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# Research on observability of two wire-guided torpedoes tracking system

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**Abstract.** Under special condition, the surface target could be attacked by the submarine with two wire-guided torpedoes with passive acoustic homing mode. When the acoustic homing device detects the target bearing and transmit it back to the submarine through the wire, a three-sensor detection system is constituted by the submarine's integrated sonar and the two wire-guided torpedoes. Whether the system can effectively track the target is related to the observability of the system. Only by satisfying the observable conditions can the moving parameters of the target could be calculated. For the nonlinear characteristics of the two wire-guided torpedoes tracking system, the observation equation is treated by pseudo-linear processing, and then the observability of the linear system is analyzed by using the observability decision theorem of the linear system, then the conditions for judging the observability are given. Finally, the correctness of the conclusion is verified by simulation.

## 1 Introduction

Under special conditions, such as obvious noise character of the target, the submarine can use two wire-guided torpedoes with passive acoustic homing mode to attack the surface ship. When the target noise is detected by the homing device of wire-guided torpedo, the position of target relative to torpedo can be transmitted back to the submarine command and control system through wire. At this time, the position of torpedo can also be obtained in real time by telemetry coordinates. Thus, the integrated sonar of submarine and two wire-guided torpedoes constitute a three-sensor tracking system. The moving parameters of target can be calculated by using bearings got by each sensor.

The system must satisfy the observability condition in order to calculate the moving parameters of target effectively. The first thing is to solve the observability problem for the two wire-guided torpedoes tracking system. In this tracking system, the observation is bearing, which is suitable for describing in polar condition. However, the state equation of system should be transformed to Cartesian coordinates when calculating the moving parameters of target. When the target parameters are transformed from polar coordinates to Cartesian coordinates, the nonlinear measurement equation is obtained, then the tracking system is a nonlinear system. Based on the observability theory of linear system, this paper

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researches the observability of two wire-guided torpedoes tracking system by pseudo-linearization, and then obtains its observability criterion.

## 2 Two wire-guided torpedoes tracking system

The target tracking by two wire-guided torpedoes tracking system is studied only in the horizontal plane, ignoring the effect in the vertical direction, and setting the tracking system and target in the same plane[1]. As shown in Fig.1, the origin of the coordinates is the position of the submarine when the first torpedo runs out of tube. At a certain time, the position of torpedo 1 is $(px_1, py_1)$ , and the measured bearing is $\alpha$ . the position of torpedo 2 is $(px_2, py_2)$ , and the measured bearing is $\beta$ . the position of integrated sonar is $(px_3, py_3)$ , and the measured bearing is $\gamma$ .

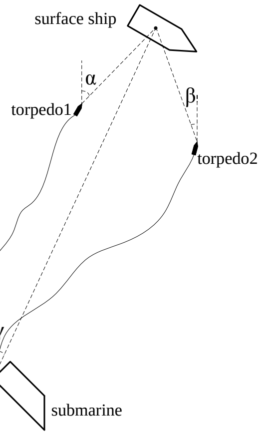


**Fig. 1.** Two wire-guided torpedoes tracking system.

The moving mode of target in water are mostly straight line at constant velocity model and straight line at constant acceleration model. After the comprehensive analysis of the advantages and disadvantages of each moving model, CV model and CA model[2-5] are selected, this is to use these two models to cover the whole movement of the target.

When target is moving in straight line at constant velocity, we use CV model. Supposed at the $k$ sampling time, the moving state vector of target relative to tracking system is $X(k) = [x(k), y(k), vx(k), vy(k)]^T$ , $x(k)$ and $y(k)$ are the distance from the target relative to the origin in the $x$ direction and $y$ direction respectively, $vx(k)$ and $vy(k)$ are its velocity in the $x$ direction and $y$ direction respectively. The equation of state of tracking system is:

$$X(k+1) = \Phi X(k) \tag{1}$$

Where $\Phi$ is the state transition matrix, $T$ is the sampling interval.

$$\Phi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

When target is moving in straight line at constant acceleration, we use CA model. Then, the moving state vector of target relative to tracking system is $X(k)=[x(k), y(k), vx(k), vy(k), ax(k), ay(k)]^T$ , $ax(k)$ and $ay(k)$ are the acceleration relative to the origin in the $x$ direction and $y$ direction respectively. The state transition matrix:

$$\Phi=\begin{bmatrix} 1 & 0 & T & 0 & \frac{T^2}{2} & 0 \\ 0 & 1 & 0 & T & 0 & \frac{T^2}{2} \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

As mentioned above, the two wire-guided torpedoes tracking system is nonlinear system, After the pseudo-linear processing is applied on the observable equation[6], the observed equation is:

$$z(t)=H(t)X(t) \quad (4)$$

For the CV model:

$$H(t)=\begin{bmatrix} \cos(\alpha(t)) & -\sin(\alpha(t)) & 0 & 0 \\ \cos(\beta(t)) & -\sin(\beta(t)) & 0 & 0 \\ \cos(\gamma(t)) & -\sin(\gamma(t)) & 0 & 0 \end{bmatrix} \quad (5)$$

For the CA model:

$$H(t)=\begin{bmatrix} \cos(\alpha(t)) & -\sin(\alpha(t)) & 0 & 0 & 0 & 0 \\ \cos(\beta(t)) & -\sin(\beta(t)) & 0 & 0 & 0 & 0 \\ \cos(\gamma(t)) & -\sin(\gamma(t)) & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

No matter which moving model is adopted, the observation vector is the same, as in Eqn.7.

$$Z(t)=\begin{bmatrix} px_1\cos(\alpha(t))-py_2\sin(\alpha(t)) \\ px_2\cos(\beta(t))-py_2\sin(\beta(t)) \\ px_3\cos(\gamma(t))-py_3\sin(\gamma(t)) \end{bmatrix} \quad (7)$$

## 3 Observability judgement

According to the theory of linear system, the observability of liner system has a clear definition. Based on the theory of obserbability of linear system, this paper analyzes the obsevability for the pseudo-linearized two wire-guided torpedoes tracking system.

The judging theorem for observability used in this paper was put forward by Claude Jaufferet[7]:

The sufficient and necessary condition of observability is that $\forall M \neq 0 \in R^n$, $\exists t \in [t_0, t_1]$, then $H(t)\Phi(t,t_0)M \neq 0$.

This theorem could be replaced by the converse negative proposition, that is:

$$\forall t \in [t_0, t_1] \text{, if } H(t)\Phi(t,t_0)M = 0 \text{, then } M = 0 .$$

It means that the system is observable during the period $[t_0, t_1]$, only for any vector $M$ which is non-zero, there is still $t \in [t_0, t_1]$ which makes $H(t)\Phi(t,t_0)M \neq 0$ or $H(t)\Phi(t,t_0)M = 0$, then $M$ must be zero.

Because whatever moving model the target chooses, the solution processes are same, and because the length of this paper is limited, we only give the solution process of observability for target tracking when the target moves in straight line with constant velocity. The solution process of target tracking in constant acceleration is similar to this.

Supposing that $M = [m_1, m_2, m_3, m_4]^T$, when the $M$ is put to $H(t)\Phi(t,t_0)M = 0$, we can get:

$$[m_1 + (t-t_0)m_3]\cos(\alpha(t)) - [m_2 + (t-t_0)m_4]\sin(\alpha(t)) = 0 \tag{8}$$

$$[m_1 + (t-t_0)m_3]\cos(\beta(t)) - [m_2 + (t-t_0)m_4]\sin(\beta(t)) = 0 \tag{9}$$

$$[y_1 + (t-t_0)m_3]\cos(\gamma(t)) - [m_2 + (t-t_0)m_4]\sin(\gamma(t)) = 0 \tag{10}$$

According to the distribution of acoustic sensors of torpedo 1, torpedo 2 and integrated sonar, two cases are considered:

Case 1: target moves on the line formed by any two of the three sonar sensors.

Supposed that $\alpha(t) - \beta(t) = k\pi$, i.e. target moves on the line formed by torpedo 2 and integrated sonar. Then Eqn.8 is equal to Eqn.9, new equation set is formed by Eqn.8 and Eqn.10, the components of vector $M$ are regards as variables, the value of determinant $\begin{bmatrix} \cos(\alpha(t)) & -\sin(\alpha(t)) \\ \cos(\gamma(t)) & -\sin(\gamma(t)) \end{bmatrix}$ is $\sin(\alpha(t) - \gamma(t))$, $\sin(\alpha(t) - \gamma(t)) \neq 0$ if exclusive solution could be got, then $\alpha(t) - \gamma(t) \neq k\pi$, because the value of determinant changes according to $t$, the equations could be solved if $m_1 + (t-t_0)m_3 = 0, m_2 + (t-t_0)m_4 = 0$.

Because the value of $t - t_0$ changes according to $t$, the above equations could be solved when $m_1 = m_2 = m_3 = m_4 = 0$, so $M = 0$, the system is observable.

According to above, we can get that the system is observable so long as $\alpha(t) - \gamma(t) \neq k\pi$ or $\beta(t) - \gamma(t) \neq k\pi$, i.e. target doesn't move on the line formed by the three sonar sensors.

Case 2: target doesn't move on the line formed by any two of the three sonar sensors.

The components of vector $M$ are regards as variables, in order to get the exclusive solution, the solution of the new equation set formed by any two of the three equations is unique, and the solutions of the three new equation sets are the same.

Similar to the last condition, we need $\sin(\alpha(t)-\beta(t)) \neq 0$, $\sin(\alpha(t)-\gamma(t)) \neq 0$, $\sin(\beta(t)-\gamma(t)) \neq 0$, i.e. any two of the three parameters $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are not equal, then the solution of the equation set is exclusive. We also could get $M=0$, the system is observable[8].

So when the target moves in straight with constant velocity, the conditions of observability for target tracking based on two wire-guided torpedoes are:

1) So long as the three sensors are not on the same line, the system is always observable.

2) When the three sensors are on the same line, only if target moves on the line, the system is not observable.

If the research is carried out on the issue of target tracking when target moves in straight with constant acceleration, we can get the same conclusions.

In a word, in the process of tracking target, it is very rare that two torpedoes and integrated sonar are co-linear, even if they do, the target can not guaranteed to move on the line. In addition, the surface ship, torpedoes, submarine are all moving in real time, which is a dynamic process. It is certain that the two wire-guided torpedoes tracking system can track the target effectively, that is, it can calculate the moving parameters of target.

## 4 Simulation and analysis

The purpose of simulation is to verify that the two wire-guided torpedoes tracking system can calculate moving parameters of target under the premise of observability, that is, to track the target effectively. The trajectories of the target, submarine and two torpedoes in the following 2 minutes are shown in Fig.2. Target velocity is 16 Kn before turning, course is 145°. Target velocity is 24Kn after turning, course is 45°, turning lasts 30s, sailing 30s after turning, initial coordinates is (-300,5000). The submarine velocity is 4 Kn, course is 270°, initial coordinates is (-300,0).The torpedo 1 velocity is 30 Kn, course is 030°, initial coordinates is (-1000,2500). The torpedo 2 velocity is 30 Kn, course is 350°, initial coordinates is (1000,2500). Bearing accuracy is 0.1°, sampling interval is 1s.

Here we use the interactive multiple models algorithm[9],100 times of Monte Carlo simulation, the results are shown in Fig3~Fig5.

The tracking trajectory in Fig.3 coincides with the target trajectory. As can be seen from Fig.4 and Fig.5, the target tracking error is large before turning. The RMS error in the x direction is less than 20m, and that in the y direction is not more than 60m during turning and after turning. This is because the filter system has a process of adaptation, in the adaptation phase the output accuracy of the algorithm is low, but with time extension, the algorithm precision greatly improved.

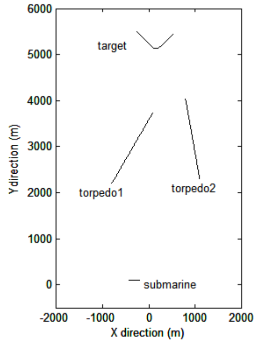


**Fig. 2.** Moving trajectories.

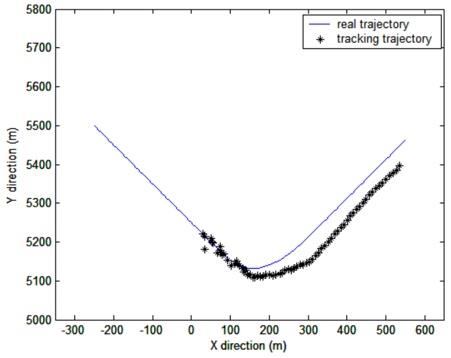


**Fig. 3.** Trajectory comparison.

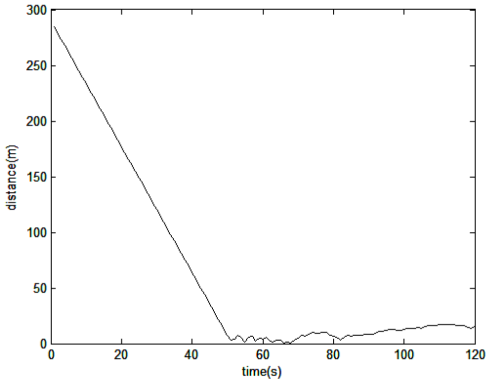


**Fig. 4.** RMS error in the x direction.

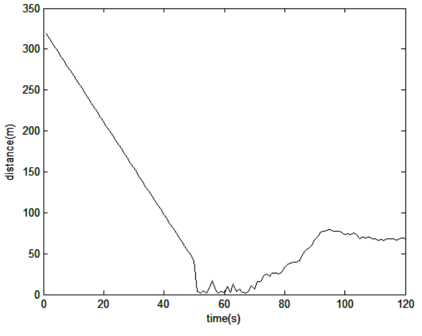


**Fig. 5.** RMS error in the y direction.

The simulation results show that the system can track the target effectively. When the interactive multiple models algorithm is adopted, the filtering convergence speed is fast and the error is acceptable.

## 5 Conclusion

In view of the obsevability of the tracking system of two wire-guided torpedoes, the decision theorem of obsevability for linear system is introduced. After linearizing the observation equation, the decision condition of the obsevability of the system is obtained by proving that the system has obsevability and can be used to track the target effectively. Conclusion is verified by simulation.

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