The relaxed asymmetric HSS-like iteration algorithms for a class of weakly nonlinear complementarity problems

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Abstract. The problem of finding solutions to a class of weakly nonlinear complementarity problems is studied. And then, the authors present the relaxed asymmetric HSS-like iteration algorithm, the asymmetric HSS-like iteration algorithm and the HSS-like iteration algorithm for a class of weakly nonlinear complementarity problems. Under suitable conditions, they establish the convergence theory of the algorithms.

Keywords: Weakly nolinear complementarity problem, Matrix multi-splitting, Hermitian and skew-Hermitian splitting, Relaxed asymmetric HSS-like iteration, Convergence.

1 Introduction

This paper focuses on a class of weakly nonlinear complementarity problems, which is to find a pair of real vectors $r = Az + f(z)$ and $z \in R^n$ such that

$$\begin{cases} z \geq 0 \\ Az + f(z) \geq 0 \\ z^T(Az + f(z)) = 0 \end{cases} \tag{1}$$

Where $A \in R^{n \times n}$ and $q \in R^n$are given real matrix and vector, respectively, $F: R^n \to R^n$ is a given nonlinear mapping, usually arises from many scientific computing and engineering applications, for instance, the network equilibrium problem, the contact problem, image processing [1,2] and the free boundary problem [3]. This kind of problems usually come from the numerical solution of some variational inequality problems with nonlinear source term. If $F(z)$ is an affine function, the nonlinear complementarity problem (1) will reduce to the linear complementarity problem. If the linear part Ax is stronger than the nonlinear part $F(z)$ under a certain norm definition, equation (1) is called a weakly nonlinear complementarity problem. We denote its solution by z^*.

Recently, a Hermitian and skew-Hermitian splitting [5-7] has drawn the authors' attention, which is $A = H + S$.Where $H = \frac{1}{2}(A + A^*), S = \frac{1}{2}(A - A^*), A^*$ is the conjugate

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transpose matrix of the matrix A.

Tang[7] generalized the asymmetric Hermitian and skew-Hermitian splitting iteration algorithm, present nonlinear relaxed asymmetric HSS-like iteration method for a class of weakly nonlinear systems:$A(z) = F(z)$:

Given an initial guess $z^{(0)}$, for $k = 0,1,2,\cdots$, until $z^{(k)}$ converges, compute

Step 1. Initialization. Let $z^{(0)} \in R^n$ be any given initial value,set $k = 0$.

Step 2. General iteration.

$$\begin{cases} (\alpha I + H)z^{(k,1)} = (\alpha I - S)z^{(k)} + F\left(z^{(k)}\right), \\ (\beta I + S)z^{(k,2)} = (\beta I - H)z^{(k,1)} + F(z^{(k,1)}). \end{cases} \tag{2}$$

where α is a given nonnegative constant and β is a given positive constant, and H is the Hermitian part of A, S the skew-Hermitian part.

Step 3. Relaxation iteration. Let $z^{(k+1)} = \omega z^{(k,2)} + (1-\omega)z^{(k,1)}$, where $\omega \geq 0$ is a given real the relaxation factor.

Step 4. Test for termination. If z^{k+1} satisfies a prescribed stopping rule, terminate. Otherwise, return to Step 2 with k replaced *by* $k + 1$.

In this paper, based on the HS splitting, we constructed the relaxed asymmetric HSS-like iteration algorithms for a class of weakly nonlinear complementarity problems. This paper is organized as follows. In the next section, we present some necessary notation and preliminary results. In Section 3 we describe the relaxed asymmetric HSS-like iteration algorithm. In Section 4, we analyze the convergence properties of the iteration algorithm under suitable conditions.

2 Preliminary

In this section we review some know results needed in section 4. To formulate them we begin with some basic notation used throughout the remaining part of this paper.

Definition 2.1: A nonsingular matrix $A = (a_{ij}) \in R^{n\times n}$ is termed as an M-matrix if $a_{ij} \leq 0$ for $i \neq j$ and $A^{-1} \geq 0$. Its comparison matrix $< A >= (b_{ij}) \in R^{n\times n}$ is defined by

$$b_{ij} = \begin{cases} |a_{ij}|, i = j \\ -|a_{ij}|, i \neq j \end{cases} (i,j = 1,2,\cdots n)$$

A is said to be an H-matrix if its comparison matrix $< A >$ is an M-matrix.

It is well known that if $A = D - B$ is an H-matrix with $D = diag(A)$, then $\rho(|D|^{-1}|B|) < 1$. It was shown in[8] that if A is an H-matrix and $A = M - N$ is an H-compatible splitting, then M is also an H-matrix and $\rho(|D|^{-1}|B|) < 1$. Note that M-matrices and strictly or irreducibly diagonally dominant matrices are contained in the class of all H-matrices.

Lemma 2.1[8] :Let $A \in R^{n\times n}$ be an H-matrix, $D = diag(A)$,and $A = D - B$. Then

(1) A is nonsingular;

(2) $|A^{-1}| \leq < A >^{-1}$;

(3) $|D|$ is nonsingular and $\rho(|D|^{-1}|B|) < 1$.

Lemma 2.2[8]: Let $A, B \in R^{n\times n}$, $|A| \leq B$. Then $\rho(A) \leq \rho(B)$ ($\rho(A)$ represents the spectral radius of matrix A).

Lemma 2.3[13] :Let A be an H-matrix with positivediagonal elements. The linear part is stronger than the nonlinear part under a certain norm definition, $x, y \in R^n, \exists\lambda > 0$,satisfies:$F(x) - F(y) \geq \lambda(x - y)$. Then, the nonlinear complementarity problem(1) has a unique solution.

3 The relaxed asymmetric HSS-like iteration algorithm

In this section, we extend the nonlinear relaxed asymmetric HSS-like iteration method for a class of weakly nonlinear systems [7] to a class of weakly nonlinear complementarity problems.

Let $A = H + S$ be the Hermitian and skew-Hermitian splitting of A, where $H = \frac{1}{2}(A + A^*), S = \frac{1}{2}(A - A^*)$. Then we can present the following extrapolated relaxed Hermitian and skew-Hermitian splitting iteration algorithm for a class of weakly nonlinear complementarity problems, which can be considered as an extension of extrapolated Hermitian and skew-Hermitian splitting iteration algorithm for linear equations proposed in [7].

Algorithm 3.1 (The relaxed asymmetric HSS-like iteration algorithm)

Step 1. Initialization. Let $z^{(0)} \in R^n$ be any given initial value, se*t* $k = 0$.

Step 2. General iteration. Let $z^{(k,1)}$ be an arbitrary solution of the following weakly nonlinear complementarity problem : find $z \in R^n$, such that

$$\begin{cases} z \geq 0 \\ (\alpha I + H)z - \phi(z^{(k)}) \geq 0 \\ z^T((\alpha I + H)z - \phi(z^{(k)})) = 0 \end{cases} \tag{3}$$

Where $(z^{(k)}) = F(z^{(k)}) + (\alpha I - S)z^{(k)}$, α is a given nonnegative constant.

Step 3. General iteration. Let $z^{(k,2)}$ be an arbitrary solution of the following weakly nonlinear complementarity problems: find $z \in R^n$, such that

$$\begin{cases} z \geq 0 \\ (\beta I + S)z - \psi(z^{(k,1)}) \geq 0 \\ z^T((\beta I + S)z - \psi(z^{(k,1)})) = 0 \end{cases} \tag{4}$$

Where $(z^{(k,1)}) = F(z^{(k,1)}) + (\beta I - H)z^{(k,1)}$, β is a given positive constant.

Step 4. Relaxation iteration. Let $z^{(k+1)} = \omega z^{(k,2)} + (1 - \omega)z^{(k,1)}$, where $\omega \geq 0$ is a given real constant.

Step 5. Test for termination. If $z^{(k+1)}$ satisfies a prescribed stopping rule, terminate. Otherwise, return to Step 2 with k replaced by $k + 1$.

Remark1. When $\omega = 1$, algorithm 3.1 is the following asymmetric HSS-like iteration algorithm 3.2 :

Algorithm 3.2 (The asymmetric HSS-like iteration algorithm)

Step 1. Initialization. Let $z^{(0)} \in R^n$ be any given initial value, set $k = 0$.

Step 2. General iteration. Let $z^{(k+\frac{1}{2})}$ be an arbitrary solution of the following weakly nonlinear complementarity problem: find $z \in R^n$, such that

$$\begin{cases} z \geq 0 \\ (\alpha I + H)z - \phi(z^{(k)}) \geq 0 \\ z^T((\alpha I + H)z - \phi(z^{(k)})) = 0 \end{cases} \tag{5}$$

Where $(z^{(k)}) = F(z^{(k)}) + (\alpha I - S)z^{(k)}$, α is a given nonnegative constant.

Step 3. General iteration. Let $z^{(k+1)}$ be an arbitrary solution of the following weakly nonlinear complementarity problems: find $z \in R^n$, such that

$$\begin{cases} z \geq 0 \\ (\beta I + S)z - \psi(z^{(k+\frac{1}{2})}) \geq 0 \\ z^T((\beta I + S)z - \psi(z^{(k+\frac{1}{2})})) = 0 \end{cases} \tag{6}$$

Where $(z^{(k+\frac{1}{2})}) = F(z^{(k+\frac{1}{2})}) + (\beta I - H)z^{(k+\frac{1}{2})}$, β is a given positive constant.

Step 4. Test for termination. If $z^{(k+1)}$ satisfies a prescribed stopping rule, terminate. Otherwise, return to Step 2 with with k replaced by $k+1$.

Remark2. When $\omega = 0$, algorithm 3.1 is the following HSS-like iteration algorithm 3.3:

Algorithm 3.3 (The HSS-like iteration algorithm)

Step 1. Initialization. Let $z^{(0)} \in R^n$ be any given initial value, set $\mathrm{k} = 0$.

Step 2. General iteration. Let $z^{(k+1)}$ be an arbitrary solution of the following weakly nonlinear complementarity problem : find $\mathrm{z} \in \mathrm{R}^{\mathrm{n}}$, such that

$$\begin{cases} z \geq 0 \\ (\alpha I + H)z - \phi(z^{(k)}) \geq 0 \\ z^T((\alpha I + H)z - \phi(z^{(k)})) = 0 \end{cases} \tag{7}$$

Where $\phi(z^{(k)}) = F(z^{(k)}) + (\alpha I - S)z^{(k)}$, α is a given nonnegative constant.

Step 3. Test for termination. If $z^{(k+1)}$ satisfies a prescribed stopping rule, terminate. Otherwise, return to Step 2 with k replaced by $k+1$.

4 The convergence of relaxed asymmetric HSS-like iteration algorithmy

In this section, we establish the convergence theory for algorithm 3.1. Before the discussion of the convergence,we first introduce some useful lemmas.

Lemma 4.1: Let $A = M - N$, *where* A *and* M are H-matrix with positive diagonal elements respective. Let z^* be the solution of the weakly nonlinear complementarity problems(1), $z^{(k,1)}$ be an arbitrary solution of the following nonlinear complementarity problem:

$$\begin{cases} z \geq 0 \\ F(z^{(k)}) - Nz^{(k)} + Mz^{(k,1)} \geq 0 \\ z^T(F(z^{(k)}) - Nz^{(k)} + Mz^{(k,1)}) = 0 \end{cases}$$

Then, for an starting vector $z^{(0)} \in R^n$, the uniquely defined sequence of iterates $z^{(k)}$ satisfies

$$< M > |z^{(k,1)} - z^*| \leq |N||z^{(k)} - z^*| \tag{8}$$

Proof. To establish the expression (13), we first remark that $z^{(k)}$ is uniquely defined because M is an H-matrix with positive diagonal elementssee Lemma 2.1. We verify (8) component by component. Consider an arbitrary index j and assume that $|z^{(k,1)} - z^*|_j = (z^{(k,1)} - z^*)_j$.

Under this assumption, the inequality (13) holds clearly if $z_j^{(k,1)} = 0$, because the *j*-th component of the left hand vector in (8) is then nonpositive and the right-hand component is always nonnegative.

Now, suppose that $z_j^{(k,1)} > 0$.Then, according to $z^{(k,1)}$ be an arbitrary solution of the following weakly nonlinear complementarity problems:

$$\begin{cases} z \geq 0 \\ (\alpha I + H)z - \phi(z^{(k)}) \geq 0 \\ z^T((\alpha I + H)z - \phi(z^{(k)})) = 0 \end{cases}$$

We have $(F(z^{(k)}) - Nz^{(k)} + Mz^{(k,1)})_j = 0$ .On the other hand, we also have $(F(z^{(k)}) - Nz^* + Mz^*)_j \geq 0$. Subtracting the last two expressions and rearranging terms, we deduce

$$(M(z^{(k,1)} - z^*))_j \leq (N(z^{(k)} - z^*))_j$$

which implies $(< M > |z^{(k,1)} - z^*|)_j \leq (|N||z^{(k)} - z^*|)_j$. (Because M is an H-matrix with positive diagonal elements and $|z^{(k)} - z^*|_j = (z^{(k)} - z^*)_j$).

In a similar fashion, we may establish the same inequality (8) if $|z^{(k)} - z^*|_j = (z^* - z^{(k)})_j$. Consequently, the inequality (8) must hold.

Lemma 4.2: Let $= M_i - N_i (i = 1,2)$, where A and $M_i (i = 1, 2)$ are H-matrix with positive diagonal elements respective. Let z^* be the solution of the weakly nonlinear complementarity problem(1), $z^{(k,1)}$ be an arbitrary solution of the weakly nonlinear complementarity problem following(9), where $\phi(z^{(k)}) = F(z^{(k)}) + (\alpha I - S)z^{(k)}$, $z^{(k,2)}$be an arbitrary solution of the weakly nonlinear complementarity problem following(9), where $\psi(z^{(k,1)}) = F(z^{(k,1)}) + (\beta I - H)z^{(k,1)}$. Then, for any starting vector $z^{(0)} \in R^n$, the uniquely defined sequence of iterates series $\{z^{(k,1)}\}$ and $\{z^{(k,2)}\}$ which is created by algorithm 3.1 satisfies

$$|z^{(k,2)} - z^*| \leq < M_2 >^{-1} |N_2| < M_1 >^{-1} |N_1||z^{(k)} - z^*| \tag{9}$$

Proof. Let z^* be the solution of the weakly nonlinear complementarity problem(1). By Lemma 4.1 we have $< M_1 > |z^{(k,1)} - z^*| \leq |N_1||z^{(k)} - z^*|$. On the other hand, we also have $< M_2 > |z^{(k,2)} - z^*| \leq |N_2||z^{(k,1)} - z^*|$.

which implies $|z^{(k,2)} - z^*| \leq < M_2 >^{-1} |N_2||z^{(k,1)} - z^*| \leq < M_2 >^{-1} |N_2| < M_1 >^{-1} |N_1||z^{(k)} - z^*|$. Hence, (9) follows.

Lemma 4.3: Let $A \in R^{n \times n}$ be a positive definite matrix, $H = \frac{1}{2}(A + A^*)$ *and* $S = \frac{1}{2}(A - A^*)$ be its Hermitian and skew-Hermitian parts, $\alpha > 0$ is a given constant. $M(\alpha, \beta) = < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|$, Then its spectral radius $\rho(M(\alpha, \beta))$ is bounded by

$$\sigma(\alpha, \beta) = \max_{\sigma_j \in \delta(S)} \frac{\sqrt{\alpha^2 + \delta_j^2}}{\sqrt{\beta^2 + \delta_j^2}} \max_{\lambda_j \in \lambda(H)} \frac{|\beta - \lambda_j|}{|\alpha + \lambda_j|} \tag{10}$$

where $\lambda(H)$ is the spectral set of H and $\delta(S)$ is the singular-value set of S.

Proof. Setting $M_1 = \alpha I + H, N_1 = \alpha I - S, M_2 = \beta I + S, N_1 = \beta I - H$ in Lemma 4.1 and Lemma 4.2. Since $\alpha I + H$ and $\beta I + S$ are nonsingular for any nonnegative constant α and positive β, we have

$$\rho(M(\alpha, \beta)) = \rho(< \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|)$$

$$\leq || < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|||_2$$

$$\leq || < \beta I + S >^{-1} |\beta I - H|||_2| < \alpha I + H >^{-1} |\alpha I - S||||_2$$

$$= \max_{\sigma_j \in \delta(S)} \frac{\sqrt{\alpha^2 + \delta_j^2}}{\sqrt{\beta^2 + \delta_j^2}} \max_{\lambda_j \in \lambda(H)} \frac{|\beta - \lambda_j|}{|\alpha + \lambda_j|}$$

Then the bound for $\rho(M(\alpha, \beta))$ is given by (10).

Applying above lemma to the relaxed asymmetric HSS-like iteration algorithm, we obtain the following convergence property.

Theorem 4.1: Let $A \in R^{n \times n}$ be an H-matrix with positive diagonal elements, $F: R^n \to R^n$. is a given continuous differentiable nonlinear mapping. $H = \frac{1}{2}(A + A^*)$ and $S = \frac{1}{2}(A - A^*)$ be its Hermitian and skew-Hermitian parts. Let α be a nonnegative constant and β be a positive constant,

$$a = \max_{\sigma_j \in \delta(S)} \frac{\sqrt{\alpha^2 + \delta_j^2}}{\sqrt{\beta^2 + \delta_j^2}}, b = \max_{\lambda_j \in \lambda(H)} \frac{|\beta - \lambda_j|}{|\alpha + \lambda_j|}$$

$$\mu = \max\{|| < \alpha I + H >^{-1} ||_2, |||\alpha I - S|||_2\}$$

where $\lambda(H)$ is the spectral set of H and $\delta(S)$ is the singular-value set of S, Suppose that $ab < 1$. When

$$0 < \omega < 1, \mu < \frac{2(1-ab)}{(a+b)+\sqrt{(a-b)^2+4}} \tag{11}$$

Or

$$0 < \omega < \frac{2}{ab+1}, \mu < \frac{2(2-\omega-ab\omega)}{\omega(a+b)+\sqrt{\omega^2((a-b)^2-4)}} \tag{12}$$

Then, the iteration sequence $\{z^{(k)}\}$ generated by Algorithm 3.1 converges to the solution of the weakly nonlinear complementarity problem(1).

Proof. Setting $M_1 = \alpha I + H, N_1 = \alpha I - S, M_2 = \beta I + S, N_1 = \beta I - H$ in Lemma 4.1 and Lemma 4.2. Since $\alpha I + H$ and $\beta I + S$ are nonsingular for any nonnegative constant α and positive β. According to Algorithm 3.1 , we get

$$|z^{(k+1)} - z^*| = |\omega z^{(k,2)} + (1-\omega) z^{(k,1)} - z^*|$$

$$= |\omega (z^{(k,2)} - z^*) + (1-\omega)(z^{(k,1)} - z^*)|$$

$$\leq |\omega (z^{(k,2)} - z^*)| + |(1-\omega)(z^{(k,1)} - z^*)|$$

$$\leq \omega |z^{(k,2)} - z^*| + (1-\omega)|z^{(k,1)} - z^*|$$

$$\leq \omega < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S||z^{(k)} - z^*|$$

$$+(1-\omega) < \alpha I + H >^{-1} |\alpha I - S||z^{(k)} - z^*|$$

$$\leq [\omega < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|$$

$$+(1-\omega) < \alpha I + H >^{-1} |\alpha I - S|]|z^k - z^*|$$

Therefor, we have $|z^{(k+1)} - z^*| = T(\alpha,\beta,\omega)|z^k - z^*|$. where

$$T(\alpha,\beta,\omega) \leq \omega < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S| + (1-\omega) < \alpha I + H >^{-1} |\alpha I - S|.$$

Let $T(\alpha,\beta) =< \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|$, we have

$$||T(\alpha,\beta)|| \leq || < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S|||_2$$

$$\leq || < \beta I + S >^{-1} |\alpha I - S|||_2|||\beta I - H| < \alpha I + H >^{-1} ||_2$$

$$\leq \max_{\sigma_j \in \delta(S)} \frac{\sqrt{\alpha^2 + \delta_j^2}}{\sqrt{\beta^2 + \delta_j^2}} \max_{\lambda_j \in \lambda(H)} \frac{|\beta - \lambda_j|}{|\alpha + \lambda_j|} = ab$$

Therefor

$$\|T(\alpha,\beta,\omega)|| = ||\omega T(\alpha,\beta)||_2$$

$$\leq ||\omega < \beta I + S >^{-1} |\beta I - H| < \alpha I + H >^{-1} |\alpha I - S| + (1-\omega) < \alpha I + H >^{-1} |\alpha I - S|||_2$$

$$\leq ||\omega T(\alpha,\beta)||_2 + ||(1-\omega) < \alpha I + H >^{-1} |\alpha I - S|||_2$$

$$\leq |\omega|ab + |1-\omega|\sqrt{|| < \alpha I + H >^{-1} ||_2||\alpha I - S||_2} \leq \omega ab + (1-\omega)\mu$$

When ω, a, b satisfies equation (11) or satisfies equation (12), we have $\omega ab + (1-\omega)\mu < 1$.

Therefor, $\rho(T(\alpha,\beta,\omega) \leq ||T(\omega,a,b)|| < 1$, Then, the relaxed asymmetric HSS-like iteration algorithm 3.1 is convergent. This completes the proof.

5 Conclusions

For a class of weakly nonlinear complementarity problems, considering the matrix multisplitting, Hermitian and skew-Hermitian splitting and parallel computing, presented the relaxed asymmetric HSS-like iteration algorithm, the asymmetric HSS-like iteration algorithm and the HSS-like iteration algorithm for solving weakly nonlinear complementarity problems, when the coefficient matrix is positive definite or diagonal elements are H-matrix, discussed the global convergence of the algorithm.

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