Joint estimation and detection method based on turbo equalization framework and VAMP

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Abstract. In this letter, we consider the single-carrier frequency domain equalization (SC-FDE) system, and propose a low-complexity joint symbol detection and channel estimation algorithm based on the recently proposed vector approximate message passing (VAMP). Specifically, we leverage VAMP twice to estimate symbols and channels, respectively, in a turbo-like way. Moreover, this algorithm organically combines the gaussian mixture model (GMM), which can accurately simulate the sparse aggregation characteristics of the channel and effectively suppress inter symbol interference (ISI). The simulation results show that compared with the traditional linear minimum mean square error (LMMSE) estimation receiving algorithm and the existing generalized approximate message passing algorithm (GAMP), the designed receiving algorithm has significant advantages in channel estimation normalized mean square error (NMSE) and bit error ratio (BER) performance, where sharing the same order of complexity.

1 Introduction

In the modern broadband wireless communication system, SC-FDE [1], [2] is an efficient broadband digital transmission scheme against multipath interference, which effectively improves the shortcomings of OFDM technology [3], and can achieve the performance similar to that of OFDM system. The anti-multipath ability is also significantly enhanced. In addition, it has the characteristics of low peak-to-average power ratio (PAPR) in single carrier transmission system. When the channel state information of the receiver is known, the complexity of the frequency domain LMMSE equalizer increases logarithmically with the increase of the number of channel taps L, which is lower than the complexity of the exponential level of the time domain receiving algorithm. The iterative receiving algorithm based on joint BP-EP effectively suppresses ISI. Its algorithm complexity is higher than that of the traditional LMMSE equalization. The frequency domain LMMSE equalization algorithm based on inter-block interference has high transmission efficiency because there is no insertion protection interval in block transmission, but the performance of the receiving algorithm is reduced due to the existence of inter-block interference.

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Turbo equalization algorithm based on MAP criterion [4], [5] has the optimal performance, but its computational complexity increases exponentially with the channel length. The Turbo equalization algorithm based on the LMMSE criterion [6-8] Gaussianizes the discrete symbols output by the decoder, uses the heuristic idea to directly calculate the mean and variance of symbols as the Gaussian pdf parameters, simplifies the nonlinear problem of MAP equalization to the linear Gaussian problem, and reduces the computational complexity. In order to further reduce the computational complexity, Rangan obtained the generalized approximate message passing algorithm (GAMP) [9] through the central limit theorem and Taylor series approximation method. As an advanced message passing algorithm, GAMP can recover the random vector in a linear system related to probability observation. For SC-FDE system, an iterative receiving algorithm based on GAMP is designed. The complexity is the same as that of the frequency domain LMMSE equalization algorithm, and its bit error rate performance is improved [10].

When the channel state information of the receiver is unknown, it is necessary to iterative between symbol estimation and channel estimation. For channel estimation, expectation maximization (EM) [11] decision directed least squares (LS), or adaptive strategies such as least mean square (LMS), Kalman filtering [12-13] can be used. Based on the combination of LS channel estimation and time domain DFE algorithm, the channel estimation error is taken into account in the filter design, but there is inversion of the filter matrix, and its complexity is too high.

The newly invented vector approximate message passing algorithm (VAMP) has the same complexity as GAMP, but it overcomes the situation that GAMP does not work when the mean value of the linear transfer matrix is not zero or the matrix is ill-conditioned. the designed receiving algorithm has significant advantages in channel estimation normalized mean square error (NMSE) and bit error ratio (BER) performance.

The paper is organized as follows. In second section, we introduce the SC-FDE system model. In third section, after a brief introduction to VAMP, we introduces the iterative algorithm design based on VAMP and Turbo equalization framework. In fourth section, we introduce the simulation results and analysis. In fifth section, we conclude.

2 SC-FDE system model

2.1 Single carrier block transmission system

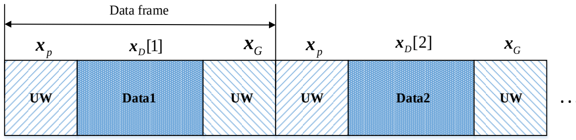


Fig. 1. Transmission data frame structure diagram.

In this section, we will briefly introduce the SC-FDE System Model. Consider a single carrier block transmission system, assuming that there are K blocks to transmit data, as shown in Figure 1, where the block transmission method based on multi-block UW is used in this paper. The pilot sequence $\boldsymbol{x}_p$ and protection interval $\boldsymbol{x}_G$ are inserted before and after the data block, and the data transmitted by the K block is $\boldsymbol{x}[k]=[x_0, x_1,..., x_{M-1}]^T = [\boldsymbol{x}_p^T, \boldsymbol{x}_D[k]^T, \boldsymbol{x}_G^T]^T$. $\boldsymbol{x}_p \in \mathbb{C}^{N_p}$ corresponding pilot sequence, $\boldsymbol{x}_D \in \boldsymbol{S}^{N_D}$ corresponding data symbol, $\boldsymbol{x}_G \in \mathbb{C}^{N_G}$ corresponding protection interval. Use $\boldsymbol{S}$ to represent the 2^A-ary symbol alphabet, e.g., QAM. As shown in Figure 2, the sender data symbol is constructed as follows. Firstly, a column of information bits $\boldsymbol{b} \in \{0,1\}^{N_b}$ with length N_b is coded and interleaved into coded bits $\boldsymbol{c} \in \{0,1\}^{N_b/R}$, where R represents the coding rate. Then, the encoding bit is mapped to the data transmission symbol $\{x_i, i \in N_D\} \in \boldsymbol{S}^{N_D}$. Further, the transmission symbol vector $\boldsymbol{x}$ can be obtained by inserting pilot symbols and protection intervals. The Chu sequence [14] is used as the pilot symbol, and the length of the pilot symbol L is the same in each data block, where L is the number of channel taps. The protection interval is the same in each data block and is equal to the post $L-1$ symbol of the pilot. Then, the symbol sequence is filtered by the transmitter pulse function to obtain the baseband transmission waveform, which is modulated by the carrier to high frequency, transmitted in the space wireless environment and passed through channel $\boldsymbol{h} = (h_1, h_2, ..., h_L)^T \in \mathbb{C}^L$. At the receiving end, the frequency domain receiving model can be obtained by discrete Fourier transform of the received signal.

$$\boldsymbol{y} = Diag(\boldsymbol{F}_M^{1:L}\boldsymbol{h})\boldsymbol{F}_M\boldsymbol{x} + \boldsymbol{w}_g, \quad (1)$$

Here, $\boldsymbol{F}_M$ represents the $M \times M$ normalized discrete Fourier transform (DFT) matrix, $\boldsymbol{F}_M^{1;L}$ denotes the front L column of the $M \times M$ normalized DFT matrix. $\boldsymbol{w}_g$ denotes additive white Gaussian noise with mean zero and variance γ_w^{-1}.

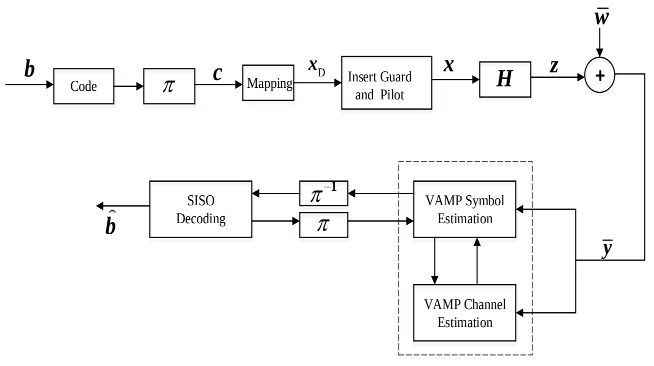


Fig. 2. The block diagram of SC block transmission and turbo reception, where π and π^{-1} represent interleaving and deinterleaving, respectively, and $\boldsymbol{H}$ represents channel propagation. Virtual frame shows reception using a SISO equalizer that iterates between symbol and channel estimation, which uses VAMP for joint symbol and channel estimation.

3 Iterative algorithm design based on VAMP and TURBO equalization framework

This chapter will introduce how to use VAMP and joint Turbo equalization framework to design joint symbol detection, channel estimation and decoding algorithm in SC-FDE system. The equalization module adopts the double VAMP iteration of channel and symbol. Before the equalization iteration, the received pilot sequence is used to estimate the channel by LMMSE algorithm, and the initial channel estimation value is obtained. Then, initialize the parameter values in the factor graph model. In Turbo iteration, the corresponding parameters are constructed according to the initial channel estimation, and the VAMP algorithm is used for symbol estimation. Then the VAMP algorithm is used for channel estimation, and the iterative relationship between symbol estimation and channel estimation is formed, which further makes the estimation algorithm more accurate. After running the

VAMP estimation algorithm until convergence, the posterior mean and variance of symbol estimation can be obtained, and the external information is output to the SISO decoder. After the decoder receives the external information from the equalizer, it outputs the corresponding external information and transmits it to the equalizer. The external information is transmitted between the equalizer and the decoder, and until the turbo algorithm converges.

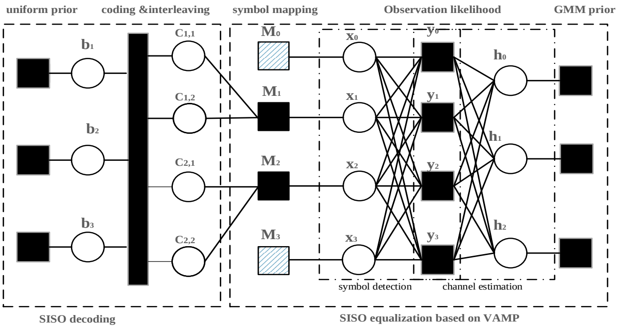


Fig. 3. Factor Diagram of SC-FDE System. There are two VAMP full connection subgraphs in the SISO equalizer, virtual box corresponding to symbol detection and channel estimation respectively.

3.1 System factor diagram

The purpose of communication is to detect the information bit $\boldsymbol{b}$ from the frequency domain observation $\boldsymbol{y}$. Due to the conditional independence and separability between variables $\boldsymbol{y}$, $\boldsymbol{h}$, $\boldsymbol{x}$, $\boldsymbol{c}$ and $\boldsymbol{b}$, the Bayesian criterion can be used to decompose the marginal posterior probability distribution $\{p(\boldsymbol{b}_i \mid \boldsymbol{y})\}_{i=1}^{N_b}$ through the following marginal factor.

$$p(b_i \mid \boldsymbol{y}) = \sum_{\boldsymbol{b}_{-i}} p(\boldsymbol{b} \mid \boldsymbol{y}) = \sum_{\boldsymbol{b}_{-i}} \frac{p(\boldsymbol{y} \mid \boldsymbol{b})p(\boldsymbol{b})}{p(\boldsymbol{y})} \propto \sum_{\boldsymbol{b}_{-i}} p(\boldsymbol{y} \mid \boldsymbol{b}) \tag{2}$$

$$= \sum_{\boldsymbol{b}_{-i}, \boldsymbol{x}, \boldsymbol{c}} \int_{\boldsymbol{h}} p(\boldsymbol{y} \mid \boldsymbol{h}, \boldsymbol{x}) p(\boldsymbol{h}) p(\boldsymbol{x} \mid \boldsymbol{c}) p(\boldsymbol{c} \mid \boldsymbol{b}) \tag{3}$$

$$= \sum_{\boldsymbol{b}_{-i}, \boldsymbol{c}} p(\boldsymbol{c} \mid \boldsymbol{b}) \sum_{\boldsymbol{x}} \int_{\boldsymbol{h}} p(\boldsymbol{y} \mid \boldsymbol{h}, x) \times \prod_{l=0}^{L-1} p(h_l) \times \prod_{n=0}^{N_D-1} p(x_{N_p+n} \mid \boldsymbol{c}_n), \tag{4}$$

Among them, define $\boldsymbol{b}_{-i} = [b_1, ..., b_{i-1}, b_{i+1}, ..., b_{N_b}]^T$, $p(\boldsymbol{c} \mid \boldsymbol{b})$ denotes coding interleaving, $p(\boldsymbol{x} \mid \boldsymbol{c})$ denotes mapping, function $p(\boldsymbol{y} \mid \boldsymbol{h}, \boldsymbol{x})$ denotes Gaussian likelihood observation, In particular

$$p(\boldsymbol{y} \mid \boldsymbol{h}, \boldsymbol{x}) = CN(\boldsymbol{y}; Diag(\boldsymbol{F}_M^{1:L} \boldsymbol{h}) \boldsymbol{F}_M \boldsymbol{x}, \lambda^{-1} \boldsymbol{I}), \tag{5}$$

$p(\boldsymbol{h})$ denotes channel prior information. The factorization represented by Formula (4) can be represented by the factor graph shown in Fig.3. In the figure, it is assumed that the number of information bits is $N_b = 3$, the number of encoding bits is 4, each symbol contains $A = 2$ bits, the number of data symbols $N_D = 2$, x_0 is the pilot, x_3 is the protection interval, the block length $M=4$, and the number of taps $L=3$. The black square in the factor graph represents the factor node, and the hollow circle represents the variable node. $\boldsymbol{M}_i$ represents modulation mapping relationship. In order to describe conveniently, the factor graph is split into two subgraphs by virtual line from left to right, corresponding to SISO decoding and SISO equalization respectively.

3.2 Introduction of VAMP algorithm

Consider recovering vector $\boldsymbol{x}\in\mathbb{R}^N$ from linear measurements with AWGN noise

$$\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{w}_g\in\mathbb{R}^M. \tag{6}$$

For the standard linear model, VAMP algorithm [15] can recover $\boldsymbol{x}$ with higher precision and lower computational complexity when $\boldsymbol{A}$ is known. Algorithm 1 summarizes the standard linear model VAMP algorithm, which is divided into the "denoising step" and the "LMMSE estimation step".

Algorithm 1 VAMP Algorithm

1:Select initial $\boldsymbol{r}_{10}$ and $\gamma_{10}\geq 0$.
2:**for** k=0,1,...K **do**
3://Denoising
4: $\hat{\boldsymbol{x}}_{1k}=\boldsymbol{g}_1(\boldsymbol{r}_{1k},\gamma_{1k})$
5: $\alpha_{1k}=\langle\boldsymbol{g}_1'(\boldsymbol{r}_{1k},\gamma_{1k})\rangle$
6: $\boldsymbol{r}_{2k}=(\hat{\boldsymbol{x}}_{1k}-\alpha_{1k}\boldsymbol{r}_{1k})/(1-\alpha_{1k})$
7: $\gamma_{2k}=\gamma_{1k}(1-\alpha_{1k})/\alpha_{1k}$
8://LMMSE estimation
9: $\hat{\boldsymbol{x}}_{2k}=\boldsymbol{g}_2(\boldsymbol{r}_{2k},\gamma_{2k})$
10: $\alpha_{2k}=\langle\boldsymbol{g}_2'(\boldsymbol{r}_{2k},\gamma_{2k})\rangle$
11: $\boldsymbol{r}_{1,k+1}=(\hat{\boldsymbol{x}}_{2k}-\alpha_{2k}\boldsymbol{r}_{2k})/(1-\alpha_{2k})$
12: $\gamma_{1,k+1}=\gamma_{2k}(1-\alpha_{2k})/\alpha_{2k}$
13: **end for**
14: Return $\hat{\boldsymbol{x}}_{1K}$.

In algorithm 1, function $\boldsymbol{g}_1(\boldsymbol{r}_{1k},\gamma_{1k})$ can be used to denoise the real signal $\boldsymbol{r}_{1k}=\boldsymbol{x}+N(0,\boldsymbol{I}/\gamma_{1k})$ containing noise by using the prior of $\boldsymbol{x}$. In the ninth line of algorithm 1, function $\boldsymbol{g}_2(\boldsymbol{r}_{2k},\gamma_{2k})$ represents the standard linear model (6) using LMMSE algorithm to estimate $\boldsymbol{x}$ under the pseudo prior $\boldsymbol{x}\sim N(\boldsymbol{r}_{2k},\boldsymbol{I}/\gamma_{2k})$, i.e.,

$$\boldsymbol{g}_2(\boldsymbol{r}_{2k},\gamma_{2k}):=(\gamma_w\boldsymbol{A}^T\boldsymbol{A}+\gamma_{2k}\boldsymbol{I})^{-1}(\gamma_w\boldsymbol{A}^T\boldsymbol{y}+\gamma_{2k}\boldsymbol{r}_{2k}) \tag{7}$$

$$\langle\boldsymbol{g}_2(\boldsymbol{r}_{2k},\gamma_{2k})\rangle=\gamma_{2k}N^{-1}tr[(\gamma_w\boldsymbol{A}^T\boldsymbol{A}+\gamma_{2k}\boldsymbol{I})^{-1}]. \tag{8}$$

3.3 Symbol detection based on VAMP

This section will show the design process of symbol detection algorithm based on VAMP. In order to apply VAMP symbol detection algorithm in SC-FDE system, we start our derivation from Equation (1). At the i-th iteration, we assume that the channel estimate $\hat{\boldsymbol{h}}$ is known, which can be obtained from the previous iteration. Replacing $\hat{\boldsymbol{h}}$ in (1) with $\hat{\boldsymbol{h}}$ and defining $\Phi = diag(\boldsymbol{F}_M^{1:L}\hat{\boldsymbol{h}})\boldsymbol{F}_M$, we reach our system model for symbol detection as:

$$\boldsymbol{y} = \Phi\boldsymbol{x} + \boldsymbol{w}_g . \tag{9}$$

We first perform LMMSE channel estimation to obtain initial channel estimate. Then we apply VAMP in (9). Generally, back and forth iteration between "denoising step" and "LMMSE estimation step" of VAMP algorithm. the complexity of the algorithm is mainly determined by the "LMMSE estimation step".

LMMSE estimation steps : In SC-FDE system, since the column of $\boldsymbol{F}$ matrix in matrix $\boldsymbol{A} = \Phi$ has a standard orthogonal property, i.e. $\boldsymbol{F}^H\boldsymbol{F} = \boldsymbol{I}$, and SVD $\boldsymbol{A} = \boldsymbol{USV}^T$ brought into equation (7) and equation (8), after which

$$\hat{\boldsymbol{x}}_{2k} = \boldsymbol{VD}_k(\tilde{\boldsymbol{y}} + \gamma_{2k}\boldsymbol{V}^T\boldsymbol{r}_{2k}) \tag{10}$$

$$\alpha_{2k} = 1/N\sum_{n=1}^{N}\gamma_{2k}/(\gamma_w\boldsymbol{s}_n^2 + \gamma_{2k}), \tag{11}$$

where $\tilde{\boldsymbol{y}} = \gamma_w\boldsymbol{S}^T\boldsymbol{U}^T\boldsymbol{y}$, $\boldsymbol{D}_k$ is a diagonal matrix with $[D_k]_{nn} = (\gamma_w s_n^2 + \gamma_{2k})^{-1}$.

Algorithm 2 VAMP Symbol Detection Algorithm in SC-FDE System

1:Select initial $\boldsymbol{r}_{10}$ and $\gamma_{10} \geq 0$.
2:**for** k=0,1,...K **do**
3://Denoising
4: $\hat{\boldsymbol{x}}_{1k} = E\{\boldsymbol{x}_{1k} \mid \boldsymbol{r}_{1k}, \gamma_{1k}\}$
5: $\alpha_{1k} = \gamma_{1k}\sum_{n=1}^{N}Var\{x_{1k,n} \mid r_{1k,n}, \gamma_{1k}\}/N$
6: $\boldsymbol{r}_{2k} = (\hat{\boldsymbol{x}}_{1k} - \alpha_{1k}\boldsymbol{r}_{1k})/(1-\alpha_{1k})$
7: $\gamma_{2k} = \gamma_{1k}(1-\alpha_{1k})/\alpha_{1k}$
8://LMMSE estimation
9: $\hat{\boldsymbol{x}}_{2k} = \boldsymbol{V}D_k(\tilde{\boldsymbol{y}} + \gamma_{2k}\boldsymbol{V}^T\boldsymbol{r}_{2k})$
10: $\alpha_{2k} = 1/N\sum_{n=1}^{N}\gamma_{2k}/(\gamma_w\boldsymbol{s}_n^2 + \gamma_{2k})$
11: $\boldsymbol{r}_{1,k+1} = (\hat{\boldsymbol{x}}_{2k} - \alpha_{2k}\boldsymbol{r}_{2k})/(1-\alpha_{2k})$
12: $\gamma_{1,k+1} = \gamma_{2k}(1-\alpha_{2k})/\alpha_{2k}$
13: **end for**
14: Return $\hat{\boldsymbol{x}}_{1K}$.

Algorithm 2 summarizes the symbol detection algorithm based on VAMP in SC-FDE system. the complexity of VAMP is dominated by two matrix-vector multiplies per iteration in nine rows.

3.4 Channel estimation based on VAMP

This section will show the design process of channel estimation algorithm based on VAMP. In order to apply VAMP channel estimation algorithm in SC-FDE transmission system, we start our derivation from Equation (1). At the i-th iteration, we assume that the symbol estimate $\hat{\boldsymbol{x}}$ is known, Redefining transfer matrix $\Phi_1 = diag(\boldsymbol{F}_M\hat{\boldsymbol{x}}_{1K})\boldsymbol{F}_M^{1:L}$, where $\boldsymbol{F}_M^{1:L}$

represents the front L column of the matrix. we reach our system model for channel detection as

$$\boldsymbol{y} = \Phi_1 \boldsymbol{h} + \boldsymbol{w}_g , \tag{12}$$

Here, $\boldsymbol{w}_g$ denotes AWGN with variance γ_w^{-1}. The VAMP algorithm in algorithm 1 can be directly applied to the standard linear model of formula (12), and the complexity of the algorithm is mainly determined by the "LMMSE estimation step".

LMMSE estimation steps : In SC-FDE system, because the matrix $\boldsymbol{A} = \Phi_1$ dimension is $M \times L$, not a square matrix, it cannot be decomposed directly by singular value. Since the column of the $\boldsymbol{F}$ matrix in matrix $\boldsymbol{A} = \Phi_1$ has a standard orthogonal property, i.e. $\boldsymbol{F}^H \boldsymbol{F} = \boldsymbol{I}$,brought into equation (7) and equation (8), the LMMSE estimation steps can be obtained as follows :

$$\hat{\boldsymbol{h}}_{2k} = \boldsymbol{F}^H (\gamma_w diag(|\boldsymbol{F}\hat{\boldsymbol{x}}_{1K}|^2) + \gamma_{2k}\boldsymbol{I})^{-1} (\gamma_w diag(\boldsymbol{F}\hat{\boldsymbol{x}}_{1K})^H \boldsymbol{y} + \gamma_{2k}\boldsymbol{F}\boldsymbol{r}_{2k}) \tag{13}$$

$$\alpha_{2k} = \gamma_{2k} N^{-1} tr[\boldsymbol{F}^H (diag(|\boldsymbol{F}\hat{\boldsymbol{x}}_{1K}|^2) + \gamma_{2k}\boldsymbol{I})^{-1} \boldsymbol{F}]. \tag{14}$$

Algorithm 3 summarizes the VAMP-based channel estimation algorithm in SC-FDE system. It can be seen from the table that the complexity of the algorithm mainly depends on the matrix vector multiplication of nine rows, so the complexity of the entire VAMP algorithm is $o(N \log(N))$.

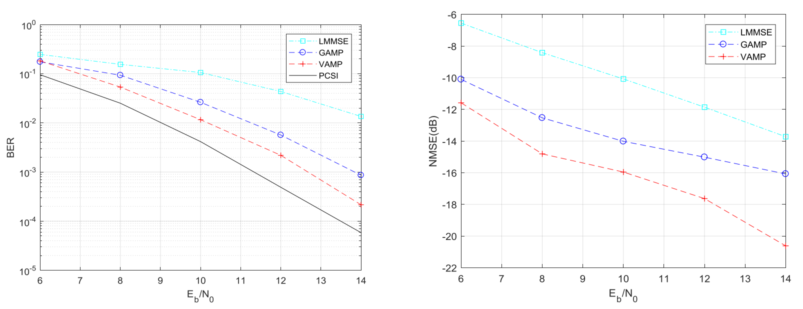
Algorithm 3 VAMP Channel Estimation Algorithm in SC-FDE System

1:Select initial $\boldsymbol{r}_{10}$ and $\gamma_{10} \geq 0$.
2:**for** k=0,1,...K **do**
3://Denoising
4: $\hat{\boldsymbol{h}}_{1k} = E\{\boldsymbol{h}_{1k} \mid \boldsymbol{r}_{1k}, \gamma_{1k}\}$
5: $\alpha_{1k} = \gamma_{1k} \sum_{n=1}^{L} Var\{h_{1k,n} \mid r_{1k,n}, \gamma_{1k}\} / L$
6: $\boldsymbol{r}_{2k} = (\hat{\boldsymbol{h}}_{1k} - \alpha_{1k}\boldsymbol{r}_{1k}) / (1-\alpha_{1k})$
7: $\gamma_{2k} = \gamma_{1k}(1-\alpha_{1k}) / \alpha_{1k}$
8://LMMSE estimation
9: $\hat{\boldsymbol{h}}_{2k} = \boldsymbol{F}^H (\gamma_w diag(|\boldsymbol{F}\hat{\boldsymbol{x}}_{1K}|^2) + \gamma_{2k}\boldsymbol{I})^{-1} (\gamma_w diag(\boldsymbol{F}\hat{\boldsymbol{x}}_{1K})^H \boldsymbol{y} + \gamma_{2k}\boldsymbol{F}\boldsymbol{r}_{2k})$
10: $\alpha_{2k} = \gamma_{2k} N^{-1} tr[\boldsymbol{F}^H (diag(|\boldsymbol{F}\hat{\boldsymbol{x}}_{1K}|^2) + \gamma_{2k}\boldsymbol{I})^{-1} \boldsymbol{F}]$
11: $\boldsymbol{r}_{1,k+1} = (\hat{\boldsymbol{h}}_{2k} - \alpha_{2k}\boldsymbol{r}_{2k}) / (1-\alpha_{2k})$
12: $\gamma_{1,k+1} = \gamma_{2k}(1-\alpha_{2k}) / \alpha_{2k}$
13: **end for**
14: Return $\hat{\boldsymbol{h}}_{1K}$.

4 Simulation results and analysis

The performance of the proposed algorithm is verified by experimental simulation. Firstly, the experimental parameter setting is introduced. Information bit length of SC-FDE system $N_b = 512$. The encoding method is irregular LDPC codes with code rate $R = 1/3$ and average column weight 3. The modulation mode is 16-QAM (Gray map). Pilot symbol length is $N_P = 64$. The protection interval length is $N_G = 63$. Length of each piece of data $M = 512$. Pilot sequence $\boldsymbol{x}_P$ using Chu sequence. Simulation using Saleh-Valenzuela channel [16]. Turbo iterations are 20. SISO LDPC decoder for decoding. Assuming that the channel tap follows an independent, non-identical zero-mean 2-GMM distribution. SNR is defined as $\text{SNR} \triangleq 1/\sigma_w^2$. Two comparison algorithms are selected and represented by "LMMSE" and "GAMP", respectively. Then, the VAMP receiving algorithm proposed in this paper is compared with the above algorithm through simulation. As a reference, the performance of VAMP receiver algorithm with known CSI is given, which is represented by "PCSI". It should be noted that when CSI is fully known, VAMP will only estimate symbols.

Figure 4 (a) shows the curve of bit error rate (BER) with E_b/N_0. We see that VAMP significantly outperform GAMP and LMMSE with about 4.2 dB and 1.3 dB gains, respectively. We also see that compared to PCSI bound, VAMP has about 1 dB performance loss.



(a) (b)

Fig. 4. (**a**) The change curve of BER with E_b/N_0 for different receiving algorithms under UW transmission. (**b**) The change curve of NMSE with E_b/N_0 for different receiving algorithms under UW transmission.

Figure 4 (b) shows the the channel estimation normalized mean square error (NMSE) versus E_b/N_0. where we can see VAMP can achieve about 4dB and 6.5dB gains compared to that of LMMSE and VAMP, respectively. The above simulations can demonstrated the superiority of the proposed algorithm.

5 Conclusions

Based on the newly proposed VAMP algorithm and the turbo framework, we propose a joint channel estimation and symbol detection. We adopt VAMP to estimate channels and detect symbols respectively in a turbo-like behavior. Compared to the LMMSE algorithms and the-state-of-art GAMP-based algorithms, the proposed algorithms shows significant performances gain in both BER and channel NMSE, while they share the same order of complexity.

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