VECTOR-VALUED NONUNIFORM MULTIRESOLUTION ASSOCIATED WITH LINEAR CANONICAL TRANSFORM

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1. Abstract A multiresolution analysis associated with linear canonical transform was defined by Shah and Waseem for which the translation set is a discrete set which is not a group. In this paper, we continue the study based on this nonstandard setting and introduce vector-valued nonuniform multiresolution analysis associated with linear canonical transform (LCT-VNUMRA) where the associated subspace V_0^μ of $L^2\big(\mathbb{R}, \mathbb{C}^M\big)$ has an orthonormal basis of the form $\left\{\mathbf{\Phi}(x-\lambda)e^{-\frac{-\iota\pi A}{B}(t^2-\lambda^2)}\right\}_{\lambda\in\Lambda}$ where $\Lambda = \{0, r/N\} + 2\mathbb{Z}, N \geq 1$ is an integer and r is an odd integer such that r and N are relatively prime. We establish a necessary and sufficient condition for the existence of associated wavelets and derive an algorithm for the construction of vector-valued nonuniform multiresolution analysis on local fields starting from a vector refinement mask with appropriate conditions.

Keywords: Non-uniform multiresolution analysis; Linear canonical transform; Scaling function; Vector-valued wavelets.

AMS Subject Classification: 42C40, 42C15, 43A70, 11S85

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