Delta system transfer function model for IEMI time domain test

Rupo Ma[1,2,*], *Qun* Wang[1], *Zhenli* Wang[1], and *Yanyan* Pan[1]

[1]Department of Computer Information and Cyber Security, Jiangsu Police Institute Nanjing, China
[2]College of Command and Control Engineering, Army Engineering University, Nanjing, China

Abstract. Intentional electromagnetic interference (IEMI) is a serious threat to the stable operation of computer and other intelligent terminal equipment. For testing research on IEMI, it is very important to build a stable and reliable test system model for accurate analysis of test data. In the time-domain test of IEMI, the z-domain discrete system transfer function is usually used to describe the response characteristics of the test system. However, with the change of sampling frequency, the parameters of the *z*-domain model should change accordingly, which leads to inconvenience in modeling the test system. In this paper, a *δ*-domain transfer function model of the test system is constructed by the transformation from *z*-domain. And the *δ*-domain model has better stability at high sampling rate and can be approximately equivalent to the system transfer function in the continuous-time domain.

1 Introduction

As a kind of transient electromagnetic phenomenon, electromagnetic pulse has a steep waveform front in time domain and a wide frequency band in frequency domain. It can be coupled into computer and other intelligent terminal equipment through various ways, which seriously threatens their normal operation [1,2]. With the development of high-power electromagnetic pulse technology, terrorists or criminals are more and more likely to use intentional electromagnetic interference (IEMI) to attack computer and intelligent terminal equipment for illegal and criminal activities [3]. To improve the anti IEMI performance of computer and other intelligent terminal equipment, it is necessary to carry out IEMI test experiment. Among them, it is very important to build a stable and reliable test system model for accurate analysis of test data. Due to the influence of parasitic circuit parameters, impedance matching and bandwidth, IEMI test results are often distorted. The time-domain dynamic calibration model based on system identification can better solve this problem [4].

Since the time-domain test results of IEMI are generally discretized data, the z-domain discretized transfer function model *H(z)* is usually adopted to represent the system transfer function. This model is simple in structure, convenient for calculation, and can be used to analyze the response characteristics in time-domain and frequency-domain. However, the *z*-domain transfer function model is closely related to the sampling interval of the measured

* Corresponding author: mrpjet@163.com

waveform. With different sampling intervals, the parameters of the *z*-domain model will change. When sampling at a high speed, the *z*-transform may appear that the poles are located on the stable boundary, and its quantization error may lead to the deterioration of the system stability [5]. To solve this problem, a δ-domain transfer function model is proposed in this paper. By applying δ transformation from the *z*-domain model *H(z)*, a stable system transfer function model with approximate continuous-time domain (*s*-domain) is constructed.

2 δ Operator and transformation

The δ transform was first proposed as Euler approximation and was used in digital filtering [6], but it was not noticed at that time. The δ transformation was carried out as a pioneering research until 1986 by Goodwin et al. and began to attract wide attention [7]. The δ transform is a new discretization method, which can solve the instability of the system model caused by the small sampling interval of the *z* model [8,9]. In the case of fast sampling, the discrete model of the δ operator approaches the continuous-time domain model. Therefore, the δ operator is used as a unified description method for continuous-time domain model and discrete-time domain model [10]. δ operator is defined as [11]

$$\delta = (q - 1)/T \tag{1}$$

where, q is the forward shift operator, and T is the sampling interval. In the δ-domain, the corresponding variable of the δ operator is γ. When mapping from *z*-domain to δ-domain, the above formula can be written as

$$\gamma = (z - 1)/T \tag{2}$$

For the *z*-domain system model

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m}} \tag{3}$$

substitute $z = T\gamma + 1$ into the above equation,

$$H(\gamma) = \frac{b_0 + b_1 (T\gamma + 1)^{-1} + b_2 (T\gamma + 1)^{-2} + \cdots + b_n (T\gamma + 1)^{-n}}{a_0 + a_1 (T\gamma + 1)^{-1} + a_2 (T\gamma + 1)^{-2} + \cdots + a_m (T\gamma + 1)^{-m}} \tag{4}$$

the δ-domain system model can be obtained by expanding the above formula

$$H(\gamma) = \frac{b'_0 + b'_1 \gamma^{-1} + b'_2 \gamma^{-2} + \cdots + b'_n \gamma^{-n}}{a'_0 + a'_1 \gamma^{-1} + a'_2 \gamma^{-2} + \cdots + a'_m \gamma^{-m}} \tag{5}$$

The coefficients of b'_i and a'_i are [12]

$$a'_i = T^i \sum_{j=i}^{m} a_j C_j^i \tag{6}$$

$$b'_i = T^i \sum_{j=i}^{n} b_j C_j^i \tag{7}$$

where, $C_j^i = j!/[i!(j-i)!]$.

When the zero-pole method is used to represent the system model, the transformation of the zero-pole from the *z*-domain to the *δ*-domain is linear. Suppose the zero-poles of the *z*-domain are z_k and p_k, the zero-poles o_k and ρ_k of the *δ*-domain are

$$o_k = (z_k - 1)/T \tag{8}$$

$$\rho_k = (p_k - 1)/T \tag{9}$$

3 Interpretation of result

3.1 Inconsistency of *H(z)*

A group of time-domain waveforms measured by the magnetic field (B) sensor are shown in Fig. 1, and the system transfer function *H(z)* curves obtained by system identification tool is shown in Fig. 2. It is found by comparison that when the sampling interval is changed, the transfer function curves of the system are obviously different, which means the sampling frequency has significant influence to *z*-model. In Fig. 2, the system transfer function curves at sampling intervals of 1ns, 0.5ns and 0.2ns are significantly different, while the system transfer function curves at sampling intervals of 0.1ns tend to be consistent with that at 0.2ns. Therefore, the sampling interval of 0.2ns is a limit value of the transfer function of the system, and the corresponding sampling frequency is about 10 times the highest frequency of the input signal.

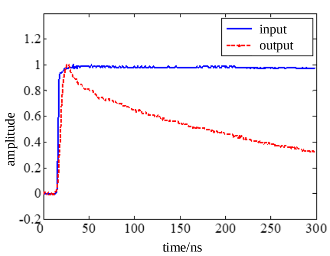

Fig. 1. Time-domain waveforms of B sensor.

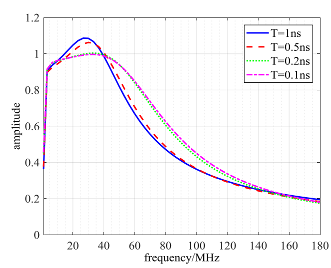

Fig. 2. *H*(*z*) curves.

3.2 Validity of the *δ* transform

Taking a given continuous-time domain system function model as an example, the validity of the transformation from *z*-domain to *δ*-domain is verified. Let the continuous-time domain system model be

$$H(s) = \frac{0.2s^2 + 0.3s + 1}{s^3 + 0.5s^2 + 0.4s + 1} \tag{10}$$

For the above formula *H(s)*, the transformation from s-domain to *z*-domain is realized through the matlab function c2dm (custom sampling interval *T*), and the transfer function *H(z)* in discrete-time domain is obtained. The zero-poles of *H(z)* are substituted into the formulas (8) and (9) to obtain the zero-poles of the *δ*-domain, and then the *δ*-domain model coefficients are obtained through the function zp2tf. Because the *δ*-domain model is approximate to the continuous-domain model at high sampling rate, the amplitude-frequency curve and step response curve of the transfer function can be obtained by functions freqs and step respectively.

When the sampling interval T of $H(s)$ is different, the numerator and denominator coefficients of the system model mapped to the δ-domain are shown in Table 1. The amplitude-frequency curves of the system transfer function are shown in Fig. 3, and the system step response curves are shown in Fig. 4.

Table 1. Numerator and denominator coefficients of δ-domain model.

T(s)	numerator coefficients			denominator coefficients			
	b_0'	b_1'	b_2'	a_0'	a_1'	a_2'	a_3'
0.1	0.2113	0.3903	0.9751	1	0.5316	0.5367	0.9751
0.01	0.2010	0.3092	0.9975	1	0.5028	0.4140	0.9975
0.001	0.2001	0.3009	0.9998	1	0.5003	0.4014	0.9998
0.0001	0.2	0.3001	1	1	0.5	0.4001	1

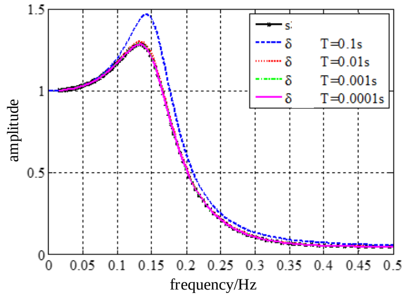

Fig. 3. Amplitude-frequency curves of δ-domain.

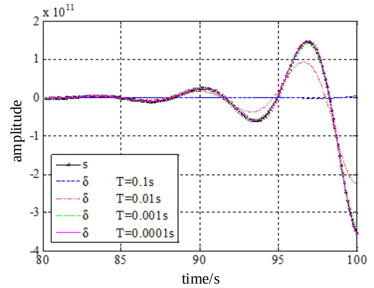

Fig. 4. Step response curves of δ-domain.

As can be seen from Table 1, with the decrease of the sampling interval, the differences between the coefficients of the δ-domain system transfer function obtained at different sampling intervals become smaller. In the table, when the sampling interval is 0.001s and 0.0001s, the numerator and denominator coefficients of the system transfer function is very close to the continuous-time domain(s-domain) function. It can also be seen from the Fig. 3 and Fig. 4 that the amplitude-frequency curves and the step response curves of the system are in good agreement with these of the continuous-time domain when the sampling interval is 0.001s and 0.0001s. It means that the frequency response and the step response remain consistent in small sampling intervals, and the δ-domain model can be approximate to the s-domain model.

Table 2. Numerator and denominator parameters of δ-domain system model of B sensor.

T(ns)	numerator coefficients			denominator coefficients			
	b_0' ($\times10^8$)	b_1' ($\times10^{17}$)	b_2' ($\times10^{23}$)	a_0'	a_1' ($\times10^8$)	a_2' ($\times10^{17}$)	a_3' ($\times10^{24}$)
1	1.9613	0.4731	0.3299	1	3.1551	0.5251	0.2419
0.2	1.5476	1.3206	2.4322	1	4.8443	1.4093	0.9502
0.1	1.5142	1.4667	2.8492	1	5.0894	1.5584	1.0809
0.05	1.5179	1.5330	3.0087	1	5.1992	1.6264	1.1351

3.3 δ-domain system model of the B sensor

As for the *z*-domain transfer function model of the B sensor shown in Fig. 2, δ transformation can be carried out to obtain the δ-domain system model. The B sensor δ-domain model with sampling intervals of 1ns, 0.2ns, 0.1ns and 0.05ns can be obtained by linear interpolation method, and the numerator and denominator parameters are shown in Table 2, the amplitude-frequency curve of the transfer function is shown in Fig. 5, and the step response is shown in Fig. 6.

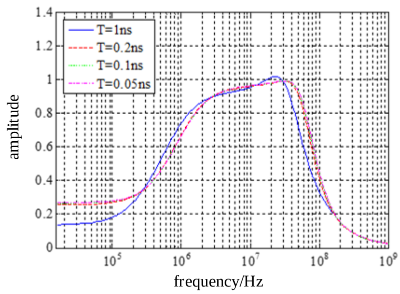

Fig. 5. Amplitude-frequency response of B sensor.

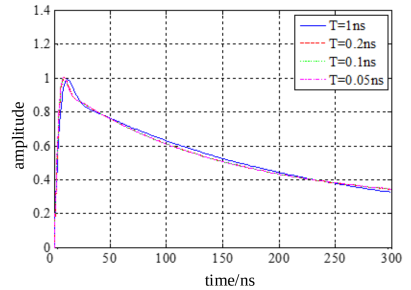

Fig. 6. Step response of B sensor.

It can be seen from the Table 2, when the sampling interval is 0.1 ns and 0.05 ns, the difference of the numerator and denominator coefficients of the system transfer function is not much. The amplitude-frequency response and the step response as also remain almost the same in Fig. 5 and Fig. 6. Therefore, the δ-domain model could approximate *s*-domain model when the sampling interval is less than 0.1 ns.

4 Conclusion

In time-domain test of IEMI for computer and other intelligent terminal equipment, discrete transfer function model in *z*-domain is usually adopted. This model can conveniently describe the response characteristics of the measurement system with limited parameters, but with the change of sampling interval, the parameters of the *z*-model changes accordingly. This paper proposes a δ transformation method to construct a δ-domain transfer function model. It is verified that the δ-domain model is more stable and approximate to the *s*-domain system transfer function at higher sampling frequencies, which provides a convenient tool for transforming *z*-domain models between different sampling frequencies.

This work is supported by the Research Project of High Level Talents of Jiangsu Police Institute (No.2911118010).

References

1. B. H. Zhou, B. Chen and L. H. Shi, *Electromagnetic Pulse and Engineering Protection*, Beijing, China: National Defense Industry Press (2003)
2. J. Lopes-Esteves and C. Kasmi, Remote and Silent Voice Command Injection on a Smartphone through Conducted IEMI - Threats of Smart IEMI for Information Security, System Design & Assessment Note SDAN 48 (2018)
3. IEC SC77C, High Power Electromagnetic (HPEM) Effects on Civilian Systems-IEC Document 61000-1-5 Working Draft (2002)

4. L. H. Shi, J. W. Tan and B. H. Zhou, Time domain calibration of pulsed current probe, *17th International Zurich Symposium on EMC*, Singapore, pp. 296-299 (2006)
5. H. J. Yang, Y. Q. Xia and H. G. Li, An Overview of Delta Operator Systems, Control Theory & Applications, vol. **32**, no. 3, pp. 569-578 (2015)
6. G. Orlandi, and G. Martinelli, Low-sensitivity recursive digital filters obtained via the delay replacement, IEEE Trans. on Circuits System, vol. **31**, no. 7, pp. 654-657 (1984)
7. G. C. Goodwin, K. S. Sin, Adaptive Filtering, Prediction and Control, Englewood Cliffs, NJ: Prentice-Hall (1984)
8. D. J. Zhang, Modeling and control of delta operator systems, Ph.D. dissertation, Nanjing University of Technology, China (2007)
9. K. L. Astrom, P. Hagander and J. Sternby, Zeros of sampled systems, Automatica, vol. **20**, no. 1, pp. 31-38 (1984)
10. R. H. Middleton and G. C. Goodwin, Improved finite word length characteristic in digital control using delta operators, IEEE Trans. on Automatica control, vol. **31**, no. 11, pp. 1015-1021 (1986)
11. R. H. Middleton and G. C. Goodwin, *Digital Control and Estimation: A Unified Approach*, Englewood Cliffs, NJ: Prentice-Hall (1990)
12. W. Y. Zhang and D. J. Zhang, Coefficient Vectors Between Transformation of δ and Z Domain Transfer Functions, Journal of Zhengzhou University, vol. **35**, no. 1, pp. 23-26 (2003)