A method for evaluating the accuracy of curved surface flattening

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Abstract. Based on the analysis of the existing methods for estimating the surface flattening error, they can be roughly classified into two types of estimation methods: the relative area error analysis method and the relative length error analysis method. The advantages and disadvantages of these methods are summarized in the paper. Combining the general surface flattening methods and sheet metal stamping forming process, a more reasonable, scientific and comprehensive evaluation system for the accuracy of surface flattening is proposed. In line with the unfolding characteristics of the grid surface, the relative average area error of triangles is used instead of the traditional area error estimation method. The accumulated length error is replaced by the relative length error of each triangle in the estimation method, and the value of the change in the orientation (direction, position) relationship between the triangles is increased to assess the deformation error of the surface flattening method. The mass attributes of the patch are introduced. The minimum displacement energy method is used to supplement the evaluation of surface flattening accuracy. Finally, a comprehensive evaluation system is established and proposed, which takes into account the accuracy of local and global surface flattening.

1 Introduction

The problem of surface expansion is often involved in the field of mechanical manufacturing, especially approximate expansion of non-deployable surfaces. The approximate expansion of non-deployable surfaces is an important technology in the aerospace, shipbuilding, and chemical equipment industries. Scholars at home and abroad have conducted extensive research for this purpose and proposed various methods and techniques for surface flattening [1-5], such as the finite element method, the geometric approximation method [6], the mechanical model method [8-9], the energy equation method [11-12], the 3D software method [7], the triangle mesh method [10,13], etc. Among them, the energy model method is utilised to triangulate a surface, using triangle points as particles and triangle edges as springs, to construct an energy equation, to calculate the binding force of the particles, and optimally unfold surface patches in the form of energy release.

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Although the analysis of the surface development error is mentioned in these surface flattening algorithms, the accuracy analysis of surface development has not been introduced as a research focus. Based on the research of the free-form surface flattening algorithm of multi-point forming plate parts, Zhang [14] theoretically analysed the reasons that caused the free-form surface expansion error. Mao [15] established a comprehensive error analysis mathematical model based on different emphases on various error requirements, and designed an error analysis system for free-form surface approximate flattening. A new Isogeometric Analysis (IGA) beam element was developed by integrating the displacement field of the element, which was approximated by the NURBS basis, with the internal work formula of Euler-Bernoulli beam theory with the small deformation and elastic assumptions. An effective alternative to standard Finite Element Analysis for shape error analysis of functional surface was proposed [16]. However, most of these studies only describe the accuracy of surface development as an evaluation of the effectiveness of the surface flattening algorithm [17-20]. For the evaluation of the development accuracy of non-developable surfaces, the effectiveness of a flattening method is evaluated by indicators such as the area change rate and the edge length change rate. However, the currently widely used evaluation methods of the area change rate and the edge length change rate have some limitations, and cannot comprehensively reflect the accuracy of surface development. Therefore, based on the geometric changes of the graphics involved in the process of surface flattening, an evaluation system for the accuracy of surface flattening is designed and established in this paper. Additionally, the errors of surface development accuracy from the shape and position changes of the graphic geometry are comprehensively measured.

2 The surface flattening deformation

In the process of surface flattening, subdivision and development are commonly used in the form of 3D mesh surfaces. The area of the triangle in the 3D mesh surface and the area of the flattened triangle will be different. Similarly, the length of the edge of the 3D mesh triangle will change before and after unfolding. These are the two most obvious deformation indicators, and hence this indicator is often used as an evaluation basis for measuring the accuracy of development in general surface flattening algorithms.

2.1 Primitive position change

Being the smallest graphic unit, points are often called primitives. Almost all complex graphics can be composed from or expressed by points. Two points can form a line, three points can form a plane, and countless free points can form a complex surface or body. Taking the common triangular surface in the calculation of surface expansion as an example, it involves many primitive deformations.

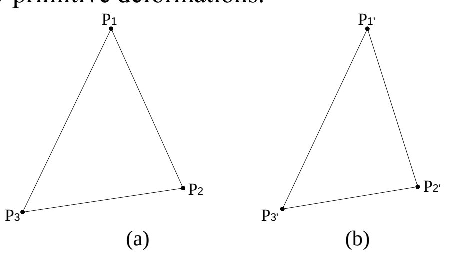


Fig. 1. Primitive deformation (dominant deformation) in triangles. (a) the triangle before deformation, (b) the triangle after deformation.

As shown in Fig.1, $\triangle p_1 p_2 p_3$ is connected by points p_1, p_2, p_3; when the position of the primitive p_1, p_2, p_3 changes, it causes the deformation of the triangle. Since $\overline{p_{1'} p_{3'}} < \overline{p_1 p_3}$, $\overline{p_{1'} p_{2'}} < \overline{p_1 p_2}$, $\overline{p_{2'} p_{3'}} < \overline{p_2 p_3}$, and $s_{\triangle p_{1'} p_{2'} p_{3'}} < s_{\triangle p_1 p_2 p_3}$, and the centroid positions of the two triangles change accordingly. This kind of deformation, caused by the change of the position of the primitive, is obvious, and its deformation can also be measured by the change of the edge length and the area.

2.2 Graphical deformation

Generally speaking, changing the position of the primitive will cause the deformation of the graphics. Still, sometimes the relative position between the primitives does not change, but it can cause another hidden deformation of the graphics. Since this kind of recessive deformation cannot be judged or measured by the commonly used indicators such as the change in edge length and area, it is called recessive deformation in this paper. As shown in Fig.2, there are two right-angled triangles $\triangle abc, \triangle def$, where $ab = ef, bc = ed, ac = df$. It can be seen that when $\triangle abc$ changes to $\triangle def$ after a certain transformation, the edge length and area of the two triangles do not change. Does this form of primitive change belong to the category of deformation? The obvious answer is yes. Clearly, $\triangle abc$ and $\triangle def$ are not the same graphic, the shapes of the two are very different, as well as their centroid positions. Therefore, the triangle has substantially deformed during this change.

The above is the analysis of the deformation of a single primitive (a point or a triangle). When multiple primitives form a graphic, its deformation meaning will be further enriched and extended. As shown in Fig.3, two triangles $\triangle s_1 s_2 s_3$ and $\triangle s_1 s_3 s_4$ intersecting in space have an angle of α, and they are flattened to a plane to obtain an expanded view of $\triangle p_1 p_2 p_3$ and $\triangle p_1 p_3 p_4$. Because the triangular sheet flattening process is an equal area expansion, the length and area of each edge of the triangular sheet do not change before and after the expansion.

However, the relative positional relationship of the two triangles has changed. The change in the positional relationship has a huge impact on the flattening result of the surface, and sometimes even exceeds the damage brought about by the deformation of a single element, which, in turn, causes the surface development to fail.

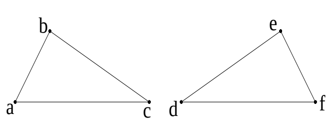


Fig. 2. Recessive deformation of triangles.

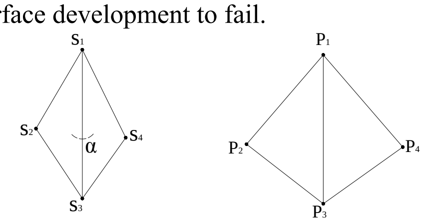


Fig. 3. Expanded deformation between triangles.

3 Establishment of evaluation method

In order to evaluate the accuracy of surface development more comprehensively and scientifically and to measure the effectiveness and practicability of a surface flattening algorithm, this chapter will introduce in detail four technical indicators that affect the accuracy of surface development. It will also use these indicators to establish the corresponding evaluation model of the surface development accuracy system, enrich the evaluation method of surface development accuracy, and provide some help to further optimize the improvement of a surface flattening algorithm.

3.1 Relative edge length error index (single object evaluation)

Since free-form surfaces are often theoretically non-developable, the flattened view will inevitably be deformed. For the evaluation of the development accuracy of non-developable surfaces, the relative edge length change rate is often used to evaluate the surface development accuracy. In the process of surface flattening, the triangular mesh model is frequently used for expansion. The length of the edge of triangles in a three-dimensional mesh surface undergoes a change before and after the expansion, which results in some edges being stretched and some others being squeezed to get shortened. This deformation is caused by the change in the position of the primitives, which is a local deformation of a single object. This edge length variable can be calculated by equation (1).

$$e_l = \sum_{i=1}^{n} \left| l_i - l_i^p \right| \Big/ \sum_{i=1}^{n} l_i \tag{1}$$

where e_l is called the average value of the relative edge length error of the surface; l_i is the actual edge length of the 3D mesh triangle; l_i^p is the flattened edge length of the triangle, and n is the number of triangle grid edges.

3.2 Relative area error index (single object evaluation)

During the development of the surface, due to the deformation force, the triangles will deform correspondingly, and cause their area to change. This is also the evaluation point that many surface flattening algorithms focus on. The effectiveness of the algorithm is directly evaluated by the area change of the entire surface before and after the expansion, or the deformation change of the entire surface is calculated by calculating the area change of each triangle. Therefore, the relative area error index is also an indispensable index in the evaluation system of the accuracy of surface development. The deformation can be calculated by equation (2):

$$e_s = \sum_{i=1}^{n} \left| s_i - s_i^p \right| \Big/ \sum_{i=1}^{n} s_i \tag{2}$$

where e_s is the average value of the relative area error of the surface; s_i is the actual area of the 3D mesh triangle; s_i^p is the area of the flattened triangle; and n is the number of triangles.

3.3 Relative displacement error index

Flattening the space surface to a plane is the process of flattening the subdivided triangular meshes to a plane one by one. The deformation of a single triangle has been included in the evaluation system of the accuracy of the surface expansion in the above process. However, due to the local deformation of a single triangle, the interaction between the triangles will occur, and this local deformation is caused by the adjacent relationship between the triangles. Propagation affects the deformation of adjacent triangles and even the entire mesh surface. As shown in Fig.4, according to a certain surface flattening method, the space curved piece is flattened. It can be seen that the relative positional relationship between the spatial triangle, and the flattened triangular piece, has changed. The relative displacement between such triangles has also been changed. Such changes are common in the process of surface flattening. They are one of the causes of surface development errors. In order to introduce such a relative displacement error between triangles into the evaluation system of surface development accuracy, as well as to globally evaluate the accuracy of surface development, it is necessary to find a constant reference point for the quantitative calculation of relative displacement. During the surface development process, a single triangle will change, and the position of the primitive will also move with it. However, a reference point or a starting point for stepwise flattening will be generally chosen for surface development, so this reference point is the most suitable reference point. As shown in Fig. 4, the relative displacement offset of the centroid of the triangular plate before and after the development is calculated using the development reference point as a base point. The specific computation method can be counted by using formula (3):

$$e_d = \sum_{i=1}^{n} \left| d_i - d_i^p \right| \Big/ \sum_{i=1}^{n} d_i \qquad (3)$$

where e_d is the average relative displacement error of the surface; d_i is the distance between the centroid of a triangle in a three-dimensional grid and the unfolding reference point; d_i^p is the distance between the centroid of the plane triangle corresponding to the flattened space triangle and the unfolded reference point; and n is the number of triangles.

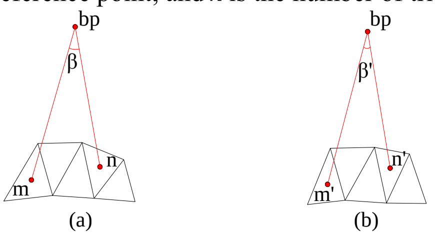


Fig. 4. Relative orientation deformation of triangle: (a) the graph before flattening, the directional angle between triangle m and n is β , (b) the graph after flattening, the directional angle between triangle m' and n' is β' .

3.4 Relative direction error index

Since the relative area error (e_s) and the relative edge length error (e_l) are only used to evaluate the development accuracy of a single triangle, they are, in fact, two local accuracy evaluation indicators, and cannot fully measure the development accuracy of a flattening method. Therefore, this article introduces a more accurate evaluation index, the relative angle change rate. Before and after the surface has been unfolded, the relative orientation between the triangles changes accordingly, and that is why e_s and e_l cannot reflect this change. Still, the change in the orientation between triangles has a huge impact on the overall development result. The flattening graphic is determined by the positional relationship between the triangles.

In order to reflect the change of this relative positional relationship, the development surface is taken as a whole, and the reference point of the development is used as a reference. The change in the relative angle between the centroid of the triangle and the development reference point (β as shown in Fig.4.) is evaluated, unfolding accuracy. The relative direction error (e_a) can be calculated by formula (4), so that the local and global comprehensive evaluation of the unfolding graph can be realized:

$$e_a = \sum_{i=1}^{n} \left| ang_i - ang_i^p \right| \Big/ \sum_{i=1}^{n} ang_i \tag{4}$$

where e_a is the average relative direction error of the surface; ang_i is the connection angle between the centroid of the three-dimensional grid triangle and the unfolding reference point; ang_i^p is the connection angle between the centroid of the flattened triangle and the unfolded reference point; and n is the number of triangles.

In order to further evaluate the accuracy of surface development, the quality attributes of patches are introduced, and the accuracy of surface development is supplemented by the minimum displacement energy of triangular patches. During the flattening process of the surface, all patches are finally located on the same plane, so the change of the displacement energy in the *z* direction can be ignored, and only the displacement energy in the *x* and *y* directions should be considered. The specific calculation method is shown in formula (5):

$$e_g = \sum_{i=1}^{n} \left| g(x,y)_i - g(x,y)_i^p \right| \cdot v_x \cdot v_y \Big/ \sum_{i=1}^{n} g(x,y)_i \tag{5}$$

where e_g is the average relative displacement energy of the surface; $g(x,y)_i$ is the displacement energy at the centroid of a triangle in a three-dimensional grid (x , y direction); $g(x,y)_i^p$ is the flattening displacement energy at the centre of the triangle; v_x, v_y are unit vectors in the *x* and *y* direction, *n* is the number of triangles.

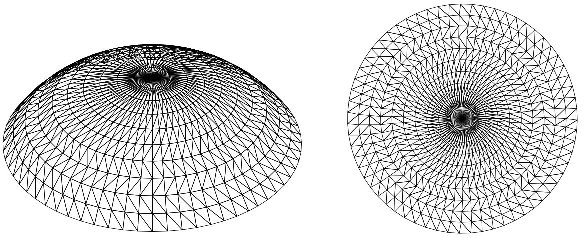
4 Quantitative design of evaluation index

In order to further quantify the evaluation index of surface development accuracy, a scientific, reasonable and comprehensive evaluation model of surface development accuracy has been constructed, and the surface development accuracy is quantified by expression (6):

$$e_{flattening} = k_1 \cdot e_l + k_2 \cdot e_s + k_3 \cdot e_d + k_4 \cdot e_a + k_5 \cdot e_g \text{ , } \quad k_1 + k_2 + k_3 + k_4 + k_5 = 1 \tag{6}$$

where, $e_{flattening}$ is the comprehensive evaluation value of surface development accuracy; k_1 is the relative edge length error weight value of the surface; k_2 is the surface relative area error weight value; k_3 is the surface relative displacement error weight value; k_4 is the surface relative direction error weight value; and k_5 is the surface relative displacement energy weight value.

According to formula (6), the error of surface development can be calculated and the effect of surface development can be evaluated, but different values of the weight coefficients k_1 , k_2 , k_3 , k_4 , k_5 will directly affect the comprehensive evaluation value of surface development accuracy, and thus affect reasonable judgment. Therefore, further selection of the reasonable weight coefficient value through surface expansion experiments is needed. This makes the evaluation system more instructive and improves its practical value.



(a) (b)

Fig. 5. Flattening of a partial sphere: (a) a mesh of partial-sphere, and (b) the flattening result of (a).

As shown in Fig.5, a partial spherical surface has been used for the flattening test, and, according to different weight coefficients, the unfolding accuracy values of the same model surface have been calculated. After plenty of tests and comparisons (some test results are listed in Table 1), a more appropriate weight reference value has finally been selected: $k_1 = 0.25$, $k_2 = 0.25$, $k_3 = 0.20$, $k_4 = 0.20$, $k_5 = 0.10$.

Table 1. The related parameters determination experiment.

Model	K_1	K_2	K_3	K_4	K_5	$e_{flattening}$
Partial-sphere	0.30	0.35	0.15	0.10	0.10	0.01087344
Partial-sphere	0.30	0.30	0.20	0.10	0.10	0.01123564
Partial-sphere	0.25	0.30	0.25	0.10	0.10	0.01824917
Partial-sphere	0.35	0.35	0.10	0.10	0.10	0.00617236
Partial-sphere	0.30	0.25	0.25	0.10	0.10	0.00712849
Partial-sphere	0.25	0.25	0.20	0.20	0.10	0.00302347
Partial-sphere	0.20	0.20	0.20	0.20	0.20	0.02346912
Partial-sphere	0.25	0.25	0.15	0.15	0.20	0.01136527

5 Conclusion

This paper has analysed the existing evaluation methods of surface development error, the deformation nature of the primitives and the graphics from the entire surface development process. It has studied the causes of a single deformation and deformation transmission from the perspective of local deformation and global deformation. A comprehensive evaluation system for the accuracy of surface development has been proposed, using five indicators, which include the relative edge length error index, the relative area error index, the relative displacement error index, and the relative direction error index. The reference values of the relevant weight coefficients are provided as parts of flattening experiments. The surface development accuracy evaluation system proposed in this paper not only overcomes the limitations of the existing area and edge length errors as evaluation methods, but also provides some reference for the reasonable evaluation of surface development accuracy and the optimization and improvement of surface flattening algorithms. Thus, it bears certain practical value.

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