Microwave diagnostics of the streamer zone of a long spark

Nikolay Bogatov¹⁺, Vladimir Syssoev², Marat Bulatov², and Dmitry Sukharevsky²

¹Institute of Applied Physics RAS, Nizhny Novgorod, Russia
²High-Voltage Research Center of Russian Federal Nuclear Center-Zababakhin All-Russian Scientific Research Institute of Technical Physics, Istra, Moscow region, Russia

Abstract. Microwave diagnostics was first applied to the study of long spark and some new parameters of this discharge not available by the traditional experimental methods in this field were measured. The conditions of applicability of the method of measuring the high-frequency conductivity of the streamer zone by absorption of probing microwave radiation were theoretically analyzed.

1 Introduction¹

Microwave diagnostics has been widely used in plasma research [1,2], including gas discharge, but it has not yet been applied to investigation of long sparks. The main difficulty of using microwave diagnostics (as well as any other active diagnostics) in studies of long sparks (and, especially, lightning) is that the position of a long spark in space is random and varies in a fairly wide (in the laboratory - up to tens of meters) range. Significant technical difficulties in the implementation of microwave diagnostics of long sparks are also associated with the features of the high voltage experiment, the minimum distances at which the equipment and microwave quasioptical elements can be approached to the discharge gap are tens of meters, and large pulse voltages (several MV) and currents (several kA) are sources of very strong electromagnetic interference.

This paper describes the microwave diagnostics of long spark carried out in the High-Voltage Research Center of Zababakhin All-Russian Research Institute of Technical Physics (http://ckp-rf.ru/usu/73578/) and used for investigation of spark discharge in the 5.5 m rod-plane gap. The results obtained with the help of this microwave diagnostics are briefly presented.

2 Experimental setup

The scheme of the experiment and microwave diagnostics are shown in Fig.1 (other measuring systems are hidden so as not to overload the picture). The object of research was the streamer zone of the breakthrough phase of positive leader, consisting of a large number, about a million, streamers simultaneously moving in the space in front of the leader’s head. The source of microwave radiation was a microwave generator G4-91

* Corresponding author: bogatov@appl.sci-nnov.ru
(labeled "35 GHz Oscillator" in Fig.1) feeding a horn antenna. The radiation frequency was 35 GHz, the output power of the generator was 5 mW, and the relative level of output power fluctuations was $10^{-3}$. The converging Gaussian microwave beam was formed by the horn antenna and dielectric lenses. The axis of the microwave beam was oriented horizontally, perpendicular to the axis of the gap. The microwave beam waist was on the axis of the gap. The diameter of the microwave beam in the waist was ~50 cm (in the region to be probed the beam was almost cylindrical; at the edges of the $18 \times 18$ m$^2$ grounded plane it was only 3% wider than at its center). The distance from the axis of the microwave beam to the grounded plane was 2.35 m. The polarization of the microwave radiation was linear (vertical). The microwave radiation that passed through the streamer zone was focused by a dielectric lens to the open end of the receiving waveguide, and the received radiation was amplified by a 20-dB microwave amplifier and detected by a microwave diode with linear volt-watt characteristics. The output signal of the microwave diode was recorded by an oscilloscope.

![Fig. 1. Schematic representation of experimental set-up.](image)

The generating and receiving parts of microwave diagnostics (with the exception of lenses) were placed in metal screens with autonomous power supply, which drastically reduced the influence of electromagnetic interference on the measurement results. The dielectric lenses closest to the discharge, one of which formed, and the other collected the microwave beam, were moved outside the flat electrode at a distance of 6 m from its edge, which was enough to exclude the influence of the lenses on the discharge. The distance between the forming and collecting lenses (microwave beam length) was 30 m. The microwave generator operated in continuous generation mode. In the absence of a discharge, the undisturbed level of the output signal of the microwave diode proportional to the power of the microwave radiation incident on the object was recorded. During the movement of the streamer zone of the spark discharge leader through the microwave beam, the transmitted radiation was weakened due to absorption in the plasma of the streamers. The measured parameter was the relative attenuation of transmitted microwave radiation. The main source of noise that determined the sensitivity of our diagnostics was fluctuations in the output power of the microwave generator, so the lower limit of the measured relative attenuation of transmitted radiation was $10^{-3}$.

As follows from the analysis given in the next paragraph, the main effect that leads to the weakening of the probing microwave radiation in the streamer zone is absorption in the
streamers plasma, and not scattering on the streamers or reflection from the boundaries of the streamer zone. In this case, the attenuation of the probing microwave radiation is proportional to the average high-frequency conductivity of the streamer zone (averaged over a volume that includes many streamers, but much smaller than the characteristic size of the streamer zone). In combination with data on the density of streamers and its distribution in the volume of the streamer zone obtained by other diagnostic methods, measurements of the attenuation of the probing microwave radiation allowed us to determine the magnitude and spatial distribution of the high-frequency conductivity of the streamer zone, the average value of the absorption of microwave radiation in one streamer and the average total number of free electrons in one streamer.

3 Microwave diagnostic measurement results

Figure 2a shows a photograph of the streamer zone of the positive leader taken with an exposure of 200 ns (the position of the microwave beam is marked with a white circle). Figure 2b shows the waveform of the relative attenuation of microwave radiation \( \delta = 1 - \frac{W_{tr}}{W_i} \), where \( W_i \) and \( W_{tr} \) are the incident and passing power of the microwave beam, respectively. The arrow in Fig.2b indicates the moment at which the discharge picture (Fig.2a) was taken.

Fig. 2. a - high-speed camera image of the breakthrough phase of positive leader. b - relative attenuation of the microwave radiation \( \delta \). The moment of the fast camera exposure is marked.

Measurements of the relative attenuation in different discharge pulses at different length of the streamer zone and different position of the streamer zone relative to the microwave beam showed that the relative attenuation of microwave radiation in the streamer zone at the time of photographing the discharge with the high-speed camera is directly proportional to the brightness of the streamer zone image at the location of the microwave beam. In order to take into account the inhomogeneity of the image brightness of the streamer zone over the cross section of the microwave beam, we calculated the average brightness over the cross section of the microwave beam \( I_a \) with a weight \( E_{mw}^2 \) proportional to the flux density of the microwave power:

\[
I_a(y_0, z_0) = \frac{\iint_{R_{mw}} I(y, z) E_{mw}^2 d\tau d\rho}{\iint_{R_{mw}} E_{mw}^2 d\tau d\rho} = \frac{1}{R_{mw}} \int_{-\infty}^{\infty} I(y, z) \exp \left\{ -\frac{(y-y_0)^2+(z-z_0)^2}{R_{mw}^2} \right\} dydz \tag{1}
\]

where \( y, z \) and \( \tau \) are coordinates and radius-vector transverse to the axis of the microwave beam, respectively, \( y_0 \) and \( z_0 \) are the coordinates of the axis of the microwave beam in the
image of the streamer zone, $I(r^+)$ is the brightness of the streamer zone, and $R_{mw}$ is the radius of the microwave beam. Fig. 3 shows the value of $\delta / I_a$ for four discharge events. The values of $\delta / I_a$ are within a factor of 2, and the relative standard deviation is 0.23.

![Graph of $\delta / I_a$ vs. $r$](image_url)

**Fig. 3.** The ratio $\delta / I_a$ at different distances $r$ between the axis of symmetry of the streamer zone and the axis of the microwave beam.

Relying on the proportionality of attenuation of microwave radiation to the brightness of the streamer zone and the axial symmetry observed in the images of the streamer zone, one can use the Abel inverse transformation to find the distribution of the absolute density of microwave radiation losses power in the streamer zone and then to determine parameters of the streamer zone.

## 4 Interpretation of microwave diagnostic measurements

### 4.1 The ratio of absorption and scattering of microwave radiation in the streamer zone (theory)

The weakening of the probing microwave radiation when passing through the streamer zone of the spark discharge is due to two effects - the absorption in the plasma of streamers and scattering on streamers. In our experiments, the distance between the streamers is several times greater than the microwave radiation wavelength, and the position of the streamers in space is random, therefore the coherent effects of the interaction of probe radiation with the streamer zone, as a combination of many streamers, can be ignored and the absorbed and scattered power can be each estimated by summing up the contributions from individual streamers. In this case, the ratio of the absorbed and scattered power of the probing microwave beam is equal to the ratio of absorbed and scattered powers for one streamer. The streamer is a thin cylindrical plasma channel, the longitudinal size of which is much larger than its diameter, therefore, the interaction of an electromagnetic wave with such a channel can be considered in the approximation of an infinitely long homogeneous circular cylinder. Streamers move approximately in one direction, along the axis of the streamer zone, which in our experiments was oriented vertically, therefore the plasma channels of streamers were also oriented approximately vertically. The axis of the microwave beam was horizontal, and the polarization was vertical. Therefore, to determine the ratio of the absorbed and scattered microwave powers for one streamer in our experiments, we need to consider the problem of diffraction of a plane electromagnetic wave on a circular cylinder with a finite conductivity when the wave vector is normal and the polarization is parallel to the axis of the cylinder [3]. The electric fields of the primary
and secondary waves in a cylindrical coordinate system \((z, r, \varphi)\) with the \(z\) axis directed along the cylinder axis have only the \(z\) component, and the magnetic field has \(r\) and \(\varphi\) components. The electric field \(E_r\) of the scattered wave and the electric field \(E_c\) and the \(\varphi\) component of the magnetic field \(H_{c\varphi}\) waves inside the cylinder are represented in the solution of the diffraction problem in the form of sums:

\[
E_r = \sum_{m=0}^{\infty} A_m H_m^{(2)}(kr) \cos(m \varphi) \quad r \geq a
\]

\[
E_c = \sum_{m=0}^{\infty} B_m J_m(k_c r) \cos(m \varphi) \quad r \leq a
\]

\[
H_{c\varphi} = \sum_{m=0}^{\infty} k_c B_m \cos(m \varphi) \frac{m}{k_c} J_m(k_c r) - J_{m+1}(k_c r) \quad r \leq a
\]

\[
A_0 = -\frac{J_1(k_0 ka) - \alpha J_0(k_0 ka) J_1(k_0 k_c a)}{\alpha h_1^{(2)}(ka) J_1(k_c a) - h_1^{(2)}(ka) J_0(k_c a)} E_i
\]

\[
B_0 = -\frac{J_0(k_0 ka) J_1(k_0 ka) - \alpha J_1(k_0 k_c a) J_0(k_0 k_c a)}{\alpha h_1^{(2)}(ka) J_1(k_c a) - h_1^{(2)}(ka) J_0(k_c a)} E_i
\]

\[
A_m = 2i \frac{m}{k_c} \frac{\left[J_{m-1}(ka) - \alpha J_{m+1}(ka)\right] J_m(k_c a) - \alpha J_m(k_0 ka) J_{m-1}(ka) - J_{m+1}(ka)}{\alpha h_m^{(2)}(ka) - h_m^{(2)}(ka)} E_i \quad m \geq 1
\]

\[
B_m = \frac{2i}{m!(ka) \left[J_{m-1}(ka) - \alpha J_{m+1}(ka)\right] J_m(k_c a) - \alpha J_m(k_0 ka) J_{m-1}(ka) - J_{m+1}(ka)} E_i \quad m \geq 1
\]

\[
a = \frac{\mu k_c}{\epsilon_c k} = \frac{\mu \epsilon_c}{\epsilon \mu}
\]

where \(a\) is the radius of the cylinder, \(k = \omega \sqrt{\epsilon \mu_0}\) is the complex wave number of the incident wave, \(k_c = \omega \sqrt{\epsilon_c \mu_0 \mu_c}\) is the complex wave number of the wave inside the cylinder, \(\epsilon\) and \(\mu\) are the complex relative permittivity and permeability of the medium outside the cylinder, \(\epsilon_c\) and \(\mu_c\) are the complex relative permittivity and permeability of the medium inside the cylinder, \(E_0\) and \(\mu_0\) are the permittivity and permeability of vacuum, \(\omega\) is the angular frequency of the waves, \(J_m\) is the Bessel function of the order \(m\), \(H_m^{(2)}\) is the Hankel function of the second kind and order \(m\). The ratio of the electromagnetic radiation power \(W_r\), scattered per unit length of the cylinder to the incident radiation power density \(S_i\) (this ratio is referred as scattering diameter) is equal to:

\[
\frac{W_r}{S_i} = \frac{1}{|E_i|^2} \int_0^{2\pi} |E_r|^2 r d\varphi \quad r \rightarrow \infty = \frac{2}{|k||E_i|^2} \sum_{m=0}^{\infty} |A_m|^2
\]

The ratio of the power \(W_a\) absorbed per unit length of the cylinder to the flow \(S_i\) of the incident power density (this ratio is referred as absorption diameter) is equal to:

\[
\frac{W_a}{S_i} = \frac{1}{E_i^* H_i + E_c H_c} \int_0^{2\pi} (E_c H_{c\varphi} + E_c H_{c\varphi})\, d\varphi = \frac{i \pi a}{k |E_i|^2} \sum_{m=0}^{\infty} |B_m|^2 \frac{k_c^2}{\mu_c} J_m(k_c a) J_m(k_c a) - \frac{k_c}{\mu_c} J_m(k_c a) J_{m+1}(k_c a) - \frac{m}{a} J_m(k_c a)^2 \left(1 - \frac{1}{m}\right)
\]

\[
\cdot \sum_{m=0}^{\infty} |B_m|^2 \frac{k_c^2}{\mu_c} J_m(k_c a) J_m(k_c a) - \frac{k_c}{\mu_c} J_m(k_c a) J_{m+1}(k_c a) - \frac{m}{a} J_m(k_c a)^2 \left(1 - \frac{1}{m}\right)
\]

If the cylinder diameter is much smaller than the wavelengths outside and inside the cylinder, then the following conditions are satisfied: \(|k| a \ll 1, |k_c| a \ll 1\). With a small value of the argument \(\xi \ll 1\), the functions \(J_m(\xi)\) and \(H_0^{(2)}(\xi)\) have the following form \([8]\):

\[
J_m(\xi) \approx \frac{1}{m!} \left(\frac{\xi}{2}\right)^m \quad m \geq 0
\]

\[
H_0^{(2)}(\xi) \approx 1 + \frac{1}{\pi} \ln \frac{2}{\xi}
\]
Substituting approximate expressions (11-13) for the Bessel and Hankel functions into expressions for coefficients (5-8), we obtain:

\[ A_0 \approx -i \frac{\pi}{4} \left( \frac{\varepsilon}{\varepsilon_0} - 1 \right) (ka)^2 E_i \quad B_0 \approx E_i \]  

\[ A_m \approx 2\pi i^{m-1} \frac{1}{m(m-1)!} \left( \frac{ka}{2} \right)^{2 \frac{m-1}{m}} \frac{1}{1+\varepsilon_0^\mu_\varepsilon} E_i \quad m \geq 1 \]  

\[ B_m \approx 4i^m \frac{k}{k_c} \frac{1}{1+\varepsilon_0^\mu_\varepsilon} E_i \quad m \geq 1 \]  

The scattering diameter (9) and the absorption diameter (10) in this case are approximately equal to:

\[ \frac{W_r}{S_i} \approx \frac{\pi^2}{8} |k|^3 \alpha^4 \left( \left| \frac{\varepsilon}{\varepsilon_0} - 1 \right|^2 + 2\pi \frac{1}{1+\varepsilon_0^\mu_\varepsilon} \right)^2 \]  

\[ \frac{W_a}{S_i} \approx \frac{\pi\alpha^2}{2\Re \left( \frac{k}{\mu_c} \right)} \Im \left( \frac{k^2}{\mu_c} \right) \]  

The ratio of the scattered power to the absorbed for a thin cylinder is approximately equal to:

\[ \frac{W_r}{W_a} \approx \frac{\pi|k|^3\alpha^2 \Re \left( \frac{k}{\mu_c} \right)}{4 \Im \left( \frac{k^2}{\mu_c} \right)} \left( \left| \frac{\varepsilon}{\varepsilon_0} - 1 \right|^2 + 2\pi \frac{1}{1+\varepsilon_0^\mu_\varepsilon} \right)^2 \]  

Let's consider a plasma cylinder in the atmospheric air. The relative permittivity and permeability of air are close to those of vacuum, \( \varepsilon = \mu = 1 \). The relative permeability of plasma is also approximately equal to unity, \( \mu_c = 1 \). The relative complex permittivity of the air plasma provided that the frequency \( \nu_{\text{em}} \) of collisions of electrons with molecules is much greater than the frequency of the electromagnetic field, \( \nu_{\text{em}} \gg \omega \), and that the movement of electrons is drift, is approximately equal to [4]

\[ \varepsilon_c \approx 1 + i \frac{\sigma}{\omega \varepsilon_0} \]  

where \( \sigma \) is the conductivity of the plasma. With such parameters, the expression (19) takes the form:

\[ \frac{W_r}{W_a} \approx \frac{\pi(ka)^2}{4} \frac{\sigma}{\omega \varepsilon_0} \]  

The condition of smallness of the scattered power with respect to the power absorbed, \( W_r \ll W_a \), imposes an upper limit on the value of conductivity:

\[ \sigma \ll \frac{4\omega \varepsilon_0}{\pi (ka)^2} \]  

The condition for the validity of the thin-cylinder approximation \( |k_c| a = ka \left[ 1 + \left( \frac{\sigma}{\omega \varepsilon_0} \right)^2 \right]^{0.25} \ll 1 \) also limits the conductivity, and this condition is a little more stringent than (22):

\[ \sigma \ll \frac{\omega \varepsilon_0}{(ka)^2} \]
The conductivity of the air plasma under the condition $v_{em} \gg \omega$ is \[\sigma \approx \frac{e^2 n_e}{mv_{em}}\] (24) where $n_e$ is the electron density, $e$ is the charge of electron, and $m$ is the mass of electron. Substituting (24) into inequality (23) and replacing $a$ with the streamer radius $r_{str}$, we obtain the condition on the parameters of the streamers, at which the absorption of the probe microwave radiation in the streamers is much more efficient than scattering: \[n_{e, str}^2 r_{str}^2 \ll \frac{mv_{em}}{\mu_0 e^2 \omega} \approx 1.2 \cdot 10^{13} \text{ cm}^{-1}\] (25)

In the calculation of the right-hand side of inequality (25), the following values were used: the value of angular frequency of microwave radiation in our experiments, $\omega \approx 2.2 \cdot 10^{11} \text{ c}^{-1}$, and the transport frequency of the collisions of electrons with molecules in the atmospheric air \[v_{em} \approx 3 \cdot 10^{12} \text{ c}^{-1}\].

4.2 Relationship between the absorption of microwave radiation in the streamer zone with the parameters of the streamer zone

4.2.1 Total number of free electrons in the streamer

As was shown in the previous paragraph, the microwave field in the streamers is approximately equal to the unperturbed field of the incident microwave beam $E_{mw}(\vec{r})$. The power absorbed by a free electron oscillating in a gas under the action of an electric field $E_{mw}(\vec{r})$, provided $v_{em} \gg \omega$, is equal to $e^2 E_{mw}^2(\vec{r})/2mv_{em}$ [4]. Then the microwave power absorbed by one streamer containing $N_e$ free electrons is equal:

$$W_i = N_e \frac{e^2 E_{mw}^2(\vec{r})}{2mv_{em}}$$

The total microwave radiation power $W_a$ absorbed in the streamer zone is determined by volume integration:

$$W_a = \frac{e^2 N_e}{2mv_{em}} \int n_{str}(\vec{r}_l, x) E_{mw}^2(\vec{r}_l) d^2 \vec{r}_l dx$$

where $n_{str}(\vec{r}_l, x)$ is the volume density of streamers, $x$ is the coordinate along the axis of the microwave beam, $\vec{r}_l$ is the radius-vector transverse to the axis of the microwave beam. In expression (27), we assume that the number of electrons in all streamers of the streamer zone is the same (so $N_e$ is a constant) and that the microwave beam is nearly parallel within the streamer zone (hence $E_{mw}^2$ is independent of $x$). The relative attenuation $\delta$ of the microwave radiation measured in the experiment is the ratio of the power absorbed in the streamer zone to the power of the incident radiation $W_i$:

$$\delta = \frac{W_a}{W_i} = \frac{e^2 N_e}{m v_{em}} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left[ \int n_{str}(\vec{r}_l) E_{mw}^2(\vec{r}_l) d^2 \vec{r}_l \right] \frac{n_s(\vec{r}_l)}{\int E_{mw}^2(\vec{r}_l) d^2 \vec{r}_l}$$

where $n_s(\vec{r}_l)$ is the surface (per unit area) density of streamer projections onto a plane parallel to the axis of symmetry of the streamer zone. The radial distribution of the power flux in the microwave beam was found to be Gaussian:

$$E_{mw}^2(\vec{r}_l) \approx \exp \left(-\frac{r_l^2}{r_{mw}^2}\right)$$


where $R_{mw}$ is the radius of the microwave beam. We can find the total number of electrons in one streamer from (29) if the values of $n_e(\vec{r}_\perp)$ and $\delta$ are known:

$$
N_e = \frac{\varepsilon_0 \, m v_{em}}{\mu_0} \frac{\pi R_{mw}^2}{\int n_e(\vec{r}_\perp) \exp\left\{ -\frac{r^2}{R_{mw}^2}\right\} d^2\vec{r}_\perp} \delta
$$

(31)

### 4.2.2 High-Frequency conductivity of the streamer zone

The total microwave power absorbed in the streamer zone can be represented as:

$$
W_a = \frac{1}{2} \int \sigma_{HF}(\vec{r}_\perp, x) E_{mw}^2(\vec{r}_\perp) d^2\vec{r}_\perp dx
$$

(32)

where $\sigma_{HF}(\vec{r}_\perp, x)$ is averaged high-frequency conductivity of the streamer zone which is conductivity averaged in the volume $V$ containing many streamers but having a size much less than the size of the streamer zone:

$$
\sigma_{HF}(\vec{r}_\perp, x) = \frac{e^2}{m v_{em}} \frac{1}{V} \int n_e(\vec{r}_\perp, x) dV = \frac{e^2 N_e n_{str}(\vec{r}_\perp, x)}{m v_{em}}
$$

(33)

where $n_e(\vec{r}_\perp, x)$ is the electron density.

Based on the proportionality of the relative absorption of microwave radiation in the streamer zone and the brightness of the image of the streamer zone at the position of the microwave beam, we can conclude that the spatial distribution of high-frequency conductivity coincides with the spatial distribution of the volume luminosity of the streamer zone, which can be found from the image of the streamer zone using the inverse Abel transform. Then the absolute value of the high-frequency conductivity can be determined from a measurement of the relative absorption of microwave radiation:

$$
\delta = \frac{W_a}{W_i} = \frac{\int \sigma_{HF}(\vec{r}_\perp, x) E_{mw}^2(\vec{r}_\perp) d^2\vec{r}_\perp dx}{c \varepsilon_0 \int E_{mw}^2(\vec{r}_\perp) d^2\vec{r}_\perp}
$$

(34)

where $W_i = c \varepsilon_0 \int E_{mw}^2(\vec{r}_\perp) d^2\vec{r}_\perp$ is the incident microwave power.

### 4.3 Results of determination of streamer zone parameters from the microwave diagnostics data

Figure 4 shows two consecutive frames of the breakthrough phase of spark discharge and the relative attenuation of the probing microwave radiation for this discharge. The high-frequency conductivity of the streamer zone shown in Fig. 4a,b was determined by the method described in paragraph 4.2.2. Figure 5 shows the dependence of the high-frequency conductivity on the axis of the streamer zone on the vertical coordinate $z$ for both images in Fig. 4.

We have determined the value of $n_e(\vec{r}_\perp)$ from the images of the streamer zone (Fig.4a,b) by the statistical analyzing method (not described in this paper) and then have found the value of the total number of free electrons in one streamer from (31) using measured value of $\delta$. We have get the following results: for the first frame (Fig.4a) $N_e = 4.6 \cdot 10^{10}$, for the second frame (Fig.4b) $N_e = 3.4 \cdot 10^{10}$.

### 5 Conclusion

Microwave diagnostics is an effective method for studying long sparks, supplementing standard methods for spark discharges and allowing obtaining new data on the parameters of long sparks that are inaccessible to standard methods in this field of research.

This research was funded by the RSF grant 19-19-00501.
**Fig. 4.** The breakthrough phase of positive spark. a and b - two consecutive high-speed camera images. The exposure time of each frame is 0.2 μs, the time delay between the end of the first exposure and the start of the second one is 10 μs. The circles indicate the position of microwave beam and its diameter at 1/e level of intensity. c - relative attenuation of probing microwave radiation δ. The times of the two frames are marked.

**Fig. 5.** High-frequency conductivity on the axis of the streamer zone for two frames presented in Fig. 4.

**References**
