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Resonances of a fractional-order biomedical model with time delay state feedback

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Abstract. In the present paper, the primary resonance of a fractional-order Willis aneurysm system with time-delay state feedback control is studied. Using the multiple scale method, the amplitude and phase equations are obtained. The first order approximate solution is derived and the influence of time delay on resonance is studied. The concept of equivalent damping related to time-delay feedback is proposed, and the reasonable selection of feedback gain and time delay is discussed from the point of view of vibration control. The frequency response and external excitation response curves of the system are given. In order to test the stability of the system, bifurcation analysis is carried out. The obtained results are very useful in the clinical diagnosis and treatment of cerebral aneurysms.

1 Introduction

The cardiovascular and cerebrovascular system can be seen as a hydraulic system that circulates blood as its working fluid and it is characterized in terms of pressure and flow variability (see [1-3]). The rupture of aneurysms is closely related to the blood flow velocity. Therefore, taking the blood flow velocity as the main research object, the dynamic model system of cerebral aneurysm based on the principle of hemodynamics plays an important role in clinical and theoretical research. The rupture of aneurysms is characterized by a great change in blood flow velocity, that is, a “spike”. Although not all spikes can cause aneurysm rupture, no peak means that the blood flow velocity is stable, the condition is stable, and the system is in a stable state ([4]). Therefore, it is necessary to study the vibration control of blood flow velocity in cerebral aneurysm model.

Over the past few years, many papers have been devoted to the control of resonant forced systems in various fields. In the aspect of vibration reduction, the common vibration reduction methods include passive control, active control and semi-active control, in which time-delay feedback control belongs to active control. In some active problems, delay feedback control is superior to traditional techniques, such as reducing crane swing [5]. The controlled systems with time delay feedback have been investigated by many researchers. In 1992, Olgac et al. [6] first proposed the application of time-delay state feedback control to the vibration reduction of dynamic systems. The main idea is to add a mass spring damper and a mechanism with time-delay state feedback device to the main system, and

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reduce and suppress the vibration amplitude of the main system by adjusting the feedback gain coefficient and time delay. Renzulli et al. [7] studied the method that the feedback gain coefficient and time delay can be automatically adjusted when the external excitation frequency changes. Stepan and Haller [8] considered quasi-periodic oscillations in robot dynamics, analysed the delayed positioning of a single degree of freedom robot arm which leads to an infinite dimensional dynamical system. Rosenblum et al. in [9] used the linear delayed feedback signal to eliminate the ill conditioned synchronization behaviour of neural networks, which has been successfully applied to ideal neural network models. The author in [10] studied vibration control of a cantilever beam with time delay state feedback by the method of multiple scales.

To the authors' knowledge, the dynamic problems of fractional-order systems with time-delay, such as the fractional order periodic forced nonlinear systems with time-delay control, have not received enough attention. This paper focuses on the fractional-order nonlinear cerebral aneurysm system with time-delay feedback control. By controlling the time-delay term (composed of feedback gain coefficient and time-delay), the blood vibration is optimized. This study has two application values: on the one hand, the research on delayed feedback control of cerebral aneurysm model provides theoretical guidance for clinical diagnosis and treatment to a certain extent; on the other hand, this model is relatively simple and can be used as the modelling basis for other practical problems.

In the following two sections, the primary resonances will be studied by using the multiple scale perturbation technique. Section 3 presents the concluding remarks.

2 Primary resonance

The fractional order Willis cerebral aneurysm system under forced excitation and delayed feedback is described by

$$\ddot{x}+\mu\,{}^{c}D_t^q x+\alpha x-\beta x^2+\gamma x^3=F\cos\omega t+G_p x(t-\tau)+G_d\dot{x}(t-\tau), \quad (1)$$

where x denotes the blood flow velocity, ${}^{c}D_t^q x(t)$ is the q-order Caputo derivative of $x(t)$ to t and $0<q<1$, $\ddot{x}$ is the acceleration of blood flow change, μ stands for the damping coefficient of the blood flow, F is the pulse pressure, ω denotes the reciprocal of heart rate, $F\cos\omega t$ is the change rate of central blood pressure, α, β, and γ are related to the blood flow resistance and the elasticity of the vessel wall, G_p and G_d are the feedback gains, τ is the time delay.

Using the coordinates transformation as follows:

$$2\varepsilon\mu_1=\mu, \omega_0=\sqrt{\alpha}, \varepsilon\beta_1=\beta, \varepsilon\gamma_1=\gamma, \varepsilon f=F, \varepsilon g_p=G_p, \varepsilon g_d=G_d,$$

Eq. (1) becomes

$$\begin{aligned}\ddot{x}+2\varepsilon\mu_1\,{}^{c}D_t^q x+\omega_0^2 x-\varepsilon\beta_1 x^2+\varepsilon\gamma_1 x^3&=\varepsilon f\cos\omega t\\ &+\varepsilon g_p x(t-\tau)+\varepsilon g_d\dot{x}(t-\tau),\end{aligned} \quad (2)$$

where $0<\varepsilon\ll 1$, ω_0 is natural frequency. In this transformation, $\mu_1, \beta_1, \gamma_1, f, g_p$ and g_d are dimensionless quantity.

In this section, we study the primary resonance of the delay controlled system (2) by using the multiple scale method which is commonly used in perturbation problems. In the theory of nonlinear vibration, the multiple scale method is a classical method. This method requires the model to meet the requirements of small damping, weak nonlinearities, weak feedback and soft excitation. Therefore, the system damping, external excitation amplitude and frequency, feedback gains and some other coefficients are limited as follows,

$$\begin{cases} \mu_1 = O(1), \beta_1 = O(1), \gamma_1 = O(1), \ f = O(1), \\ g_p = O(1), g_d = O(1), \omega = \omega_0 + \varepsilon\sigma, \sigma = O(1), \end{cases} \tag{3}$$

Where σ is the detuning parameter. We rewrite (2) as

$$\begin{aligned} \ddot{x} + \omega_0^2 x = \varepsilon[-2\mu_1 \ {}^c D_t^q x + \beta_1 x^2 - \gamma_1 x^3 + f \cos(\omega_0 + \varepsilon\sigma)t \\ + g_p x(t-\tau) + g_d \dot{x}(t-\tau)]. \end{aligned} \tag{4}$$

Only two time scales are needed to study the first approximation of the solution, thus we assume a two scales expansion of the form

$$x(t) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) + O(\varepsilon^2), (T_r = \varepsilon^r t, r = 0,1), \tag{5}$$

and use the following differential operators:

$$\begin{aligned} \frac{d}{dt} &= \frac{\partial}{\partial T_0} + \varepsilon \frac{\partial}{\partial T_1} + \cdots = D_0 + \varepsilon D_1 + \cdots, \\ \frac{d^2}{dt^2} &= D_0^2 + 2\varepsilon D_0 D_1 + \cdots. \end{aligned} \tag{6}$$

Substituting (5) and (6) into (4) and equating the coefficients of the same power of ε, we obtain a set of linear partial differential equations:

$$O(\varepsilon^0): D_0^2 x_0 + \omega_0^2 x_0 = 0, \tag{7}$$

$$\begin{aligned} O(\varepsilon^1): D_0^2 x_1 + \omega_0^2 x_1 = -2D_0 D_1 x_0 - 2\mu_1 \ {}^c D_{T_0}^q x_0 + \beta_1 x_0^2 - \gamma_1 x_0^3 \\ + g_p x_0(T_0 - \tau, T_1) + g_d D_0 x_0(T_0 - \tau, T_1) + f \cos(\omega_0 T_0 + \sigma T_1). \end{aligned} \tag{8}$$

Solving (7), we have

$$x_0(T_0, T_1) = a(T_1)\cos[\omega_0 T_0 + \theta(T_1)] = A(T_1) e^{i\omega_0 T_0} + cc, \tag{9}$$

where cc denotes the complex conjugate of the preceding terms and

$$A(T_1) = \frac{a(T_1)}{2} e^{i\theta(T_1)}. \tag{10}$$

For the convenience of the following derivation, we give the following formula for fractional derivative (see [15,16]). When the time T_0 is large, ${}^{c}D_{T_0}^{q} A e^{i\omega_0 T_0}$ approximately reduces to

$$ {}^{c}D_{T_0}^{q} A e^{i\omega_0 T_0} = (i\omega_0)^q A e^{i\omega_0 T_0}. \tag{11} $$

Substituting (9) into (8) and using (11), we obtain

$$ \begin{aligned} D_0^2 x_1 + \omega_0^2 x_1 &= -[2i\omega_0 D_1 A + 2\mu_1 (i\omega_0)^q A + 3\gamma_1 A^2 \bar{A}] e^{i\omega_0 T_0} + \beta_1 A^2 e^{2i\omega_0 T_0} \\ &- \gamma_1 A^3 e^{3i\omega_0 T_0} + g_p A e^{i\omega_0 T_0} e^{-i\tau} + g_d i A e^{i\omega_0 T_0} e^{-i\tau} + \frac{f}{2} e^{i\omega_0 T_0} e^{i\sigma T_1} + cc, \end{aligned} \tag{12} $$

where $\bar{A}$ is the complex conjugate of A. To eliminate the secular term in (12), we let

$$ 2i\omega_0 D_1 A + 2\mu_1 (i\omega_0)^q A + 3\gamma_1 A^2 \bar{A} - g_p A e^{-i\tau} - g_d i A e^{-i\tau} - \frac{f}{2} e^{i\sigma T_1} = 0. \tag{13} $$

By substituting (10) into (13) and separating the real and imaginary parts, we can obtain a set of autonomous differential equations that govern the amplitude $a(T_1)$ and the phase $\theta(T_1)$:

$$ \begin{cases} D_1 a = -\mu_1 a \omega_0^{q-1} \sin\dfrac{q\pi}{2} - \dfrac{a g_p}{2\omega_0} \sin\tau + \dfrac{a g_d}{2\omega_0} \cos\tau + \dfrac{f}{2\omega_0} \sin(\sigma T_1 - \theta), \\ a D_1 \theta = \mu_1 a \omega_0^{q-1} \cos\dfrac{q\pi}{2} + \dfrac{3}{8\omega_0} \gamma_1 a^3 - \dfrac{a g_p}{2\omega_0} \cos\tau - \dfrac{a g_d}{2\omega_0} \sin\tau - \dfrac{f}{2\omega_0} \cos(\sigma T_1 - \theta). \end{cases} \tag{14} $$

Let $\varphi = \sigma T_1 - \theta,$ in order to determine the amplitude and phase of the steady solution corresponding to the steady-state motion, let $D_1 a = 0, D_1 \varphi = 0$, we get a set of algebraic equations of the steady-state fundamental resonance:

$$ \begin{cases} -2\bar{a}\mu_1 \omega_0^q \sin\dfrac{q\pi}{2} - \bar{a} g_p \sin\tau + \bar{a} g_d \cos\tau + f \sin\bar{\varphi} = 0, \\ 2\omega_0 \sigma \bar{a} - 2\bar{a}\mu_1 \omega_0^q \cos\dfrac{q\pi}{2} - \dfrac{3}{4}\gamma_1 \bar{a}^3 + \bar{a} g_p \cos\tau + \bar{a} g_d \sin\tau + f \cos\bar{\varphi} = 0. \end{cases} \tag{15} $$

Eliminating $\bar{\varphi}$ from (15), we can get the relationship between amplitude $\bar{a}$ and excitation frequency offset σ:

$$ \begin{aligned} &[(-2\mu_1 \omega_0^q \sin\frac{q\pi}{2} - g_p \sin\tau + g_d \cos\tau)^2 + (2\omega_0 \sigma - \frac{3}{4}\gamma_1 \bar{a}^2 \\ &-2\mu_1 \omega_0^q \cos\frac{q\pi}{2} + g_p \cos\tau + g_d \sin\tau)^2] \bar{a}^2 = f^2. \end{aligned} \tag{16} $$

From (15), we can also get the relationship between amplitude $\bar{\varphi}$ and excitation frequency offset σ :

$$\bar{\varphi} = \arctan \frac{-2\mu_1\omega_0^q \sin\frac{q\pi}{2} - g_p \sin\tau + g_d \cos\tau}{2\omega_0\sigma - 2\mu_1\omega_0^q \cos\frac{q\pi}{2} - \frac{3}{4}\gamma_1\bar{a}^2 + g_p \cos\tau + g_d \sin\tau}. \tag{17}$$

Substituting the parameters with the original ones, (16) and (17) become

$$\begin{aligned}&[(-\mu\omega_0^q \sin\frac{q\pi}{2} - G_p \sin\tau + G_d \cos\tau)^2 + (2\omega_0\omega - 2\alpha \\ &-\mu\omega_0^q \cos\frac{q\pi}{2} - \frac{3}{4}\gamma\bar{a}^2 + G_p \cos\tau + G_d \sin\tau)^2]\bar{a}^2 = F^2,\end{aligned} \tag{18}$$

$$\bar{\varphi} = \arctan \frac{-\mu\omega_0^q \sin\frac{q\pi}{2} - G_p \sin\tau + G_d \cos\tau}{2\omega_0\omega - 2\alpha - \mu\omega_0^q \cos\frac{q\pi}{2} - \frac{3}{4}\gamma\bar{a}^2 + G_p \cos\tau + G_d \sin\tau}. \tag{19}$$

Accordingly, the first order approximate solution of (9) is expressed as

$$x(t) = \bar{a}(T_1)\cos[\omega_0 T_0 + \sigma(T_1) - \bar{\varphi}(T_1)] = \bar{a}(\varepsilon t)\cos[\omega t - \bar{\varphi}(\varepsilon t)].$$

3 Conclusion

Since the fractional derivative can better describe the complex hemodynamics, we established the Willis ring cerebral aneurysm model based on the fractional derivative. In this paper, the dynamic response of fractional delay feedback Willis aneurysm system is studied. Using the multiple scale method, the approximate analytical solution and amplitude frequency equation are obtained. This study has certain significance for the clinical diagnosis and treatment of cerebral aneurysms. There is an obvious time lag relationship between the medication delay and the time required for drug absorption. This time lag is related to personal constitution and can be regarded as a constant in statistical sense. The further research will analyze the effects of fractional order, feedback coefficient and time delay, compare the analytical solution and numerical solution, verify the correctness and accuracy of the approximate analytical solution, and take appropriate control methods. It is also found that although the average method is simple and easy, some useful information may be lost in the discussion. Next, other perturbation methods will be used for further discussion.

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