Delta = 0.1$, and low relaxation rate of a polarization, $\Gamma_2 = 0.02$, when three modes with $T_E \nu_c \sim 3$ becomes unstable (electromagnetic modes decay and not shown).

Below two important examples are discussed to show how filling of a cavity with a resonant active medium changes the spectrum of modes (from cold to hot ones) depending on the magnitude of the inhomogeneous broadening of the spectral line of an active medium. If the inhomogeneous broadening of the spectral line of a medium is larger than the cooperative frequency, the lasing takes place for some of the most high-quality electromagnetic modes with the highest growth rates (Fig. 2a) [4, 5]. This is true both for the class B lasers with low cooperative frequencies, and for the class C and D lasers with fairly high values of ν_c . The opposite situation appears at high densities or sufficiently large dipole moments of active centers, when the cooperative frequency becomes comparable to or larger than the polarization relaxation rate. In this case the ensemble of radiating active centers exhibits its collective properties and the polariton modes are lasing (Fig. 2b). In the latter case, the maximum growth rate of mode reaches the value ν_c (under a complete inversion of the populations of energy levels of an active medium).

If the inhomogeneous broadening dominates, $\Delta > 1 > \Gamma_2$, the dispersion of electromagnetic modes is weakly changed by active centers, while the maximum growth rate decreases to the so-called effective cooperative frequency [4, 5], ν_c / Δ , determined by the cooperative frequency of not all but only a part of the active centers with close frequencies occupying a spectral band with width $2\nu_c / \Delta$ and having no time to dephase during the formation time of a superradiant pulse, $\delta t \sim \Delta / \nu_c$. This minimal duration of expected field pulses can be achieved due to a special mode selection and is almost independent of the coupling parameter b of counter-propagating waves and the relaxation times T_2 until $\Delta \leq \Gamma_2^{-1}$.

It is obvious that for obtaining collective spontaneous emission with the highest-power pulses (without strong oscillations) the active samples are needed with length on the order of the optimal length $L \sim 2\Delta \gg 1$ and $L \sim 2$ determined in the cases of strongly inhomogeneous and homogeneous broadenings by the effective cooperative length $B_c\Delta$ and the cooperative length B_c , respectively. In the above estimates we assume that the inversion specified by pumping (n_p) is on the order of unity. Otherwise, it is necessary to take into account that maximal growth rates are obtained by the multiplication of ν_c / Δ and ν_c , and the cooperative lengths by the division of $B_c\Delta$ and B_c by the inversion and the root from it, respectively.

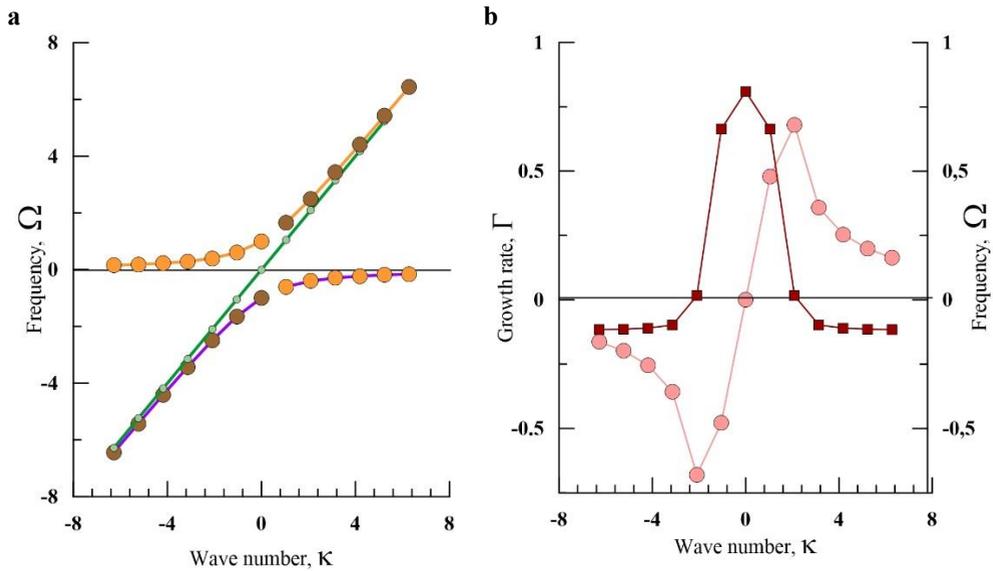


Fig. 3. Dependences of growth rates, Γ , (squares) and frequencies, Ω , (circles) on normalized wavenumbers, k , for hot modes of a pure Fabry-Perot cavity with reflection factor $R = 0.45$ and length $L = 3$. (a) Frequencies of cold modes (solid line with dots) and two branches of decaying hot modes (light circles are polariton modes, dark circles are electromagnetic modes) in the presence of an active (absorbing) medium in the ground state, $n_p = -1$. (b) Frequencies (circles) and growth rates (squares) of polariton modes in the case of completely inverted active medium, $n_p = 1$, with small inhomogeneous broadening, $\Delta = 0.1$, and weak relaxation rate of a polarization of active centers, $\Gamma_2 = 0.02$. Only five modes are unstable, $\Gamma > 0$.

3 Polariton modes of an active medium with homogeneous broadening

Let us discuss in more detail the features of the spectrum of polariton modes in a medium with a small inhomogeneous broadening of a spectral line, $\Delta < 1$, and a long relaxation time of the polarization, $T_2 \gg T_E$, when the dispersion profiles of wave in an absorbing and amplifying media are qualitatively different [5, 6]. In the case of an amplifying medium placed in a cavity with sufficiently high-quality modes, it turns out that only the corresponding polariton (but not electromagnetic) modes can have positive growth rates, i.e. be unstable, as it has already been demonstrated in Fig. 2b. According to [4, 5], these modes are formed by the so-called polarization waves, the instability of which, due to their

negative energy in the inverted active medium, can lead to collective spontaneous emission or lasing of one or several superradiant modes.

A typical example is shown in Fig. 3b for a Fabry-Perot cavity filled with an inhomogeneous broadened active medium, $\Delta = 0.1$. For a medium in the ground state (Fig. 3a, $n_p = -1$), the linear dispersion profile of an empty cavity (Fig. 2a, circles; Fig. 3a, dots) splits into two branches of the polariton and electromagnetic modes. Upon excitation of the active medium by pumping ($n_p = 1$), the flat parts of these branches transform into an asymmetric curve of the polariton modes (Fig. 3b, circles), and the parts that close to the bisector are not changed practically and form a branch of damped electromagnetic modes (not shown in Fig. 3). When the threshold is slightly exceeded, stationary lasing will occur at the central mode ($\Omega = 0$), which is nearest to the center of the spectral line of the active medium, i.e. $(\omega - \omega_{21}) / \nu_c = 0$, and has a maximum growth rate.

In Fig. 4, the typical spectral features of a combined FP-DFB cavity are compared to that of a pure Fabry-Perot cavity. The latter has a symmetric linear spectrum of cold modes (Fig. 4a), for which, in a certain range of pump powers, most of the hot modes around the highest-Q central one have a decay rate or a very low growth rate, that ensures stationary single-mode lasing in a reasonably wide range of inhomogeneous broadening of the spectral line of an active medium, up to the value $\Delta \sim 0.1$. The combined FP-DFB cavity of the same Q-factor has an asymmetric spectrum of cold modes of various Q-factors with a bandgap of order 2β . In such a situation, a monochromatic lasing may be obtained by means of a non-zero detuning of the Bragg resonance from the frequency of the working transition, $\Phi \neq 0$, the proper choice of which ensures amplification of a mode with the highest Q-factor at the edge of the bandgap (Fig. 4b) and excludes noticeable excitation of the neighboring modes.

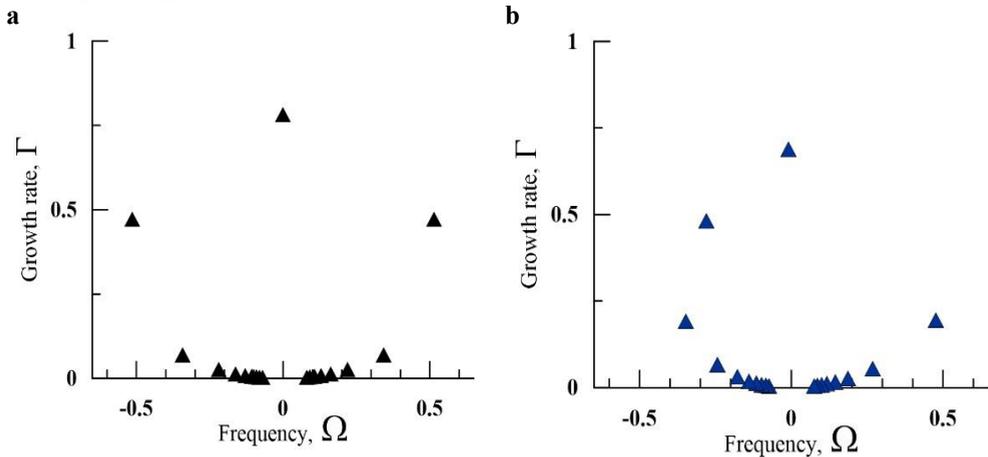


Fig. 4. Spectra of the hot modes of (a) the pure Fabry-Perot cavity with length $L = 2$, reflection factor $R = 0.37$, $\Phi = 0$ and (b) the combined FP-DFB cavity of the same length and Q-factor, caused by the combination of the following parameters: reflection factor $R = 0.2$, DFB parameter $b = 1$, and frequency detuning between the Bragg resonance and the spectral line $\Phi = 0.7$. In both cases, there is a completely inverted active medium, $n_p = 1$, with small both inhomogeneous, $\Delta = 0.1$, and homogeneous, $\Gamma_2 = 0.02$, broadenings.

In a general case for a Fabry-Perot cavity with the reflection factor R of the end mirrors, the photon lifetime is equal to $T_E \nu_c = L / \ln R^{-1}$ and the real parts of wave numbers are described as $\kappa = l\pi / L$, $l = 1, 2, \dots$. The most unstable mode having the maximum growth rate

Γ_{l_0} is the one with the number l_0 giving the value $\kappa=l_0\pi/L$ closest to the resonance frequency of the spectral line of active centers. For a short inverted sample with $L \ll 1$ and $R \leq 0.5$ in the case of weak relaxation of polarization and strong pumping of an active medium, when $\Gamma_2 \ll \sqrt{n_p}$ and $\Gamma_2 \ll n_p T_E \nu_c$, the radiative (dissipative) instability is developed in fact only for this single polariton mode having the growth rate $\Gamma_{l_0} \approx n_p T_E \nu_c - \Gamma_2$. This directly follows from the equations (1) - (2) and remains qualitatively valid for a combined Fabry-Perot cavity with a small integrated Bragg reflection factor up to the value $b \sim 1$. In particular, the growth rate for this resonance polariton mode is still determined by the above expression for Γ_{l_0} if the value $\ln R^{-1}$ in the expression for the photon lifetime $T_E \nu_c = L / \ln R^{-1}$ is replaced by the approximate expression $\ln |R - ib / \ln R^{-2}|^{-1}$ (for $R \ll b/2$, the replacement $\ln R^{-1} \rightarrow \ln |b/2|^{-1}$ should be made).

4 Electromagnetic modes of an active medium with strong inhomogeneous broadening

In the case of the large inhomogeneous broadening, $\Delta > 1 \gg \Gamma_2$, of a spectral line tuned to the Bragg resonance, $|\Phi| < \Delta$, the well-marked superfluorescence or superradiant lasing require a low-Q Bragg cavity with $b \sim 1$ or a related combined cavity (in the case of finite reflections R from the ends of a sample) with the considerable difference between the growth rates of the most unstable hot modes. By excluding cavities that are too long, $n_p \nu_c / \Delta \gg T_E^{-1} > T_2^{-1}$, and don't suit the mode superradiance [4, 5], we consider only low-Q cavities with rather short photon lifetimes restricted by the inequality $T_E \nu_c < \Delta / n_p < \Gamma_2^{-1}$.

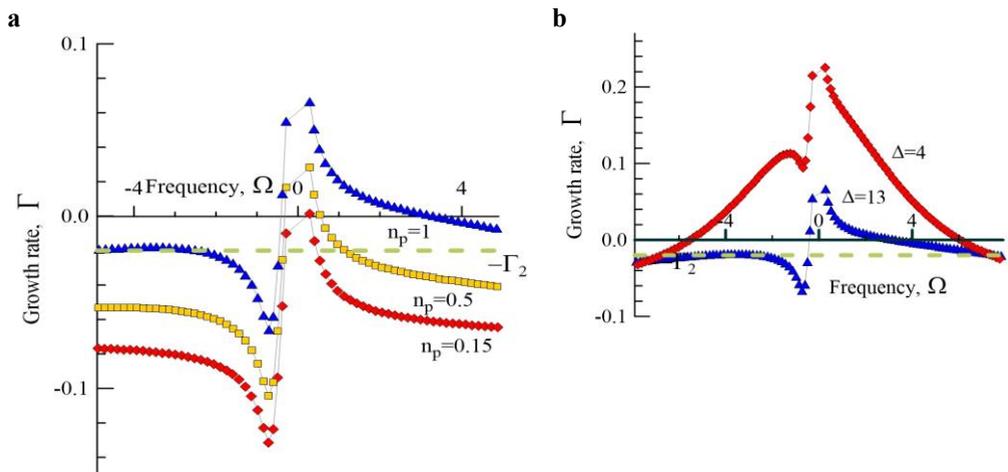


Fig. 5. Frequency dependences of the growth rates of hot modes of the combined FP-DFB cavity with length $L = 20$, reflection factor $R = 0.2$ and DFB parameter $b = 5$. (a) Influence of the population inversion level ($n_p = 0.15; 0.5; 1$) of the active medium with the inhomogeneous broadening of a spectral line $\Delta = 13$. (b) Influence of the inhomogeneous broadening of a spectral line ($\Delta = 4; 13$) under complete population inversion, $n_p = 1$. In all cases, there is just the same weak relaxation rate of the polarization of active centers, $\Gamma_2 = 0.02$.

Then, the mode selection preventing the dephasing of dipole oscillations of active centers during collective spontaneous emission is optimal if, for spectral regions with enhanced interaction of active centers with the electromagnetic field of this mode, the formally calculated cooperative frequency of active centers from a separated spectral region singled out by a particular cold mode will be about the relaxation rate T_E^{-1} of this mode and will considerably exceed the incoherent relaxation rate T_2^{-1} of their individual dipole oscillations. In other words, the growth rate of the corresponding resonance hot electromagnetic mode, which has the form $\Gamma = n_p \Delta^{-1} - T_E^{-1} \nu_c^{-1}$, should be on the order of the relaxation rate $T_E^{-1} \nu_c^{-1}$ of the initial cold mode. This requirement, taking into account the inequality $T_E < T_2$, gives $2\Delta \sim n_p \nu_c T_E \approx n_p L / \ln |R - ib / \ln R^{-2}|^{-1}$ and guarantees that mode superfluorescence will be maximally fast and powerful.

However, for extended samples with the optimal length on the order of the effective cooperative length, $L \sim 2\Delta / n_p$, required in the latter case, a great number of modes $\sim \Delta^2$ will be excited in a Fabry-Perot cavity which does not provide the frequency dispersion. Therefore, without taking special precautions, a superfluorescence pulse will be a superposition of the same number of incoherent random emission pulses of separate modes, i.e., it will be referred to the quasichaotic type with a broad spectrum.

In order to avoid this problem in the case of strong inhomogeneous broadening of a spectral line, $\Delta \gg 1$, one can involve an appropriate DFB of the counter-propagating waves which results in the underlying of some highest-Q electromagnetic modes. Their growth rates can be controlled using an optical or current pumping of a high working level (see an example in Fig. 5a). The spectrum of these modes of a combined FP-DFB cavity is asymmetric and considerably differs from the spectrum of a pure Fabry-Perot cavity, especially in the region close to the bandgap (Fig. 5), that is caused Bragg resonance [3-5]. According to the analysis of the solution to the dispersion and characteristic equations (1) - (2) in the case of interest $R \ll b \equiv \beta L \sim 1$, when moving away from the highest-Q central modes at the edges of the photonic band gap, the growth rates of the modes with large numbers $m \gg 1$ fall approximately according to the law

$$\frac{n_p}{\Delta} - \frac{\ln(R + b / 2\pi m)^{-1}}{L},$$

so that the total number of unstable modes can be estimated as

$$M < \frac{b / \pi}{\exp(-n_p L / \Delta) - R}.$$

This estimate is consistent with spiking of the number of unstable hot modes with an increase of the pumping level, n_p , and a decrease of the inhomogeneous broadening of the spectral line, Δ (Fig. 5). Thus, the application of the DFB into an active sample even with a small integrated reflection factor $b \sim 1$ considerably changes the growth rates of modes at frequencies close to the photonic bandgap, even weakly manifested. Depending on the reflection factors at the sample ends, this change can be nonsymmetric, differently increasing or decreasing mode growth rates on different sides of this band. Such a selection makes it possible to obtain highly coherent single-mode superfluorescence in a rather broad range of parameters by using the highest-Q modes with the maximum growth rate and suppressing the emission of other modes.

5 Conclusions

The analysis of the hot modes of the combined FP-DFB cavity filled with a resonant medium with a homogeneous or inhomogeneous broadening of a spectral line shows the possibility of efficient selection of the modes responsible for the interaction of the polarization oscillations of the medium with the electromagnetic field and allows one to judge about various lasing regimes, including the superradiant lasing, which depend on the properties of the active medium, pump level, phases ratio and reflection factors, formed by both distributed feedback inside the cavity and local mirrors on its edges. The obtained results are important for the development of new quantum and classical oscillators based on the active media demonstrating coherent properties.

This work was supported in the framework of State Order for R&D Works at the Institute of Applied Physics (Nizhny Novgorod) [project number 0035-2019-0002]; and the Presidium of the Russian Academy of Sciences, program of fundamental research “Nanostructures: Physics, Chemistry, Biology, Fundamentals of Technologies”.

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