Research on the construction of the referral networks of city hospitals and invulnerability in response to major public health emergency

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Abstract. In face of major public health emergencies, how to ensure the orderly and stable operation of city hospitals? Based on the theory and method of complex network, in this paper, we put forward the construction method of city hospital referral networks, analyze the static characteristics of the constructed network, discuss the invulnerability of networks with four attacking modes, and propose two ways (or integrated) to optimize the invulnerability of networks, which are: (i) Identifying and protecting key hospitals that can increase network invulnerability, (ii) Adding hospitals to the network. Taking hospitals of Wuhan as an example and using the proposed construction method for networks, in this paper, a directed-referral network I with 219 major hospitals in Wuhan is constructed. On the basis of network I, 16 mobile cabin hospitals, Huoshenshan hospital and Leishenshan hospital have been added, the referral-hospital network II of Wuhan is achieved. Compared with network I, network II has better referral ability and invulnerability.

Keywords: Major public health emergency, Referral networks of city hospitals, Complex network, Static feature quantity, Invulnerability.

1 Introduction

Dealing with major public health emergencies is a very large social system engineering, involving the coordinated development of people, the public health system, and the medical service system[1]. In 2020, Li Deren et al. based on the big data of temporal and spatial position stated that considering the hospital facility system of a region as a whole is the top priority in dealing with major public health emergencies[2]. In recent years, a series of research results have been achieved on the layout of city hospitals and hospital facilities[3-5].

Complex networks are closely related to our daily lives[6]. In a complex network, nodes are related to each other. Once these functional nodes are attacked by external or internal attacks, they may cause the rest of the system to malfunction and affect the stability of the network[7-8]. For the city hospital system, whether it is within a hospital or between hospitals, it can be regarded as a very complex network. In 2020, Jennifer I Lather et al.

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proposed to use graph theory to implement and evaluate layout options for hospital layout problems, and proposed a building information modeling (BIM) algorithm to obtain the optimal layout[9]. In 2020, MA Miranda et al. studied the efficiency of hospital operation and management from the dynamics of patient flow based on complex network theory, which can well evaluate the efficiency of hospital services and improve process performance[10]. However, the above two documents are only based on the complex network theory to study the various departments within the hospital, and solve the related problems of the layout of the various departments within the hospital. They do not analyze the rationality of the entire city hospital system as a whole. So far, in response to major public health emergencies, there have been few studies on the layout and location of city hospitals based on complex network methods, and the rational evaluation and management of hospital facility systems.

This article will use the complex network method to study the construction of the directed referral network of city hospitals and the invulnerability of the network, analyze the robustness and fragility of the network structure, and make the referral network system of city hospitals reasonable under major public health emergencies. Build to provide reference. The main contributions of this paper are as follows: (i) From the perspective of complex network theory and methodology, research on the construction method of city hospital referral network; (ii) Discussed how to analyze the statistics and invulnerability of the newly constructed network, and discussed how to optimize and improve the invulnerability of the network; (iii) Taking Wuhan hospital as an example, combining the main hospitals in Wuhan with mobile cabin hospitals, Huoshenshan and Leishenshan hospitals, constructing two types of networks according to the construction method proposed in this article, and comparing and analyzing the static status of the two types of networks statistics and invulnerability.

2 Construction of city hospitals referral network

2.1 Construction method of city hospitals referral network

According to the medical encyclopedia website, hospitals can be divided into one, two, and three levels, and each level can be divided into three levels: A, B, and C. The national health and construction commission[11] and the "National Medical and Health Service System Planning Outline (2015-2020)"[12] specify that the service radius of city hospitals is 10-15 km. This article considers that the service radius of the hospital is 10 km. Of course, depending on the size of the city, the service radius can be changed according to the actual situation. The data in this article is based on the planned driving distance of the Gaode Map[13], that is, the actual distance from a hospital to another hospital. When patients need to be referred between hospitals, it is reasonable to use the distance between hospitals as the basis for referral.

Suppose $G=(V,E)$ represents the referral network of city hospitals, where V is the node set and E is the edge set. The elements in V are various hospitals, and $|V|=N$. The elements in the E edge set indicate whether there is a referral relationship between hospitals. In constructing the referral network G of city hospitals, this article is mainly based on the following two construction principles.

Principle 1: According to the planned driving distance between any two hospitals, if the distance is greater than 10km, no edge will be generated between the two hospitals; if the distance is less than or equal to 10 km, an edge may be generated.

Principle 2: Under the premise that patients can accept, it is stipulated that low-level hospitals can refer to high-level hospitals, but not vice versa; no referral between hospitals of the same level.

Based on the above two principles, any pair of hospitals in the pair will be connected in the V . Let a_{ij} denote the referral relationship between any two hospitals $v_i \in V$ and $v_j \in V$ in the network G . If hospital v_i and hospital v_j conform to the referral relationship, there is a connection between hospital v_i and hospital v_j , and $a_{ij} = 1$; otherwise, $a_{ij} = 0$.

Let $A = \left(a_{ij}\right)_{N \times N}$ and $a_{ii} = 0$, then A reflects the topology of network G . As a result, the construction of the city hospital directed referral network G is completed.

2.2 Construction of Wuhan hospital referral network and analysis of its characteristics

2.2.1 Construction of Wuhan hospital referral network

This section will take Wuhan hospitals as an example to discuss how to build a directed referral network in Wuhan hospitals. Through the Gaode Map, this paper obtains the name and geographic location data of 219 major hospitals currently within the scope of Wuhan City[13]. The 219 main hospitals are the main general hospitals and community hospitals in Wuhan. According to the construction method in the previous section, the Wuhan hospital referral network I is obtained, which contains 219 nodes and 3042 edges. This network is a directed and unauthorised network, as shown in Figure 1. For convenience, each major hospital in Figure 1 is represented by a number from 1 to 219; the arrow in the figure indicates that it can be referred. The different colors of the nodes in network I represent the 13 different administrative regions of Wuhan. For example, the green nodes in Figure 1 represent hospitals in Wuchang District.

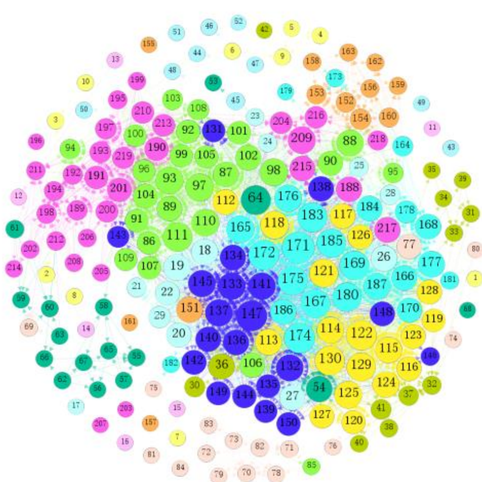
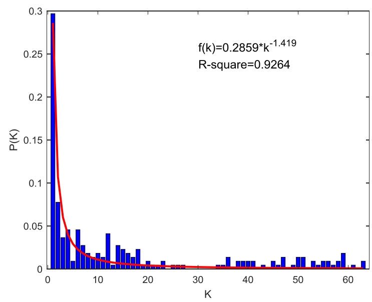


Fig. 1. Hospital referral network I of Wuhan.

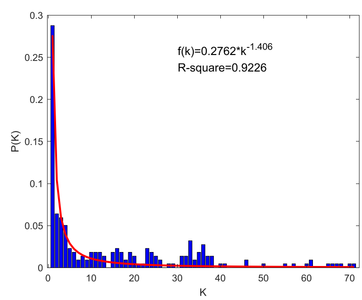
2.2.2 Characteristic analysis of network I

(1) Node degree and degree distribution fitting

The greater the degree value of a node, the more important the node is. By analyzing the degree (including in-degree and out-degree) of referral network I, we can see that 18 nodes in network I are isolated nodes with a degree value of 0. The isolated node corresponds to the hospital with the lowest level or the hospital with a higher level, but is far away from other hospitals, and there is no connection in the network. Patients at these hospital nodes will have more problems when they are referred. The average degree of nodes in network I is about 27.78, which means that each node in the referral network of Wuhan hospitals is connected to it by an average of about 28 edges, that is, patients can be referred between 28 hospitals on average when they are being referred. Nodes with higher than average degrees account for more than 50%, that is, more than half of the hospitals can guarantee that there are more than 28 hospitals connected to it.



(a) In-degree distribution



(b) Out-degree distribution

Fig. 2. Degree distribution for network I.

The top 5 hospitals in the order of entry are all top-tier hospitals in Wuhan, indicating that they play a very important role in the referral network; The top 5 hospitals with the highest outpatient rates are all first-class hospitals in Wuhan. This finding is related to the method of network construction, which verifies that low-level hospitals can be referred to high-level hospitals, and high-level hospitals with high accessibility must have the highest admission rate.

Figure 2 describes the in-degree and out-degree distribution of network I and the fitting curve. The blue histogram in the figure represents the probability of degree k. They show that the out-degree or incoming degree of network I nodes are widely distributed, and most hospitals are connected, and they can provide suitable hospitals when patients need to be referred. The in-degree and out-degree distributions of the nodes of network I are fitted, and the fitting graph is as shown in the red curve in the figure. The fitting function can be expressed as:

$$\mathrm{f}(k) = 0.2859 \, * k^{-1.419}, \quad \mathrm{f}(k) = 0.2762 * k^{-1.406}$$

The results of degree distribution fitting are shown in Table 1. Table 1 shows that the mean square error of the in-degree and out-degree distribution fitting curves of network I is close to 1. Therefore, we believe that the in-degree and out-degree distributions of network I are approximately obeying the power-law distribution, and both have scale-free characteristics.

Table 1. The fitting results of degree distribution (95% confidence level).

Network Type	SSE	RMSE	R-square	Adjusted R-square
In-degree of network I	0.006772	0.01054	0.9264	0.9252
Out-degree of network I	0.006773	0.009908	0.9226	0.9215

(2) Average path length, clustering coefficient

Calculated that the average path length of network I is $L = 1.605$ can be obtained. It shows that under the condition of accessibility, starting from any hospital in network I, an average of 1.605 hospitals can reach any other hospital in network I. It can be seen that the average clustering coefficient of network I is $C = 0.2396$. At this time, the average clustering coefficient is relatively low, and emergencies have little impact on the stability of the network.

3 Invulnerability analysis and optimization of the referral network of city hospitals

3.1 Invulnerability analysis of referral network in city hospitals

When major public health emergencies occurs, the referral network of city hospitals will be attacked. How to ensure the orderly operation of city hospitals or how to ensure the invulnerability of the network is particularly important. Generally, attacks on the network are divided into attacks on nodes and attacks on connected edges. When major public health emergencies occurs, such as the outbreak of the new crown pneumonia, some hospitals will influx a large number of patients in a short period of time, which will paralyze these hospitals and cause a fatal blow to the referral network of the entire city's hospitals. This situation is similar to attacking a node in a complex network. Therefore, this article only considers attacks on nodes, and does not consider attacks on connected edges. This section selects four types of attacks on network nodes: (i) random attacking; (ii) attacking according to degree; (iii) attacking according to in-degree; (iv) attacking according to out-degree. In the four attack methods, the attacked nodes are deleted at a rate of 2% in each step. These attack methods can simulate situations where random nodes or key nodes are destroyed due to major public health emergencies.

Since the total number of network nodes will be changed after attacking the node, it is convenient for comparison. In this section, the relative size of the largest strongly connected subgraph is selected as the network invulnerability index. The formula for the relative size of the largest strongly connected subgraph is as follows:

$$S = \frac{|V_d|}{|V_D|}, \tag{1}$$

Among them, $|V_d|$ is the scale of the largest connected subgraph when nodes are deleted proportionally, and $|V_D|$ is the scale of the largest connected subgraph in the original network.

Based on the above four node attack methods, in order to explore the impact of the outbreak of the new crown epidemic on the layout of Wuhan hospitals, on the basis of the network I constructed in section 2.2, the following is an analysis of the invulnerability of the network I.

Using R software, Figure 3 shows the change in the relative size of the largest connected subgraph in network I with the proportion of deleted nodes, where the abscissa R in the figure represents the proportion of deleted nodes.Figure 3 shows that: for network I, random attacking cause the network's invulnerability to decline slowly, and the rate of decline is relatively uniform; with the increase in the deletion ratio, deliberate attacks decrease faster than random attacking. In particular, when about 40% to 50% of the nodes are deleted, network I is deliberately attacked, and the relative size of the largest connectivity subgraph has a cliff-like change; and as the proportion of attacks continues to increase, the network soon approaches collapse.

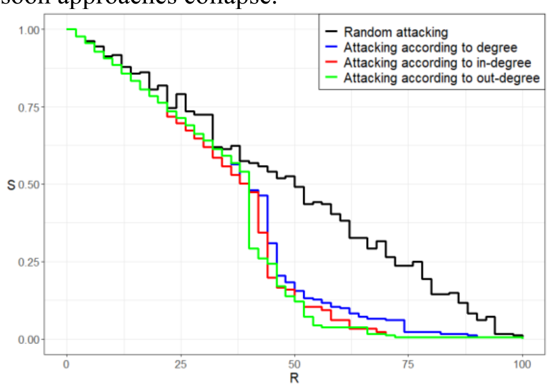


Fig. 3. The relative results of maximum connected subgraph in network I.

3.2 Optimization of network invulnerability

In complex network analysis, the invulnerability of the network is a hot issue to ensure network security. Especially when there is major public health emergency, how to ensure the stability of the referral network of city hospitals and how to optimize and improve the invulnerability of the network has important practical significance. In the previous section, we found that if an attack is carried out on network I, random attacking, the network's anti-destroy performance is better. If it is the other three attack methods, we need to identify key nodes that can increase the invulnerability of the network before the outbreak occurs, and improve the invulnerability of the network. For example, when ranking attacks according to in-degree, out-degree, and degree, about 40% of the hospitals in the first order need to formulate relevant policies for key protection.

The above is to attack the nodes of the network, that is, to remove the hospital. In order to optimize the anti-destroy performance of the city hospital network, in addition to the above method of finding and protecting key hospitals in the network, we can also add hospitals in a certain city, that is, increase the nodes of the network, and formulate special referral policies to achieve The purpose of optimizing network resilience.

As we all know, after the outbreak of the new crown epidemic, the state added 16 shelter hospitals, Huoshenshan and Leishenshan hospitals in Wuhan[14]. Still taking Wuhan hospital as an example, according to the network construction method and construction principle 3 in section 3.1, the following analyzes the anti-destroy performance of the Wuhan hospital referral network after adding a hospital from the perspective of a complex network.

Principle 3: 219 major hospitals can refer to the corresponding 16 mobile cabin hospitals in the administrative area; 219 major hospitals and 16 mobile cabin hospitals can respectively refer to Huoshenshan and Leishenshan hospitals; Huoshenshan and Leishenshan hospital does not refer each other.

Using the construction method and principle 3 in section 2.1, Figure 8 is the referral network II for hospitals with and without rights. There are 237 nodes and 3783 edges in network II. For convenience, each major hospital in Figure 4 is still numbered 1 to 219, while mobile cabin hospitals are represented by names; the arrows in the figure indicate that they can be referred. Similar to Figure 1, the node numbers represent 13 different administrative regions in Wuhan. The red color represents mobile cabin hospital and Huoshenshan Leishenshan hospital. 16 mobile cabin hospitals are represented by A1-A16, Huoshenshan is represented by H, and Leishenshan hospital is represented by L.

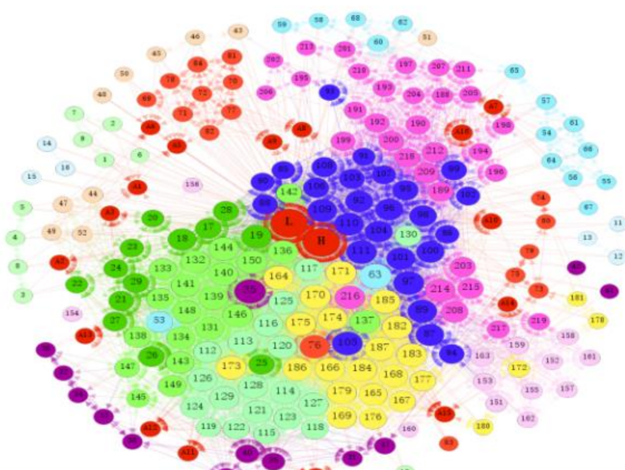
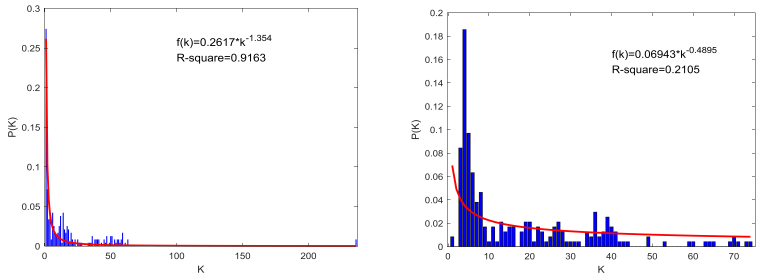


Fig. 4. Hospital referral network II of Wuhan.

3.2.1 Characteristic analysis of network II

(1) Node degree and degree distribution fitting

The average degree of network II nodes is about 31.92, indicating that patients can be referred between 32 hospitals on average when they are referred. Nodes with higher than average degrees account for more than 50%, that is, more than half of the hospitals can guarantee that there are more than 32 hospitals connected to it. Table 3 shows the list of the top 5 hospitals in Wuhan Hospital Referral Network II in terms of in-degree and out-degree. The top 5 hospitals in the ranking of Network II, except for Huoshenshan and Leishenshan hospitals, are all hospitals in Wuhan with the grade three, and the top 5 hospitals with the highest out-degree are all with the Wuhan grade. A first-level hospital.



(c) In-degree distribution **(d)** Out-degree distribution

Fig. 5. Degree distribution for network II.

The distribution of in-degree and out-degree of network II and the fitting curve (see the red curve) are shown in Figure 5. The degree distribution fitting function can be expressed as:

$$\mathrm{f}(k) = 0.2617 * k^{-1.354}, \quad \mathrm{f}(k) = 0.06943 * k^{-0.4895}.$$

The results of degree distribution fitting are shown in Table 2. Table 2 shows that the mean square error of the in-degree distribution fitting curve of Network II is close to 1. Therefore, we believe that the in-degree distribution of Network II approximately obeys the power-law distribution and has scale-free characteristics, while the out-degree distribution of Network II is not. Has this characteristic.

Table 2. The fitting results of degree distribution (95% confidence level).

Network Type	SSE	RMSE	R-square	Adjusted R-square
In-degree of network II	0.007567	0.005687	0.9163	0.916
Out-degree of network II	0.0413	0.02395	0.2105	0.1995

(2) Average path length, clustering coefficient

It can be seen that L=1.359 of network II. The average path length is smaller than the average path length of network I, mainly because of the existence of Huoshenshan and Leishenshan hospitals and 16 other mobile cabin hospitals. It can be seen that the average clustering coefficient of network II is $C = 0.5654$, which indicates that network II is more convenient for patient transfer than network I in the face of emergencies. This conclusion verifies the principle of building the network on the side, that is, the addition of mobile cabin hospital, Huoshenshan hospital and Leishenshan hospital facilitates the referral between hospitals.

3.2.2 Invulnerability analysis of network II

In order to analyze the invulnerability of Network II in more detail, the following is a detailed comparison of the four attack methods of Network I and Network II (Figure 6 to Figure 9). Figure 6, Figure 7, and Figure 8 show that whether it is a random attacking, or attacking according to degree and in-degree, as the deletion ratio increases, the invulnerability of the two networks is almost the same. In Figure 7 and Figure 8, the original red line obviously fell faster than the blue line, mainly because the Huoshenshan and Leishenshan hospitals were deleted.

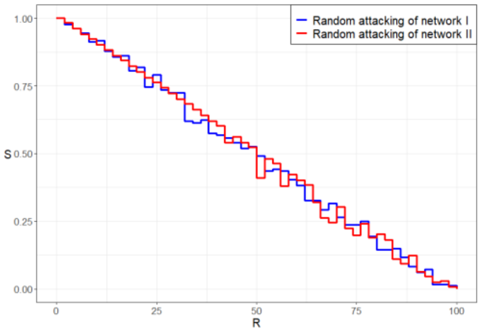


Fig. 6. The relative results of maximum connected subgraph(random attacking).

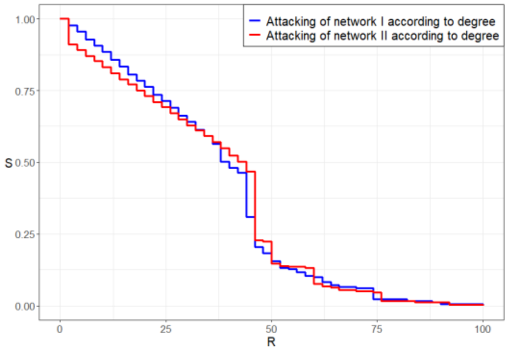


Fig. 7. The relative results of maximum connected subgraph (attacking according to degree).

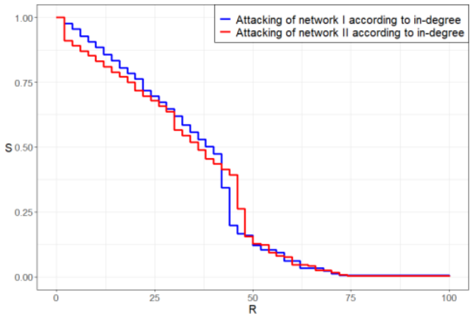


Fig. 8. The relative results of maximum connected subgraph (attacking according to in-degree).

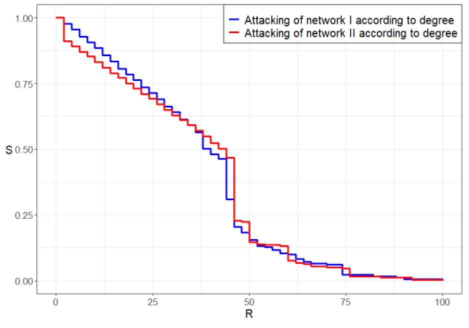


Fig. 9. The relative results of maximum connected subgraph (attacking according to out-degree).

Figure 9 shows that the proportion of the number of nodes deleted according to the out-degree is less than about 40%, and the invulnerability of network I and network II is similar. However, when the percentage of deletion exceeds about 40%, the invulnerability of Network I declines sharply. At this time, approximately 88 hospitals have been deleted, of which nearly 17 are Grade a hospitals. And at about 70%, network I has collapsed, while the invulnerability of network II has been declining almost uniformly. The main reason for this situation is the existence of mobile cabin hospital, Huoshenshan and Leishenshan hospitals, especially because Huoshenshan and Leishenshan hospitals have a 0 out-degree.

Therefore, with the addition of mobile cabin hospital, Huoshenshan and Leishenshan hospitals, Network II showed better stability according to the maximum degree of attack or random attacking. In addition, in comparison, the performance of the two networks is similar in random attacking, attacking according to degree or in-degree; however, in accordance with the attacking according to out-degree, Network II is more stable because of the existence of mobile cabin hospitals.

4 Conclusion

Sudden major public health emergencies will have a great impact on society. Based on the complex network theory, this paper studies the construction method of the referral network of city hospitals, and discusses how to analyze and optimize the invulnerability of the network when major public health emergencies occurs. Combining with the new crown pneumonia epidemic, this article takes Wuhan hospitals as an example, constructs a directed referral network I of Wuhan hospitals, and discusses the static statistics and invulnerability analysis of the network I. In addition to protecting key hospitals, this article also discusses the method of adding network nodes (hospitals) to improve the stability of the network. In general, when cities face major public health emergencies, they can study and build a three-dimensional, better-performing city hospital referral system from the following aspects:(i) Analyze and construct a directed referral network in city hospitals from the perspective of complex network theory and methods; (ii) In the event of major public health emergencies, for the constructed network, identify the key hospitals in the network according to the degree, the out-degree, and the in-degree, and provide more protection and effective prevention and control to ensure the stability of the entire network sex; (iii) In response to sudden major public health emergencies, according to the actual situation, add a batch of standby wartime hospitals and dual-use public buildings for peace and war, so as to optimize the invulnerability of the network.

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