Analysis of polarization characteristics of freeform surface optical system

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Abstract. Freeform surfaces are widely used in off-axis optical systems with large aperture, large field of view, and long focal length. The polarization effect caused by the non-rotationally symmetrical shape has an impact on the system's polarization imaging quality and measurement accuracy. Based on Jones' notation, this paper proposes a polarization aberration analysis method for fringed Zernike polynomial freeform optical systems and constructs a full-field polarisation aberration analysis model of non-rotationally symmetric freeform reflective optical systems. The light propagation vector $\dot{\boldsymbol{k}}$ is added on the basis of the two-dimensional ray tracing algorithm. By tracing the full-field polarized light of the off-axis optical system in the field of view, the Jones pupil diagram is obtained. The phase aberration, diattenuation and retardance are separated by Pauli decomposition and SVD decomposition. The off-axis freeform surface optical system is designed. The analysis results show that the phase aberration of the off-axis freeform surface optical system is directly related to the surface shape of the freeform surface. The changes of the freeform surface to the diattenuation and retardance are both 56% of the diattenuation and retardance of the sistem. For deep-space telescopes and lithography systems, it is of great significance for improving system accuracy to master this change.

Keywords: Polarization aberration, Jones matrix, Freeform surface, Polarization ray tracing.

1 Introduction

Polarization aberration represents the change in the polarization state of light as it passes through the optical system.Designers usually use free-form non-rotationally symmetrical optical systems to meet the design requirements of large aperture, large field of view, and long focal length, but at the same time, polarization aberrations are also

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introduced, which affects the imaging quality and measurement accuracy of the system[1].Therefore, grasping the mechanism of the free-form surface on the polarization aberration of the non-rotationally symmetric optical system is helpful to analyze and control the polarization aberration distribution of the optical system.

In 1987, Chipman proposed the theory of polarization aberration and decomposed diattenuation and retardance using the eigenvalues of the Jones matrix[2]. James P decomposed and expanded the polarization aberration function into a polarizationaberration matrix[3]. At present, there is no theoretical model method for the analysisof the polarization aberration characteristics of non-rotationally symmetric optical systems with freeform[4]. It is difficult for designers to predict the impact of polarization aberrations on the imaging quality of such optical systems. Therefore, the study of polarization aberration distribution characteristics of non-rotationally symmetric optical systems with freeform can not only further perfect the theoretical system of polarization aberration, but also provide important guidance for the application of freeform surfaces in non-rotationally symmetric polarization imaging systems.

2 Polarization aberration analysis method of freeform surface optical system

2.1 Jones matrices

This section focuses on the relative changes in the phase and amplitude of polarized light in optical systems. Compared with the Muller matrix, the Jones representation can more intuitively express the relationship between phase aberration, diattenuation, retardance, and freeform surfaces. Therefore, the polarization theory based on the Jones vector and Jones matrix is employed in this study [5].

A Jones matrix is typically used to describe the entire beam cross-section through the optical element. The polarization change of the optical system along different optical paths can be represented as a polarization aberration function $J(\boldsymbol{h},\boldsymbol{\rho},\lambda)$:

$$\boldsymbol{J}(\boldsymbol{h},\boldsymbol{\rho},\lambda)=\begin{pmatrix} j_{11}(\boldsymbol{h},\boldsymbol{\rho},\lambda) & j_{12}(\boldsymbol{h},\boldsymbol{\rho},\lambda) \\ j_{21}(\boldsymbol{h},\boldsymbol{\rho},\lambda) & j_{22}(\boldsymbol{h},\boldsymbol{\rho},\lambda) \end{pmatrix} \tag{1}$$

Where J is the Jones matrix related to the optical path, which is a function of the object coordinate $\boldsymbol{h}$, aperture coordinate $\boldsymbol{\rho}$, and wavelength λ. The polarization aberration function $\boldsymbol{J}(\boldsymbol{h},\boldsymbol{\rho},\lambda)$ comprehensively describes the polarization characteristics of the optical system, where the phase term $\phi_0(\boldsymbol{h},\boldsymbol{\rho},\lambda)$ is related to the wave aberration function $W(\boldsymbol{h},\boldsymbol{\rho},\lambda)$ of geometrical optics as follows[6]:

$$W(\boldsymbol{h},\boldsymbol{\rho},\lambda)=\frac{\lambda}{2\pi}\phi_0(\boldsymbol{h},\boldsymbol{\rho},\lambda) \tag{2}$$

The phase aberration is closely related to the optical path difference, which changes owing to the introduction of the freeform surface. Diattenuation and retardance are also related to factors such as the angle of incidence of light. Owing to the non-rotational symmetry of the freeform surface, the angle of incidence of light changes slightly.

Therefore, studyingthepolarization aberrations in non-rotationally symmetric optical systems with freeform is essentially an in-depth analysis and calculation of phase aberrations, diattenuation, and retardance.

2.2 Establishment of the freeform polarization aberration analysis method

The phenomenon of the difference in the angle of incidence of light at different points on the interface is caused by the multi-degree-of-freedom characteristic of the freeform surface. Theoretically, the traditional paraxial polarization aberration plays a role in analyzing the polarization of the coaxial system, but it is very difficult to directly analyze the polarization characteristics of the freeform surface non-rotationally symmetric system.In this paper, the freeform surface and the phase aberration expressed by fringe Zernike polynomials are combined. The diattenuation and retardance of the full field of view and the full aperture are analyzed through the correlation between the two, and then the overall influence of the Jones matrix of the system is analyzed.

2.2.1 Phase aberrations introduced by freeform surfaces

This paper uses the advantage of Zernike polynomials to fit full-field non-rotationally symmetric surfaces to represent freeform surfaces. According to formula (2), it is observed that there is a quantitative relationship between wave aberration and phase aberration. The multiple degrees of the shape of the freeform surface will affect the optical path difference of the system to varying degrees, thereby affecting the wave aberration. Therefore, the free-form surface expressed by fringe Zernike polynomial is related to phase aberration through wave aberration. The Zernike polynomial expression is vectorized, and the wave aberration theory is used to analyze the aberration produced by the Zernike polynomial free surface. In addition, we discovered the relationship between the Zernike free-form surface and the phase aberration. Therefore, the phase aberration analysis method of Zernike polynomial freeform surface is established. The specific process is as follows:

First, vectorize the Zernike polynomial originally expressed in polar coordinates. The higher-order oscillation part of the Zernike polynomial free-form surface can be expressed as

$$\vec{\boldsymbol{C}}_{\boldsymbol{x}/\boldsymbol{y}} = C_{x/y} e^{im\alpha_{x/y}} \tag{3}$$

where $C_{x/y}$ represents the magnitude of the coefficient vector, $\alpha_{x/y}$ represents the direction of the coefficient vector, and m represents the multiple of the azimuth angle ϕ of the Zernike polynomial. The specific relationship is as follows:

$$\begin{cases} \mathrm{C}_{x/y} = \sqrt{\mathrm{C}_x^2 + \mathrm{C}_y^2} \\ \alpha_{x/y} = \dfrac{\pi}{2} - \dfrac{1}{m}\arctan\left(\dfrac{C_y}{C_x}\right) \end{cases} \tag{4}$$

$\alpha_{x/y}$ has been modified. It is stipulated that $\alpha_{x/y}$ represents the angle of clockwise rotation from the +Y axis in the right-hand coordinate system.

According to the formula (2), when the freeform surface is far from the diaphragm position, the contribution of the phase aberration is

$$\phi_0(\boldsymbol{h},\boldsymbol{\rho},\lambda)=\frac{2\pi}{\lambda}\boldsymbol{\delta}_{x/y}(\boldsymbol{\rho})=\frac{2\pi}{\lambda}\cdot\frac{(n_2-n_1)}{\lambda}\vec{\boldsymbol{C}}_{x/y}\cdot Z(\boldsymbol{\rho}) \tag{5}$$

where n_1 and n_2 are the refractive indices of the medium in which the incident and outgoing rays are located, λ is the wavelength, and $Z(\boldsymbol{\rho})$ is the Zernike polynomial vectorized according to the formula $\boldsymbol{\rho}=\rho\begin{pmatrix}\sin(\phi)\\ \cos(\phi)\end{pmatrix}$.

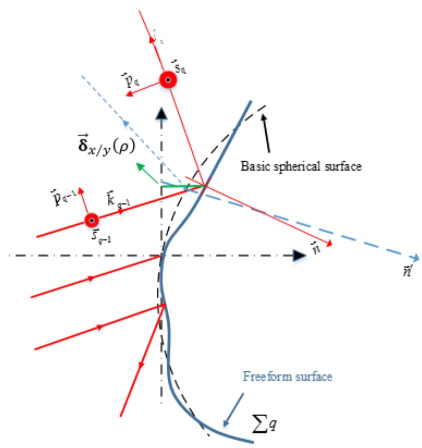

Fig. 1. Schematic diagram of polarized light refraction, reflection, and optical path difference on the qth plane.

It is worth noting that the wave aberration distribution of an optical system is closely related to the position of the diaphragm. Then the phase aberration distribution is also close to the diaphragm position. When the entrance pupil diameter is constant, and the diaphragm position is located on the spherical surface and away from the spherical surface, there is a pupil offset vector $\Delta\vec{\boldsymbol{h}}$ between the two regions of the off-axis field of view incident on the mirror through the diaphragm. The pupil offset vector will redistribute the phase aberration of the optical system, as shown in formula (6):

$$\phi_0(\boldsymbol{\rho})=\phi_0(\boldsymbol{\rho}'+\Delta\boldsymbol{h}) \tag{6}$$

When the freeform surface is far from the diaphragm position, the aperture of the light irradiated onto it is shifted. The aperture is shifted by vector $\Delta\boldsymbol{h}$. The phase aberration contribution of the freeform surface will correspondingly become

$$\phi_0(\boldsymbol{h},\boldsymbol{\rho},\lambda)=\frac{2\pi}{\lambda}\boldsymbol{\delta}_{x/y}(\boldsymbol{\rho})=\frac{2\pi}{\lambda}\boldsymbol{V}_{x/y}\cdot Z(\boldsymbol{\rho}+\Delta\boldsymbol{h}) \tag{7}$$

Equation (7) shows that although the Zernike polynomial is only related to the aperture, when it is far from the diaphragm position, its aberration contribution may also be related to the field of view.

2.2.2 Diattenuation and retardance introduced by freeform surfaces

The traditional polarization ray-tracing algorithm uses two-dimensional polarization ray tracing [7-9], which uses the Jones matrix to characterize the polarization properties of the optical interface [10-12]. Because the normal vector of each point on the freeform surface is different, the direction of the emitted ray corresponding to the light incident at different angles is also different. Adding the propagation vector $\boldsymbol{k}$ of the light to the traditional two-dimensional matrix can improve analysis of the polarization properties of optical systems containing freeform surfaces. In the following, based on the Zernike freeform surface expression method, a freeform surface polarization ray-tracing algorithm is constructed, and the influence of the freeform surface on the diattenuation and retardance is analyzed.

The Jones matrix added to the propagation vector $\boldsymbol{k}$ as a matrix $\boldsymbol{P_q}$ of 3×3. In the global coordinate system, the polarization transformation matrix $\boldsymbol{P_{total}}$ representing the polarization transformation of the optical system for the incident light is obtained by tracing the freeform polarized ray.

$$\boldsymbol{E_{out}} = \boldsymbol{P_{total}} \cdot \boldsymbol{E_{in}} = \prod_{q=1}^{Q} \boldsymbol{P_q} \cdot \boldsymbol{E_{in}} \tag{8}$$

In (8), $\boldsymbol{E_{in}}$ and $\boldsymbol{E_{out}}$ represent the Jones vectors of the incident and emerging light beams, and the matrix $\boldsymbol{P_q}$ represents the change in the polarization state of the incident beam at the *q*th interface, as shown in Fig. 4. The relationship between $\boldsymbol{P_q}$ and $\boldsymbol{k}$, $\boldsymbol{s}$ and $\boldsymbol{p}$ components of the light beam at the optical interface is

$$\boldsymbol{P_q} = \begin{pmatrix} s_{x,q} & p_{x,q} & k_{x,q} \\ s_{y,q} & p_{y,q} & k_{y,q} \\ s_{z,q} & p_{z,q} & k_{z,q} \end{pmatrix} \begin{pmatrix} a_{s,q} & 0 & 0 \\ 0 & a_{p,q} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_{x,q\text{-}1} & s_{y,q\text{-}1} & s_{z,q\text{-}1} \\ p_{x,q\text{-}1} & p_{y,q\text{-}1} & p_{z,q\text{-}1} \\ k_{x,q-1} & k_{y,q-1} & k_{z,q-1} \end{pmatrix} \tag{9}$$

Where $a_{s,q}$ and $a_{p,q}$ are the amplitude transmission (reflection) coefficients of the $\boldsymbol{s}$ and $\boldsymbol{p}$ components of the qth interface, respectively. $s_{m,q\text{-}1}$, $p_{m,q\text{-}1}$, and $k_{m,q\text{-}1}$ $(m = x, y, z)$ represent the coordinates of the $\boldsymbol{k}$, $\boldsymbol{s}$ and $\boldsymbol{p}$ components of the incident light in the global coordinate system. $(m = x, y, z)$ represent the coordinates of the $\boldsymbol{k}$, $\boldsymbol{s}$ and $\boldsymbol{p}$ components of the emerging light in the global coordinate system.

Employing the freeform surface polarization ray-tracing algorithm for the incident beam in a specific direction, the polarization conversion matrix $\boldsymbol{P_{total}}$ of the beam can be calculated for each interface of the system. The singular value decomposition of $\boldsymbol{P}_{total}$ can be performed as follows.

$$\boldsymbol{P}_{total} = \boldsymbol{UDV}^{\dagger} = \begin{bmatrix} k_{x,Q} & u_{x,1} & u_{x,2} \\ k_{y,Q} & u_{y,1} & u_{y,2} \\ k_{z,Q} & u_{z,1} & u_{z,2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \Lambda_1 & 0 \\ 0 & 0 & \Lambda_2 \end{bmatrix} \begin{bmatrix} k_{x,0} & k_{y,0} & k_{z,0} \\ v_{x,1} & v_{y,1} & v_{z,1} \\ v_{x,2} & v_{y,2} & v_{z,2} \end{bmatrix} \quad (10)$$

The polarization transformation matrix $\boldsymbol{P}_{total}$ is decomposed into two unitary matrices $\boldsymbol{U}$, $\boldsymbol{V}$ and a diagonal matrix $\boldsymbol{D}$. Λ_1, Λ_2 ($\Lambda_1 \geq \Lambda_2$) contained in the diagonal matrix $\boldsymbol{D}$ are the eigenvalues of the polarization transformation matrix $\boldsymbol{P}_{total}$. $\boldsymbol{k_0}$, $\boldsymbol{k_Q}$ in the matrix correspond to the propagation direction of the incident light and the propagation direction of the emerging light after Q refraction and reflection. $\boldsymbol{v_1}$ $\boldsymbol{v_2}$ and $\boldsymbol{u_1}$ $\boldsymbol{u_2}$ correspond to the eigenpolarization states in the entrance pupil and the exit pupil of the optical system, respectively, and they satisfy the following relationship:

$$\mathrm{P_{total}} \cdot \mathrm{v_1} = \Lambda_1 \mathrm{u_1}, \mathrm{P_{total}} \cdot \mathrm{v_2} = \Lambda_2 \mathrm{u_2}, \mathrm{P_{total}} \cdot \mathrm{k_0} = \mathrm{k_Q} \quad (11)$$

According to the definition of diattenuation and retardance[13-14], The magnitude and direction of the diattenuation and retardance of the optical system can be calculated.

3 Design and polarization characteristics of the off-axis three-refelection system of field of view

In this section, a large field-of-view off-axis triple mirror optical system is designed and its polarization characteristics are analyzed. The above theory is verified.

The steps are as follows: First, establish a connection between Matlab and Zemax, transfer the system parameters obtained in Zemax to Matlab, and divide the aperture and field of view into several equal parts. Secondly, select a certain field of view point, and perform polarization ray tracing on several sampling points of the pupil under the field of view to obtain the Jones pupil diagram and the three-dimensional polarization transmission matrix under the field of view. Third, decompose the singular value of the matrix to obtain the attenuation and delay of the system in the field of view; at the same time, convert the matrix to Jones matrix and perform Pauli decomposition to obtain the phase aberration of the system in the field of view . Fourth, switch to other fields of view and perform the above operations. Finally, the polarization characteristics of the full field of view and full aperture of the optical system are obtained and analyzed.

Taking into account the need to reflect the influence of the freeform surface shape on the polarization aberration, when the optical design is carried out, each element is in a state of no eccentricity and no tilt. In order to eliminate the obstruction of the center of the system, an off-axis view field is adopted. The optical specifications of the system are shown in Table 1.

The initial structure of the optical system selects three mirrors, all of which are quadric surfaces. The edge field of view imaging is selected in the design, which can effectively eliminate the central occlusion and primary stray light, but at the same time it also brings

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about theproplem of the degradation of imaging quality and the increase of polarization aberration. A freeform surface will be introduced on the primary mirror to balance the aberration levels in each field of view. The initial structure is a Y-axis symmetric system. The off-axis direction of the field of view should be the Y-axis direction. At the same time, the selection of freeform surfaces should also consider the issue of axis symmetry.Therefore, we selected the freeform surface Z_5 term to modulate the system. The theoretical analysis in the previous section shows that the surface shape of the freeform surface has a certain effect on the size and distribution of the polarization aberration. Considering that the field of view is 20°×2°, the central field of view is selected as the Y-axis offset 8 °. After optimization, the modulation transfer function (MTF) of each field of view of the system is greater than 0.6 at 38.5 lp/mm.

Table 1. Main optical indicators.

Specification	Parameters
Focal length	850 mm
F number	6.5
Field of view	20°*2°
Distortion	<0.5%
Pixel size	13um

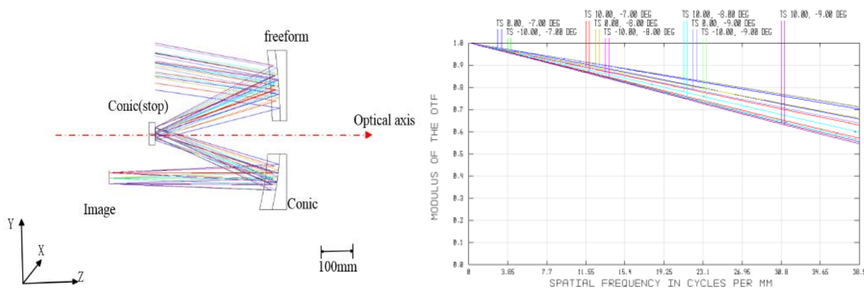

Fig. 2. Optical system diagram and MTF diagram.

The full-field and full-aperture freeform surface polarization ray tracing of the off-axis three-reflection system can calculate the overall diattenuation (Figure 3) and retardance (Figure 5) distribution of the system. At the same time, the distribution of diattenuation (Figure 4) and retardance (Figure 6) caused by the freeform surface are calculated. The result is consistent with the previous theoretical derivation, and the values of the diattenuation and retardance increase with the increase of the field of view. The polarization aberration caused by the freeform surface accounts for 56% of the polarization aberration of the entire system.

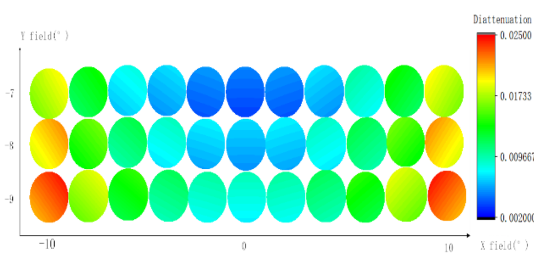

Fig. 3. Diattenuation pupil in full field of view.

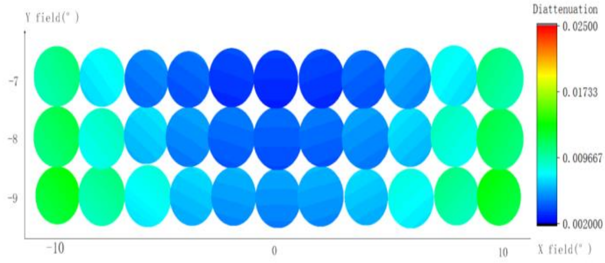

Fig. 4. Diattenuation pupil of freeform surface in full field of view.

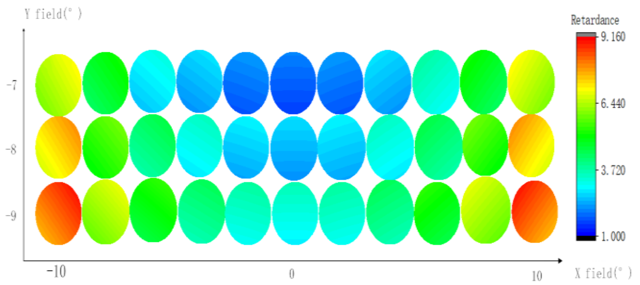

Fig. 5. Retardance pupil in full field of view.

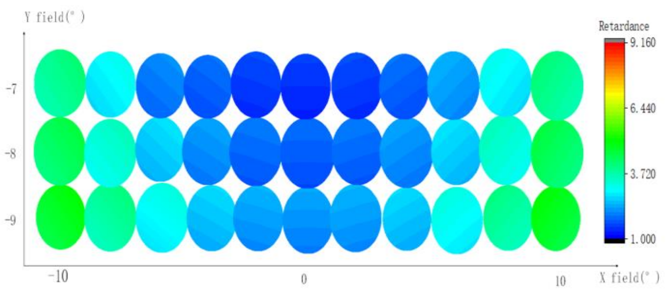

Fig. 6. Retardance pupil of freeform surface in full field of view.

4 Conclusions

Based on Jones notation, this paper proposes a freeform surface optical system polarization aberration analysis method, and constructs a polarization aberration analysis model of the off-axis free-form surface optical system in the field of view. Based on the above theory, the Zernike coefficient was selected in a targeted manner, and an off-axis optical system with free-form surface field of view was designed. The design results show that the introduction of free-form surfaces not only affects the wave aberration, but also the freeform surface has a great influence on the distribution of the overall polarization aberration of the system. The polarization aberration caused by the free-form surface accounts for 56% of the polarization aberration of the entire system. The system has a 20°×2° field of view, and the modulation transfer function (MTF) is greater than 0.6 at 38.5 lp/mm. Meet the design requirements.

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