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Tribonacci graphs

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Abstract. Special numbers have very important mathematical properties alongside their numerous applications in many fields of science. Probably the most important of those is the Fibonacci numbers. In this paper, we use a generalization of Fibonacci numbers called tribonacci numbers having very limited properties and relations compared to Fibonacci numbers. There is almost no result on the connections between these numbers and graphs. A graph having a degree sequence consisting of t successive tribonacci numbers is called a tribonacci graph of order t. Recently, a new graph parameter named as omega invariant has been introduced and shown to be very informative in obtaining combinatorial and topological properties of graphs. It is useful for graphs having the same degree sequence and gives some common properties of the realizations of this degree sequence together with some properties especially connectedness and cyclicness of all realizations. In this work, we determined all the tribonacci graphs of any order by means of some combinatorial results. Those results should be useful in networks with large degree sequences and cryptographic applications with special numbers.

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1 Significance of the work

Graph theory is getting more popular each day due to its numerous applications in many areas including some industrial applications. Therefore all aspects of graph theory is receiving more attention and require new methods. Especially graph indices play an important role in the study of molecular graphs effecting chemistry, physics, pharmacology, etc. One of the most important graph index is the Wiener index. It is calculated by means of the distances between the vertices of the graph and recently edge version of this index have been defined and studied. In this paper, we consider the entire Wiener index of graphs which is defined by means of all distances between vertices and edges as many aspects in application requires knowledge of both.

2 Introduction

Let $G = (V, E)$ be a graph having order n and size m. The degree of a vertex $v \in V(G)$ is denoted by d_v. A vertex of degree one will be called a pendant vertex and an edge having a

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pendant vertex will be called a pendant edge. The largest vertex degree in a graph is denoted by Δ. If u and v are two adjacent vertices of G, then the edge e connecting these vertices is denoted by $e = uv$ and also the vertices u and v are called adjacent vertices. e is said to be incident with the vertices u and v. A graph is connected if there is a path between every pair of vertices and disconnected if not.

Graphs are sometimes classified according to the number of cycles they have. Graphs having no cycle like all trees are called acyclic. The remaining graphs are called cyclic graphs. In particular, a graph having one, two, three cycles is called uni-, bi- and tri-cyclic, respectively.

An edge connecting a vertex to itself is called a loop, and at least two edges connecting two vertices will be called multiple edges. When there are no loops nor multiple edges, the graph will be called simple.

A degree sequence is a set $D = \{1^{(a_1)}, 2^{(a_2)}, 3^{(a_3)}, \cdots, \Delta^{(a_\Delta)}\}$, where some of a_i's could be zero. Let $D = \{d_1, d_2, d_3, \cdots, \Delta\}$ be a set of non-negative integers. If D is the degree sequence of a graph G, then D is said to be realizable. The most popular and effective test for determining realizability of a given set is Havel-Hakimi criteria, see [6, 7].

Probably the most popular number sequence is the Fibonacci sequence and the tribonacci sequence $1, 1, 2, 4, 7, 13, 24, \cdots$ is defined similarly. Let T_n be the n-th tribonacci number. While $T_1 = 1$, $T_2 = 1$, $T_3 = 2$ are the initial values, the recurrence relation $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$ holds. Tribonacci numbers have got lots of properties and there are many identities on them. The sum $T_1 + T_2 + \cdots + T_n$ of the first n tribonacci numbers given by

$$\Sigma_{i=1}^{n} T_i = \frac{1}{2}\left(T_{n+2} + T_n - 1\right).$$

A tribonacci graph of order n is a graph with vertex degrees being n successive tribonacci numbers. We study the existence of tribonacci graphs.

3 Ω invariant

For a realizable degree sequence D, a new graph invariant called Ω invariant has been introduced in [2]. We briefly recall some fundamental properties of it, see [2, 3]: The $\Omega(D)$ is introduced in terms of D as $\Omega(D) = a_3 + 2a_4 + 3a_5 + \cdots + (\Delta - 2)a_\Delta - a_1 = \sum_{i=1}^{\Delta}(i-2)a_i$. Ω is additive over the set of the components of G and $\Omega(G) = 2(m - n)$. Hence, for a graph G, $\Omega(G)$ is even. Therefore when $\Omega(D)$ is odd for a set D of non-negative integers, D is not realizable.

The number r of independent (non-overlapping) cycles in G known as the cyclomatic number of G is given by

$$r = \frac{\Omega(G)}{2} + c \tag{1}$$

where c is the number of components of G. The fact that the maximum number of components amongst all realizations of a given degree sequence is

$$c_{max} = \sum_{d_i \ even} a_i + \frac{1}{2} \sum_{d_i \ odd} a_i \tag{2}$$

was given in [3]. Several properties of omega invariant are studied in [1, 4, 8, 9].

In this paper, we shall use some special graph classes of order one, two, three or four. The former two were already used in [3] in solving the extremal problem of finding the maximum number of components amongst all realizations of a given degree sequence. L_q is constructed by adding q loops to a single vertex. First few L_q graphs are seen in Fig. 1:

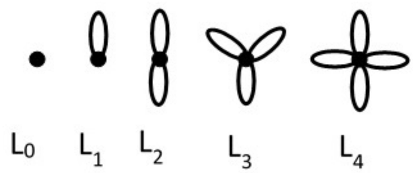

Figure 1 First few L_q graphs

Second graph class denoted by $B_{r,s}$ for $r,\ s \in N$ is defined by adding r and s loops to the ends of an edge. The first few $B_{r,s}$ are seen in Fig. 2:

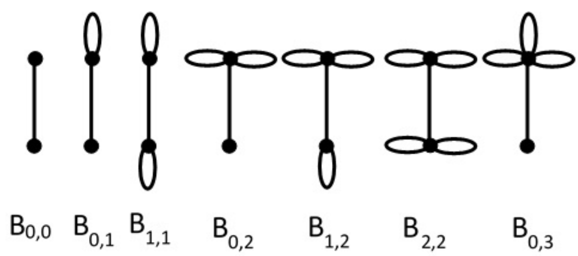

Figure 2 First few $B_{r,s}$ graphs

A connected graph having three vertices u, v, w of degrees $2a + 1$, $2b + 2$ and $2c + 1$ with a, b, c are integers, respectively, is a graph $T_{a,b,c}$ consisting of a path $P_3 = \{u, v, w\}$ so that a loops are incident to u, b loops are incident to v and c loops are incident to w, see Fig. 3.

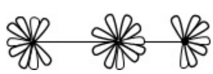

Figure 3 $T_{9,10,7}$

The last type of graphs we shall need is a connected graph $Q_{a,b,c,d}$ on four vertices u, v, w, z of degrees $2a + 1$, $2b + 2$, $2c + 2$ and $2d + 1$ with a, b, c, d are integers. It is the graph consisting of a path $P_4 = \{u, v, w, z\}$ so that a loops are incident to u, b loops are incident to v, c loops are incident to w and d loops are incident to z, see Fig. 4.

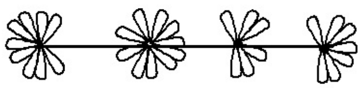

Figure 4 $Q_{9,10,6,7}$

For positive integers r and k, we use the notation "$r[k]$" for a graph where k loops are attached to a new vertex on one of the r loops incident to others. For positive integers $r, k_1, k_2, \cdots, k_t$, the symbol "$r[k_1, k_2, \cdots, k_t]$" means that k_1 loops are attached to one of the loops, k_2 loops are attached to another loop, and continuing this way, k_t loops are attached to yet another loop at a vertex of degree r. Finally, the notation "$r[k_1; k_2; \cdots; k_t]$" will mean that k_1 loops are attached to one of the loops of degree r at a new vertex, k_2 loops are attached to the same loop at another new vertex, and continuing this way, k_t loops are attached to the same loop at another new vertex. To clarify these definitions, the notations $B_{2[3],1}$, $B_{2[3,1],1}$ and $T_{2[3;1],1}$ corresponds to graphs shown in Fig. 5:

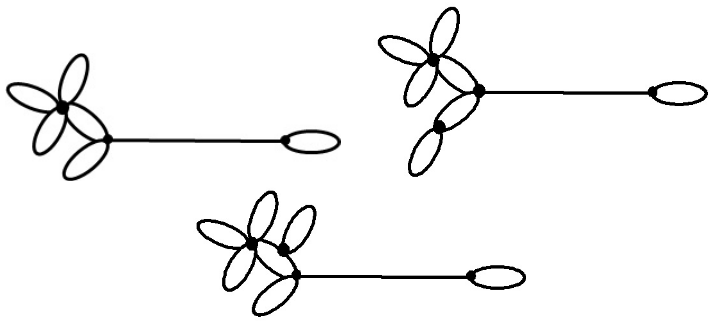

Figure 5 The graphs $B_{2[3],1}$, $B_{2[3,1],1}$ and $B_{2[3;1],1}$

4 Existence conditions for tribonacci graphs

In [5, 10], the Fibonacci and Lucas graphs have been studied and their existence conditions have been given. The general results obtained for the existence of Fibonacci and Lucas graphs of order n were given in modulo 3. In this work, we shall obtain the existence relation of tribonacci graphs in modulo 4. It seems that such an existence result depends on the number of terms in the recurrence relation. As the recurrence relations $F_n = F_{n-1} + F_{n-2}$, $L_n = L_{n-1} + L_{n-2}$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for Fibonacci, Lucas and tribonacci numbers have 2, 2 and 3 terms on their right hand sides with total giving the number on the left, their existence conditions are in modulo 3, 3 and 4, respectively. We have not tried to prove the general case for k-bonacci graphs, but it is not difficult to state the following:

Conjecture 4.1 *The existence condition for a k-bonacci graph can be given in modulo* $k + 1$.

In this section we determine the existence of tribonacci graphs having n consecutive tribonacci numbers as the vertex degrees. That is, we are looking for graphs having degree sequence consisting of n consecutive tribonacci numbers. This problem can differently be stated as follows: Which sets consisting of n tribonacci numbers are realizable? We shall solve this problem by means of the graph invariant Ω and related results given in [2].

4.1 Tribonacci graphs of order 1

Recall that the tribonacci numbers follow the rule odd, odd, even, even, odd, odd, even, even $\cdots$. That is, if the index of a tribonacci number is 1 or 2 modulo 4, then it is odd, otherwise it is even. Therefore only the tribonacci numbers $T_3,\ T_4,\ T_7,\ T_8,\ T_{11},\ T_{12},\ \cdots$ are even. We easily conclude the following:

Theorem 4.1 *A tribonacci graph of order 1 with degree sequence* $\{T_n\}$ *is realizable iff* $n \equiv 3$ *or 0 modulo 4.*

4.2 Tribonacci graphs of order 2

Secondly, we study the case of two successive tribonacci numbers as vertex degrees of a graph. That is, we want to determine which sets of two consecutive tribonacci numbers are realizable as graphs. By the observation above, we can deduce that the sum of these two consecutive tribonacci numbers must be even. This is only possible when the index of the smaller term is odd. So we proved

Theorem 4.2 *A degree sequence consisting of two consecutive tribonacci numbers is realizable iff the index of the smaller term is odd.*

Hence we can take the degree sequence of such a graph consisting of two consecutive tribonacci numbers as

$$D = \left\{T_{2n-1}^{(1)},\ T_{2n}^{(1)}\right\}.$$

For such a graph, we have

$$\begin{aligned}\Omega(D) &= (T_{2n-1} - 2) \cdot 1 + (T_{2n} - 2) \cdot 1 \\ &= T_{2n-1} + T_{2n} - 4.\end{aligned}$$

Hence the number r of faces of a connected realization G of D would be

$$\begin{aligned}r &= \tfrac{\Omega(D)}{2} + 1 \\ &= \tfrac{T_{2n-1}+T_{2n}-4}{2} + 1 \\ &= \tfrac{T_{2n-1}+T_{2n}}{2} - 1.\end{aligned}$$

By Eqn. (2), the realization G of D might have c_{max} components where

$$\begin{aligned}c_{max} &= \textstyle\sum_{d_i\ even} a_i + \frac{1}{2}\sum_{d_i\ odd} a_i \\ &= \begin{cases}\dfrac{1}{2}(1+1) = 1, & \text{if n is odd,} \\ 1 + 1 = 2, & \text{if n is even.}\end{cases}\end{aligned}$$

That is, we proved the following:

Theorem 4.3 *A tribonacci graph of order two could be either connected or disconnected with two components.*

In the first possible case, if we use above formula for $T_1 = 1$ and $T_2 = 1$, then there is just one connected graph as below:

Figure 6 The graph with degree sequence $\{T_1 = 1,\ T_2 = 1\}$

This graph is $B_{0,0}$. Because of $r = \frac{2-2}{2} = 0$, there is no region.

For the second possible pair $T_3 = 2,\ T_4 = 4$, there are two graphs one is connected and the other is disconnected:

Figure 7 One connected and one disconnected graph with degree sequence $\{T_3 = 2,\ T_4 = 4\}$

The connected graph is $L_{2[0]}$ and the disconnected one is $L_1 \cup L_2$. By Eqn. (1), we have $r = \frac{\Omega}{2} + 1 = 2$ for the connected graph $L_{2[0]}$, and there are two closed regions. Again by the same equation, we have $r = \frac{\Omega}{2} + 2 = 3$ faces for the disconnected graph $L_1 \cup L_2$.

For the remaining cases where a graph with degree sequence $D = \left\{T^{(1)}_{2n-1},\ T^{(1)}_{2n}\right\}$ with $n \geq 3$, we have the following possibilities: If n is odd, G must be connected and $G = B_{\frac{T_{2n-1}-1}{2},\frac{T_{2n}-1}{2}}$. If n is even, then either G is connected or has two components. In the former case, the graph is $L_{\frac{T_{2n-1}}{2}\left[\frac{T_{2n}-2}{2}\right]} = L_{\frac{T_{2n}}{2}\left[\frac{T_{2n-1}-2}{2}\right]}$. In the disconnected realization case, we have two components when n is even and hence it has

$$\begin{aligned} r &= \tfrac{\Omega(D)}{2} + 2 \\ &= \tfrac{T_{2n-1}+T_{2n}-4}{2} + 2 \\ &= \tfrac{T_{2n-1}+T_{2n}}{2} \end{aligned}$$

faces. The corresponding graph is $L_{\frac{T_{2n-1}}{2}} \cup L_{\frac{T_{2n}}{2}}$.

4.3 Tribonacci graphs of order 3

By the parities of tribonacci numbers, we observe that the three succesive tribonacci numbers must be chosen either as $T_{4n},\ T_{4n+1}$ and T_{4n+2} or as $T_{4n-3},\ T_{4n-2}$ and T_{4n-1} for $n \geq 1$, to have even sum. That is, to have a realizable degree sequence D consisting of three consecutive tribonacci numbers, it must be either

$$D_1 = \left\{T^{(1)}_{4n-3},\ T^{(1)}_{4n-2}, T^{(1)}_{4n-1}\right\}.$$

or

$$D_2 = \left\{T^{(1)}_{4n},\ T^{(1)}_{4n+1}, T^{(1)}_{4n+2}\right\}.$$

The omega values of these are

$$\begin{aligned} \Omega(D_1) &= (T_{4n-3} - 2) \cdot 1 + (T_{4n-2} - 2) \cdot 1 + (T_{4n-1} - 2) \cdot 1 \\ &= T_{4n} - 6 \end{aligned}$$

and

$$\begin{aligned} \Omega(D_2) &= (T_{4n} - 2) \cdot 1 + (T_{4n+1} - 2) \cdot 1 + (T_{4n+2} - 2) \cdot 1 \\ &= T_{4n+3} - 6. \end{aligned}$$

Hence the number of faces of any connected realization of D must be

$$\begin{aligned} r_1 &= \tfrac{T_{4n}-6}{2} + 1 \\ &= \tfrac{T_{4n}}{2} - 2 \end{aligned}$$

or

$$\begin{aligned} r_2 &= \tfrac{T_{4n+3}-6}{2} + 1 \\ &= \tfrac{T_{4n+3}}{2} - 2. \end{aligned}$$

Let us now calculate the maximum number c_{max} of components that any realization could have as follows: In both D_1 and D_2, there are two odd and one even integers. So $c_{max} = 2$ by Eqn. (2). Hence we proved that

Theorem 4.4 *A tribonacci graph of order three is either connected or could have two components.*

The first case is $T_1 = 1,\ T_2 = 1,\ T_3 = 2$. There are two realizations having these vertex degrees. In Fig. 8, the connected realization P_3 is on the left and the disconnected realization $B_{0,0} \cup L_1$ is on the right.

Figure 8 One connected and one disconnected graph with degree sequence $\{T_1 = 1,\ T_2 = 1,\ T_3 = 2\}$

The former one has $r = \frac{T_4-6}{2} + 1 = 0$ faces, as easily seen, and the latter one has $r = \frac{T_4-6}{2} + 2 = 1$ face.

The second case is $T_4 = 4,\ T_5 = 7,\ T_6 = 13$. There are four realizations having these vertex degrees. In Fig. 9, the disconnected realization is on the left and three connected realizations are on the right.

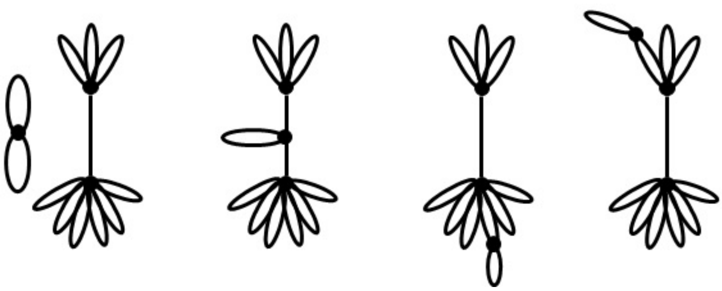

Figure 9 One disconnected and three connected graphs with degree sequence $\{T_4 = 4,\ T_5 = 7,\ T_6 = 13\}$

The first one has $r = \frac{T_7-6}{2} + 2 = 11$ faces and the latter ones have $r = \frac{T_7-6}{2} + 1 = 10$ faces.

The remaining cases are $\{T_{4n-3},\ T_{4n-2},\ T_{4n-1}\}$ and $\{T_{4n},\ T_{4n+1},\ T_{4n+2}\}$. In the former case, there are four realizations having these vertex degrees which are $L_{\frac{T_{4n-1}}{2}} \cup B_{\frac{T_{4n-3}-1}{2},\frac{T_{4n-2}-1}{2}}$, $T_{\frac{T_{4n-3}-1}{2},\frac{T_{4n-1}-2}{2},\frac{T_{4n-2}-1}{2}}$, $B_{\frac{T_{4n-3}-1}{2}[\frac{T_{4n-1}-2}{2}],\frac{T_{4n-2}-1}{2}}$ and $B_{\frac{T_{4n-3}-1}{2},\frac{T_{4n-2}-1}{2}[\frac{T_{4n-1}-2}{2}]}$. The first one which is the only disconnected one with two components has $r = \frac{T_{4n-1}}{2} + \frac{T_{4n-3}-1}{2} + \frac{T_{4n-2}-1}{2}$ faces. The other three connected ones have $r = \frac{T_{4n-1}-2}{2} + \frac{T_{4n-3}-1}{2} + \frac{T_{4n-2}-1}{2}$ faces. In the latter case, again there are four realizations having this degree sequence which are $L_{\frac{T_{4n}}{2}} \cup B_{\frac{T_{4n+1}-1}{2},\frac{T_{4n+2}-1}{2}}$, $T_{\frac{T_{4n+1}-1}{2},\frac{T_{4n}-2}{2},\frac{T_{4n+2}-1}{2}}$, $B_{\frac{T_{4n+1}-1}{2}[\frac{T_{4n}-2}{2}],\frac{T_{4n+2}-1}{2}}$ and $B_{\frac{T_{4n+1}-1}{2},\frac{T_{4n+2}-1}{2}[\frac{T_{4n}-2}{2}]}$. The first one which is the only disconnected one with two components has $r = \frac{T_{4n}}{2} + \frac{T_{4n+1}-1}{2} + \frac{T_{4n+2}-1}{2}$ faces. The other three connected ones have $r = \frac{T_{4n}-2}{2} + \frac{T_{4n+1}-1}{2} + \frac{T_{4n+2}-1}{2}$ faces.

4.4 Tribonacci graphs of order 4

According to the parities of tribonacci numbers, the sum of any four consecutive tribonacci numbers is always even. Therefore

Theorem 4.5 *Any set consisting of four consecutive tribonacci numbers is realizable.*

Hence if the four consecutive tribonacci numbers are $T_{4n+k},\ T_{4n+k+1},\ T_{4n+k+2},\ T_{4n+k+3}$, then they form a tribonacci graph G of order 4 and its omega invariant is

$$\begin{aligned}\Omega(G) &= T_{4n+k} + T_{4n+k+1} + T_{4n+k+2} + T_{4n+k+3} - 8\\ &= 2T_{4n+k+3} - 8.\end{aligned}$$

Hence considering Eqn. (2), any realization of this degree sequence could have maximum three components. That is, a tribonacci graph of order 4 could be either connected or have two or three components. Let us name the odd tribonacci numbers by O_i and O_j which are two successive ones amongst $T_{4n+k},\ T_{4n+k+1},\ T_{4n+k+2},\ T_{4n+k+3}$ and even ones by E_r and E_s. Actually, there is only one realization $B_{\frac{O_i-1}{2},\frac{O_j-1}{2}} \cup L_{\frac{E_r}{2}} \cup L_{\frac{E_s}{2}}$ having three components.

There are six realizations having two components which are

$$\begin{array}{lll} L_{\frac{E_r}{2}} \cup T_{\frac{O_i-1}{2},\frac{E_s-2}{2},\frac{O_j-1}{2}}, & L_{\frac{E_r}{2}} \cup T_{\frac{O_i-1}{2}[\frac{E_s-2}{2}],\frac{O_j-1}{2}}, & L_{\frac{E_r}{2}} \cup T_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_s-2}{2}]}, \\ L_{\frac{E_s}{2}} \cup T_{\frac{O_i-1}{2},\frac{E_r-2}{2},\frac{O_j-1}{2}}, & L_{\frac{E_s}{2}} \cup T_{\frac{O_i-1}{2}[\frac{E_r-2}{2}],\frac{O_j-1}{2}}, & L_{\frac{E_s}{2}} \cup T_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_r-2}{2}]} \end{array}.$$

Finally there are sixteen realizations having one component only, that is, there are sixteen connected realizations as below:

$$\begin{array}{lll} Q_{\frac{O_i-1}{2},\frac{E_r-2}{2},\frac{E_s-2}{2},\frac{O_j-1}{2}}, & T_{\frac{O_i-1}{2}[\frac{E_s-2}{2}],\frac{E_r-2}{2},\frac{O_j-1}{2}}, & T_{\frac{O_i-1}{2},\frac{E_r-2}{2},\frac{O_j-1}{2}[\frac{E_s-2}{2}]}, \\ T_{\frac{O_i-1}{2},\frac{E_r-2}{2}[\frac{E_s-2}{2}],\frac{O_j-1}{2}}, & T_{\frac{O_i-1}{2}[\frac{E_r-2}{2}],\frac{E_s-2}{2},\frac{O_j-1}{2}}, & T_{\frac{O_i-1}{2},\frac{E_s-2}{2},\frac{O_j-1}{2}[\frac{E_r-2}{2}]}, \\ T_{\frac{O_i-1}{2},\frac{E_s-2}{2}[\frac{E_r-2}{2}],\frac{O_j-1}{2}}, & B_{\frac{O_i-1}{2}[\frac{E_r-2}{2},\frac{E_s-2}{2}],\frac{O_j-1}{2}}, & B_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_r-2}{2},\frac{E_s-2}{2}]}, \\ B_{\frac{O_i-1}{2}[\frac{E_r-2}{2}],\frac{O_j-1}{2}[\frac{E_s-2}{2}]}, & B_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_r-2}{2}[\frac{E_s-2}{2}]]}, & B_{\frac{O_i-1}{2}[\frac{E_r-2}{2}[\frac{E_s-2}{2}]],\frac{O_j-1}{2}}, \\ B_{\frac{O_i-1}{2}[\frac{E_s-2}{2}[\frac{E_r-2}{2}]],\frac{O_j-1}{2}}, & B_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_s-2}{2}[\frac{E_r-2}{2}]]}, & B_{\frac{O_i-1}{2}[\frac{E_r-2}{2};\frac{E_s-2}{2}],\frac{O_j-1}{2}}, \\ & B_{\frac{O_i-1}{2},\frac{O_j-1}{2}[\frac{E_r-2}{2};\frac{E_s-2}{2}]} & \end{array}.$$

The number of faces such a realization would have is $r = T_{4n+k+3} - 4 + c$ where c is the number of components which could be 1, 2 or 3, according to above calculations.

4.5 Tribonacci graphs of order $n \geq 5$

In Subsections 3.1-3.4, we determined the conditions for sets of $n = 1, 2, 3$ and 4 consecutive tribonacci numbers to be realizable. Similar discussions using the fact that the sum of any four consecutive tribonacci numbers is always even gives the following generalizations:

Corollary 4.1 ***i)*** *A set consisting of* $n = 4s + 1$ *consecutive Fibonacci numbers is realizable iff the index of the first tribonacci number is congruent to 3 or 0 modulo 4.*

ii) *A set consisting of* $n = 4s + 2$ *consecutive Fibonacci numbers is realizable iff the index of the first tribonacci number is odd.*

iii) *A set consisting of* $n = 4s + 3$ *consecutive Fibonacci numbers is realizable iff the index of the first tribonacci number is congruent to 1 or 0 modulo 4.*

iv) *A set consisting of* $n = 4s$ *consecutive Fibonacci numbers is always realizable.*

5 Conclusion

In the existing literature, there are results giving a large number of formulae, equalities, bounds and relations on special numbers such as Fibonacci, Lucas and tribonacci numbers. But there is almost no result on the connections between these numbers and graphs. In this work, we determined all the tribonacci graphs of any order by means of omega invariant and some combinatorial results. We believe that the results obtained here can be applied to several areas of combinatorics, cryptology and graph theory in relation with other special number classes.

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