

# Application of GOA-GDOP in aircraft collaborative detection

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**Abstract.** The cooperative detection of aircraft has been widely concerned by scholars at home and abroad in recent years. This paper mainly studies the positioning accuracy of the sensor-carrying aircraft in collaborative detection. By using Grasshopper Optimisation Algorithm (GOA), Taking Geometrical Dilution of Precision (GDOP) as the objective function, the relative position relationship between aircraft is determined, the optimal detection configuration is planned, and the feasibility of the algorithm is analyzed under limited flight area conditions, the positioning accuracy of collaborative detection is further improved.

## 1 Introduction

Cooperative detection technology is a key technology in the field of cooperative warfare. This paper focuses on the detection configuration design in cooperative detection by building the aircraft dynamics model (taking missile as an example), and designing the most suitable detection configuration to locate the target through GOA-GDOP algorithm.

The research on airborne cooperative detection and unmanned aerial vehicle cooperative detection is extensive at home and abroad<sup>[1]-[7]</sup>. The E-2C Hawkeye is the most advanced carrier-borne early warning aircraft in the world. It has successively used three types of surveillance system radars, AN/APS-138, 139 and 145, which can achieve all-round coverage of road, sea and air<sup>[8]</sup>. Russia's A-50 "Core" aircraft, which entered service in the mid-1980s, can track up to 50 targets<sup>[9]</sup>. In the field of airborne infrared, The F-14D fighter is equipped with aAS-42 infrared search and tracking system, which has strong anti-interference capability. In the field of airborne infrared, THE American F-14D fighter is equipped with aAS-42 infrared search and tracking system, which has strong anti-interference capability<sup>[10]</sup>. In the field of UAV cooperative detection, radar detection, photoelectric detection and acoustic detection are mainly used<sup>[10]</sup>. In photoelectric detection, mainly the application of infrared, visible light and laser detection, compared with radar detection, photoelectric detection has higher detection accuracy and reliability<sup>[12]</sup>. There is no direct description of the types of radar and infrared sensors unloaded by UAV at home and abroad in the public literature.

It is necessary to use certain parameters as reference to measure the positioning accuracy. At present, there are several positioning accuracy analysis methods as follows: Geometrical Dilution of Precision (GDOP), Cramer–Rao Lower Bound(CRLB), Circular Error Probable(CEP), etc. In essence, GDOP can be regarded as the least squares estimation, while CRLB is the maximum likelihood estimation. When the system is linear and the high noise is Gaussian independent white noise, the two are equivalent, but in the nonlinear system, the two will produce certain differences. This is due to the fact that GDOP ignores the error covariance diagonal element and is more sensitive to the sensor geometric position than CRLB, which is used as an analysis tool for the geometric uncertainty of the sensor target, while CRLB is used as a statistical performance evaluation tool based on sensor observations derived from the target<sup>[13] [14]</sup>. The CEP is an uncertain measure of the positioned estimator relative to the estimated mean, which contains half of the circle radius realized by the random vector centered on the mean<sup>[15]</sup>.

Shahrzad Saremi et al first proposed the Grasshopper optimization algorithm in 2017<sup>[16]</sup>. This algorithm can solve the practical problem of unknown search space well. In the collaborative detection of stationary targets on the ground, different positions between them, namely different configurations, will also have a certain impact on the positioning accuracy of the target. Therefore, Grasshopper optimization algorithm can be used to traverse and search the best position of aircraft in a certain range of space, and form the best detection configuration, so as to achieve the highest precision of collaborative detection.

The establishment of aircraft dynamics model, equation of state and observation equation has been described in detail in my other papers. This paper only analyzes and experiments the proposed algorithm. 1.1 Formatting the title, authors and affiliations.

## 2 GOA-GDOP algorithm

### 2.1 Grasshopper algorithm (GOA)

The algorithm utilizes the behavioral characteristics of Grasshopper larva and adult population gathering for large-scale migration, and the larva usually moves slowly with small amplitude, while the adult mostly moves for a long time. By establishing a mathematical model to simulate the movement rule of Grasshopper in a certain area, the best foraging area of Grasshopper can be explored and searched. Further optimize the optimal position of the target.

The mathematical model of grasshoppers aggregation and migration can be expressed as:

$$P_j = S_j + G_j + A_j \tag{1}$$

$P_j$  represents the position of the  $j$ th grasshoppers, and  $S_j$  represents the interaction between grasshoppers, namely the force, which is also the main control force affecting grasshoppers position. When grasshoppers see the distance is too large, it will produce attraction, conversely, it will produce repulsion. Grasshoppers are in their comfort zone only if they keep the right distance between them.  $G_j$  and  $A_j$  represents the force of gravity and the force of wind on the grasshoppers. In general, the force of gravity on the grasshoppers is not taken into account and the wind is assumed to always be in the direction of the target. The interactions  $S_j$  between grasshoppers can be expressed as follows:

$$S_j = \sum_{\substack{k=1 \\ k \neq j}}^N s(d_{jk}) \hat{d}_{jk} \tag{2}$$

$d_{jk}$  represents the  $j$ th and  $k$ th the distance between grassoppers. The calculation formula can be expressed as:  $d_{jk} = |P_j - P_k|$ ,  $P_j, P_k$  are the locations of the two grassoppers.  $s$  represents the force function between locusts, It is described in formula (3).  $\hat{d}_{jk}$  is the unit vector between  $j$  and  $k$ :

$$\hat{d}_{jk} = \frac{P_j - P_k}{d_{jk}} \tag{3}$$

For  $s$  function, its calculation formula can be expressed as:

$$s(r) = fe^{-r/l} - e^{-r} \tag{4}$$

It has been proved in [16] that  $l=1.5$  and  $f=0.5$  are the optimal parameters for the calculation and solution of this algorithm.

However, the above model cannot be directly applied to the optimization problem, because locusts will quickly reach the "comfort zone" and the population will not converge to the specified point. Therefore, the above equation is modified as follows to solve the optimization problem:

$$P_j^d = c \left( \sum_{\substack{k=1 \\ k \neq j}}^N c \frac{ub_d - lb_d}{2} s(|P_j^d - P_k^d|) \frac{P_k - P_j}{d_{jk}} \right) + \hat{T}_d \tag{5}$$

Here,  $ub_d$  is the upper bound of the  $d$ th dimension,  $lb_d$  is the lower bound of the  $d$ th dimension.  $\hat{T}_d$  is the current optimal solution.  $C$  can be expressed as:

$$c = c_{\max} - l \frac{c_{\max} - c_{\min}}{M} \tag{6}$$

$M$  represents the maximum number of iterations,  $c_{\max}$  and  $c_{\min}$  Represents the maximum and minimum value of  $c$  respectively,  $c_{\max}$  is 1 and  $c_{\min}$  is 0.00004.

## 2.2 GDOP Model

Assume that the position of the aircraft under the earth fixation is  $(x_i, y_i, z_i)$ , the observed distance, azimuth and pitch angle are respectively  $r_i, a_i, e_i$ :

$$\begin{cases} r_i = \sqrt{x_q^2 + y_q^2 + z_q^2} \\ a_i = \arctan \frac{y_q}{x_q} \\ e_i = \arctan \frac{z_q}{x_q^2 + y_q^2} \end{cases} \tag{7}$$

$r_i = f_i(x_T, y_T, z_T, x_i, y_i, z_i)$ ,  $a_i = g_i(x_T, y_T, z_T, x_i, y_i, z_i)$ ,  $e_i = h_i(x_T, y_T, z_T, x_i, y_i, z_i)$ , The derivative of the above formula can be obtained:

$$\begin{cases} dr_i = \frac{\partial f_i}{\partial x_T} \Delta x_T + \frac{\partial f_i}{\partial y_T} \Delta y_T + \frac{\partial f_i}{\partial z_T} \Delta z_T + \frac{\partial f_i}{\partial x_i} \Delta x_i + \frac{\partial f_i}{\partial y_i} \Delta y_i + \frac{\partial f_i}{\partial z_i} \Delta z_i \\ da_i = \frac{\partial g_i}{\partial x_T} \Delta x_T + \frac{\partial g_i}{\partial y_T} \Delta y_T + \frac{\partial g_i}{\partial z_T} \Delta z_T + \frac{\partial g_i}{\partial x_i} \Delta x_i + \frac{\partial g_i}{\partial y_i} \Delta y_i + \frac{\partial g_i}{\partial z_i} \Delta z_i \\ dh_i = \frac{\partial h_i}{\partial x_T} \Delta x_T + \frac{\partial h_i}{\partial y_T} \Delta y_T + \frac{\partial h_i}{\partial z_T} \Delta z_T + \frac{\partial h_i}{\partial x_i} \Delta x_i + \frac{\partial h_i}{\partial y_i} \Delta y_i + \frac{\partial h_i}{\partial z_i} \Delta z_i \end{cases} \quad (8)$$

The jacobian matrix is used to solve the first partial derivative, and a more accurate value can be obtained.

Without considering the site error, the measurement error variances of distance, pitch angle and azimuth angle are respectively set as  $\Delta r_i^2, \Delta a_i^2, \Delta e_i^2$ ,  $i = 1, 2, \dots, n$ , the measurement error matrix is set as R, which is diagonal matrix.

$$J = \begin{bmatrix} \frac{\partial r_1}{\partial x} & \frac{\partial r_1}{\partial y} & \frac{\partial r_1}{\partial z} \\ \frac{\partial a_1}{\partial x} & \frac{\partial a_1}{\partial y} & \frac{\partial a_1}{\partial z} \\ \frac{\partial e_1}{\partial x} & \frac{\partial e_1}{\partial y} & \frac{\partial e_1}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial r_n}{\partial x} & \frac{\partial r_n}{\partial y} & \frac{\partial r_n}{\partial z} \\ \frac{\partial a_n}{\partial x} & \frac{\partial a_n}{\partial y} & \frac{\partial a_n}{\partial z} \\ \frac{\partial e_n}{\partial x} & \frac{\partial e_n}{\partial y} & \frac{\partial e_n}{\partial z} \end{bmatrix} \quad (9)$$

According to the maximum likelihood criterion, the fusion error covariance matrix and positioning accuracy GDOP can be expressed as:

$$P = (J^T R^{-1} J)^{-1}$$

$$GDOP = \sqrt{\text{trace}(P)} \quad (10)$$

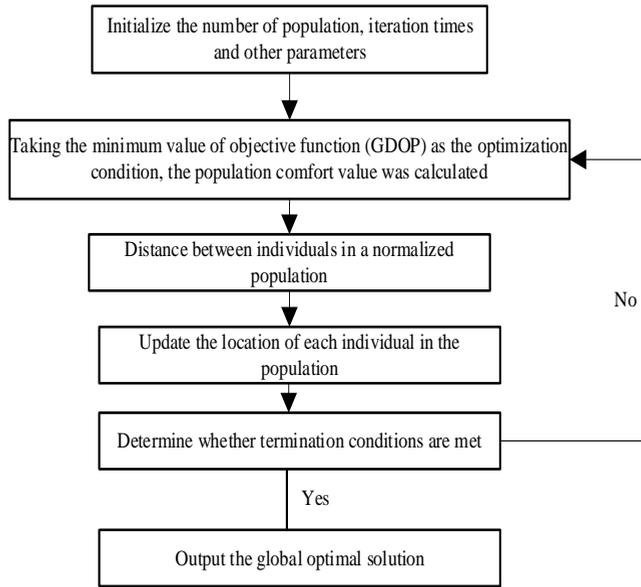
Taking GDOP as the objective function, when the value of GDOP is the smallest, the detection accuracy reaches the highest. The coordinate parameters of different aircraft in the launching system are taken as the unknown parameters of locust search to determine the best locust position and the best launching position of aircraft. The flow chart can be expressed as:

### 3 Simulation analysis

#### 3.1 Parameter settings

Assume that the initial altitude of the first aircraft (taking the missile as an example) is 6000m, latitude 40°, longitude 100°, launch azimuth -1°, initial velocity in x direction 50, initial velocity in Y direction -0.1 (to prevent singular value generated by calculation), and

initial velocity in Z direction 0. The target altitude is 1500m, latitude 39° and longitude 101°. The X-axis coordinates of the other aircraft in the launch system are set as the same as the first aircraft (that is, launch at the same altitude), and the coordinates of y and Z directions are set as unknown parameters to be searched in GOA-GDOP algorithm. The radar pitch and azimuth error are 0.2° and the range error is 0.3m. Constraints: Keep a distance of more than 1km between aircraft to prevent collisions in flight. The simulation software was Matlab2016a. Set the number of iterations for scenario 1, 2, and 3 to 30, and set the population to 50.



**Fig. 1** Flow chart of GOA-GDOP.

### 3.2 Scenario 1 The relationship between aircraft spacing and GDOP

Under the condition of cooperative detection by three aircraft, the upper bound of search range is changed, GDOP value is calculated, and the influence of different aircraft distances on positioning accuracy is analyzed.

**Table 1.** The relationship between aircraft spacing and GDOP.

Aircraft spacing	GDOP
2km	4.7947
3km	3.2764
4km	2.5103
5km	2.1168

It can be seen from the above table, the farther the distance between aircraft, the smaller the GDOP value and the higher the positioning accuracy.

### 3.3 Scenario 2 The relationship between the number of aircraft and GDOP.

Change the number of aircraft (radar), analyze the GDOP values of 3-5 aircraft, and analyze the influence of different numbers of aircraft on positioning accuracy. (Search boundary is set to 3km)

**Table 2.** The relationship between the number of aircraft and GDOP.

The number of aircraft	GDOP
3	3.2764
4	2.5386
5	2.3524

As can be seen from the above table, with the increase of the number of aircraft, the higher the observation accuracy of aircraft to the ground, the smaller the GDOP value.

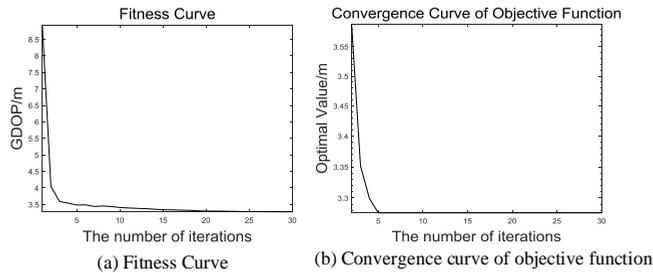
### 3.4 Scenario 3 The relationship between the aircraft configuration and GDOP

Under the condition of 3-5 aircraft (radars), GOA-GDOP method was used to analyze the configurations between aircraft and verify the optimal configuration conditions.( The maximum search distance is set to 3000m, and the coordinates of the first aircraft in the launch coordinate system is (0,0,0).)

#### 3.4.1 Three aircrafts

The coordinates of the other two aircrafts obtained by simulation are as follows: (0,3000, 0), (0,3000,3000).

Convergence curve of objective function and average fitness curve of population are shown as follows:

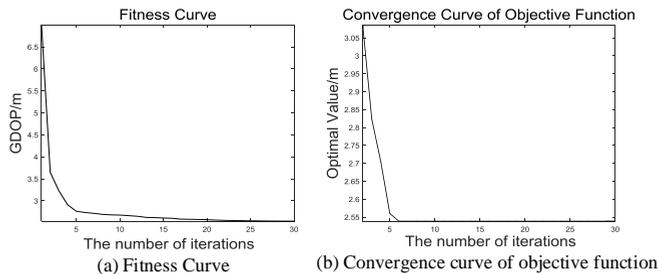


**Fig. 2.** Population mean fitness and target convergence curves -3 aircrafts.

#### 3.4.2 Four aircrafts

The coordinates of the other three aircrafts obtained by simulation are as follows: (0,3000, 0), (0,3000,3000), (0,0,3000).

Convergence curve of objective function and average fitness curve are shown as follows:

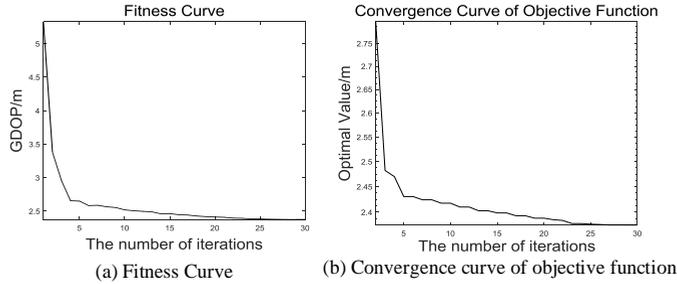


**Fig. 3.** Population mean fitness and target convergence curves -4 aircrafts.

### 3.4.3 Five aircrafts

The coordinates of the other four aircrafts obtained by simulation are as follows: (0, 3000, 0), (0, 3000, 3000), (0, 0, 3000), the coordinates of the fifth aircraft have randomness on the premise of satisfying constraint conditions.

Convergence curve of objective function and average fitness curve are shown as follows:



**Fig. 4.** Population mean fitness and target convergence curves -5 aircrafts.

It can be concluded from the above simulation that when three aircraft cooperate in detection, three aircraft form a right triangle relationship, that is, when the line area between three points is the largest, the detection accuracy is higher. When the four aircraft cooperate in the detection, the higher the detection accuracy is when the lines between the four points form a square (covering the entire search area). When five aircraft cooperate in the detection, the location selection of the fifth aircraft can be random because the area formed by four of them has occupied all the search space, which can ensure the optimal detection accuracy.

Considering the relationship between cost and accuracy, the cooperative detection accuracy of 5 aircraft is improved relatively slowly compared with that of 4 aircraft. Therefore, better positioning effect can be achieved when 3-4 aircraft carry out cooperative detection and maintain the above configuration.

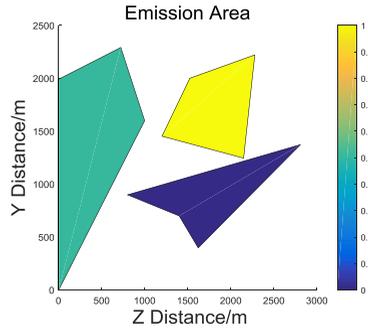
### 3.5 Simulation analysis under flight area limitation

In the launching process, it is difficult to meet the specific configuration requirements of the aircraft. Therefore, how to design the configuration under the condition of limited flight area is also an important research content.

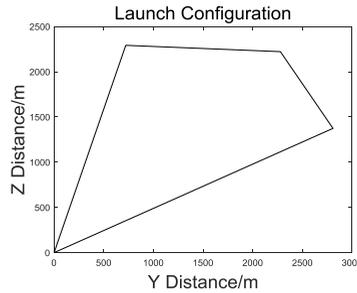
Therefore, based on the assumption that the launching points are in the same plane, this paper takes the range of 3 km as the reference and introduces random quantity to form an irregular polygon in the plane formed by the launching points of the aircraft, and uses GOA-GDOP algorithm to design and plan the configuration of the aircraft in the random plane. Simulation parameter Settings are the same as section 3.4.2. Taking the collaborative detection of four radars as an example, the number of iterations is 50, and the number of populations is 50. Each vehicle launch area is assumed to be a quadrilateral, and the coordinates of one of the vehicles are set to (0,0,0). The range of emission area formed by the other three random quadrilateral is shown in the figure below (position coordinates in the X direction are all set to 0):

According to the calculation, the coordinates of the four transmitting points are : (0,0,0),(2807.1926,0,0), (0,0,2291.6938), (2277.9821,0, 2221.9441) and the GDOP value is 3.7169.

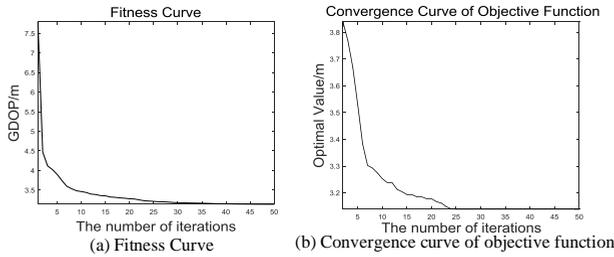
The launch configuration of the aircraft is shown in Fig. 6, and the convergence curve of average population fitness and objective function is shown in Fig. 7:



**Fig. 5.** Schematic diagram of aircraft launch area.



**Fig. 6.** Launch configuration.



**Fig. 7.** Population mean fitness and target convergence.

As can be seen from the simulation results, Figure 10(a) shows that with the increase of iteration number, the average fitness of the population gradually becomes stronger, and (b) shows the relationship between iteration number and convergence of objective function. Compared with the planning in regular configuration, the convergence speed in this simulation is relatively slow, because the configuration is irregular and the region and path searched by the population in the process of finding the optimal solution are relatively more complex, but the optimal solution of the objective function can still be found. When the total number of iterations is 50, the objective function can converge after about 25 iterations, so that the GDOP can reach the minimum value under the special configuration, and then the configuration planning and design can be completed. To sum up, when each aircraft is different in the launch range, that is, it cannot meet the requirements of specific configurations, the goA-GDOP algorithm can be used to quickly and effectively plan and design configurations between aircraft, and the average fitness and objective function of the population can achieve convergence well, so as to achieve the lowest GDOP value and the highest accuracy.

## 4 Conclusion

GOA-GDOP algorithm can effectively plan the configuration of aircraft cooperative detection. The analysis shows that when the space between aircraft is larger and the area formed by the connecting line between aircraft is larger, the detection accuracy is higher. Goa-gdop algorithm can still effectively and quickly plan the configuration relationship between aircraft under the condition of small number of locust population and number of iterations and limited launch area requirements. It can also meet the real-time requirements of structural programming.

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