A new multimodal planning algorithm based on PRM

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Abstract. The robot motion planning problem has a unique multimodal structure, where the space of feasible configuration consists of intersecting submanifolds. The planning problem of reconfigurable robot leg motions was considered as a multimodal structure composed of intersecting submanifolds in different dimensions. After that, a new multimodal planning algorithm based on PRM was proposed. Simulation results showed that the new algorithm had a shorter running time than PRM in different modes.

Keywords: Robot, PRM, Submanifold, Multimodal.

1 Introduction

The PRM (Probabilistic Roadmap) planning algorithm is applied for high-dimensional configuration space motion planning under geometrically complex feasibility constraints [1]. With a suitable sampling configuration of probability measure, the connectivity of the robot's feasible space is approximated and connected by simple paths. However, the PRM planning algorithm is not suitable for multimodal structures consisting of intersecting submanifolds in different dimensions [2]. This multimodal structure appears in the planning problem of manipulating leg motions of reconfigurable robots [3-4]. In this structure, each submanifold corresponds to a mode. A set of fixed contact points is maintained between the robots and the environment (or between the robots). The planning algorithm consists of discrete modal sequences and continuous single-mode paths (namely joint spatial motions for contact changes). In a multimodal structure, the PRM planning algorithm cannot explain whether a path exists or not. When a single-modal query fails, the feasibility of the query cannot be judged. It needs more time to query whether the path exists. Therefore, the work put forward MM-PRM for finite multimodal problems. The new algorithm builds a cross-modal PRM by sampling configurations in transitions between modalities (corresponding to intersections of submanifolds). If all modes are extensible when being restricted to insert submanifolds, the new algorithm, as the classical PRM planner, will find

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a feasible path with an improved convergence rate. It is determined that the expected running time is finite, with finite variance.

2 PRM planning algorithm and non-extended space

The PRM planning algorithm approximates the connectivity of a feasible subset of robot configuration space $\mathbb{Z}$ using a configuration roadmap (called milestone) connected by simple paths (usually straight lines). Meanwhile, the concept of expansion degree is introduced to characterize the convergence rate of the roadmap.

The expansion degree of $\mathbb{Z}$ is measured by three parameters $\varepsilon, \alpha, \beta$ ($\varepsilon, \alpha, \beta \in [0,1]$). These values depend only on the relative volume of some subset of $\mathbb{Z}$, rather than the dimensionality. If all three parameters have positive values, then $\mathbb{Z}$ is extended; otherwise, $\mathbb{Z}$ is non-extended [5-6]. In all extension spaces, the query probability of the basic PRM planning algorithm decreases exponentially as the roadmap grows. Larger $\varepsilon, \alpha, \beta$ leads to a larger convergence rate.

In a multimodal structure, $\mathbb{Z}$ is non-extended if a cusp is included ($\varepsilon = 0$). However, PRM still works well in a position far from the cusp because the structure is expanded to maintain its connectivity after removing the tiny areas around the cusp. If it contains the regions in different dimensions ($\alpha = 0$ or $\beta = 0$), then $\mathbb{Z}$ is also non-extended. For example, a PRM planner cannot find a path connecting the 2D regions even in an arbitrary time in a space consisting of two 2D regions connected by a 1D curve.

Variable dimensions are inherent in the multimodal problem structure. If the number of modalities is finite, then the submanifolds with joint forms can be enumerated from the problem definition. However, the number can be enormous.

3 Definition of robot multimodal problem

A hybrid system must be planned in major planning problems, where the state space consists of a discrete finite (or countably finite) set called the modal space. In other words, the system moves between sets and modalities. Depending on different current modalities, the robot is constrained to traverse some lower-dimensional subspace of the actual space.

A robotic system with contact moves between modalities, where each modality defines a contact state. For example, this concept was applied for leg movements in Ref. [10], where the modality refers to a set of footholds constraining the foot on the ground. The mutual transition of the modalities ensures walking. Manipulation planning also exhibits a multimodal structure. When a robot pushes an object at a fixed point of contact, it traverses the same modality. However, the robot has to go through the specified modal without contact before reaching the modal of new points. Different from traditional robotic motion planning algorithms, multimodal planner reaches the goal through discrete mode conversion sequence, thus achieving continuous path within each modality.

It is considered that a robot can move between finite sets $\sum$ of modalities, where each modality $\psi \in \sum$ corresponds to a specific feasible space $\mathbb{Z}_{\psi}$ ($\mathbb{Z} = \cup_{\psi \in \sum} \mathbb{Z}_{\psi}$). It is denoted that q is a certain configuration of the robot in the modality ψ. The robot must plan a path in the feasible space $\mathbb{Z}_{\psi^*}$ for the configuration q^* in $\mathbb{Z}_{\psi} \cap \mathbb{Z}_{\psi^*}$, thus switching to the new modality $\psi *$. Wherein $\mathbb{Z}_{\psi} \cap \mathbb{Z}_{\psi^*}$ is the transition region between ψ and $\psi *$; q^* the transition configuration. A modality has two feasibility constraints.

(1) Dimensionality reduction constraint is often expressed as Eq. $\delta_{\psi}(q) = 0$, which defines submanifold $\mathbb{R}_{\psi}$ as a set of configurations satisfying these constraints.

(2) Volume reduction constraint is often expressed as $\square_{\psi}(q) > 0$. This may lead to null of $\mathbb{Z}_{\psi}$. Otherwise, it has the same dimension but a smaller volume.

In robot leg motions, ψ is defined by a set of fixed steps; $\mathbb{Z}$ can be embedded in the configuration space $\mathbb{C}$, consisting of the base parameters of the free-floating robot and the joint angles of the robot. A submanifold of lower dimension excluding singularities $\mathbb{C}_{\psi}$ is defined in turn by the kinematic constraints imposed at the foothold [7-8]. Collision avoidance and static stability constraints are reduced in volume [9-10]. Besides, $\mathbb{Z}_{\psi}$ is restricted in Subset $\mathbb{C}_{\psi}$.

4 Multimodal planning algorithm (MM-PRM)

A multimodal planning algorithm (MM-PRM) builds a PRM across all modalities. It is assumed that there are *m* modalities ($\sum = \{\psi_1, \dots, \psi_m\}$). For each modality ψ_i, the algorithm maintains a roadmap $\Re_{\psi_i}$ of $\mathbb{Z}_{\psi_i}$. The modal sampler ψ_i is used to randomly sample the configuration q from $\mathbb{C}_{\psi_i}$. If q belongs to $\mathbb{Z}_{\psi_i}$, the sampler returns to q; otherwise, it fails to return. The multimodal PRM gives the robot's starting and target configurations q_{start} and q_{goal}, and an iteration limit N. The algorithm steps are described as follows.

(1) Add q_{start}, q_{goal}, corresponding modalities ψ_{start} and ψ_{goal} as milestones to the roadmap.

(2) For each modality ψ_i, modal sampler ψ_i is used to sample configuration q. If the sampling is successful, then q is added to roadmap $\Re_{\psi_i}$ as a new milestone.

(3) For each pair of adjacent modalities ψ_i and ψ_j, configuration q is sampled by sample transmitter (ψ_i, ψ_j). If it is successful, then q is added to $\Re_{\psi_i}$ and $\Re_{\psi_j}$, and connected to the corresponding milestone.

(4) Repeat Steps (2) and (3) for N times.

An aggregated roadmap is built by connecting the roadmaps when matching transition configurations. If q_{start} and q_{goal} are connected by the paths in $\Re$, then the connection is successful; otherwise, it fails to return.

5 Expected runtime

If there is a solution, then the MM-PRM algorithm will find a path with a probability of 1, which is probabilistically complete (see Ref. [2]). However, this condition is not sufficient to guarantee the bounded average runtime. For example, if the probability of failure in N iterations is $1/N$, then the expected runtime is infinite. Since MM-PRM is exponentially convergent, the number of iterations to find a solution has a finite expected value and variance.

The expected runtime of MM-PRM is restricted with the worst behavior. Random variable N_W is denoted as the number of iterations at which MM-PRM terminates under the worst behavior. In MM-PRM, the failure probability e^{-dn} has a tight bound; the cumulative distribution function of N_W can be expressed as $\max\left\{0,1-ce^{-dn}\right\}$. If $N \le \log c/d$, then ce^{-dn} is not less than 1. Therefore, the probability of $N_W \le \log c/d$ is zero. For simplicity, $\log c/d$ is assumed as an integer. If N is larger than $\log c/d$, the boundary decreases geometrically. Therefore, $N_W - \log c/d$ can be defined as random variable X. For X, the geometric probability accumulation function is e^{-dx}; the expected value $E\left[X\right]=1\left(1-e^{-d}\right)$. Therefore, the expected number of iterations is bounded.

$$E\left[N\right] \le E\left[N_W\right] = \log c/d + 1\left(1-e^{-d}\right) \tag{1}$$

The expected runtime is limited. The definition of variance is applied to obtain Eq. (2), thus restricting the variance in runtime.

$$\begin{aligned} Var(N) &= E\left[N^2\right] - E\left[N\right]^2 \\ &\le E\left[N^2\right] \le E\left[N_W^2\right] \end{aligned} \tag{2}$$

Let $N_0 = \log c/d$. $E\left[N_W^2\right]$ is expressed as follows.

$$\begin{aligned} E\left[N_W^2\right] &= \sum_{N=N_0+1}^{\infty} N^2 e^{-d(N-N_0)} \\ &= \sum_{N=1}^{\infty}\left(N+N_0\right)^2 e^{-dN} \\ &= \sum_{N=1}^{\infty}\left(N^2+2NN_0+N_0^2\right)e^{-dN} \\ &= \sum_{N=1}^{\infty} N^2 e^{-dN} + 2N_0\sum_{N=1}^{\infty} Ne^{-dN} \\ &+ N_0^2\sum_{N=1}^{\infty} e^{-dN} \end{aligned} \tag{3}$$

First summation $E\left[X^2\right]$ in Eq. (3) is calculated to obtain

$$E\left[X^2\right] = Var(X) + E[X]^2 \\ = \frac{e^{-d}}{(1-e^{-d})^2} + \frac{1}{(1-e^{-d})^2} \quad (4)$$

The second summation in Eq. (3) is equal to $E\left[X\right] = 1/\left(1-e^{-d}\right)$; the third summation is 1. These values in Eq. (3) are substituted to obtain the boundary of $Var(N)$ after some additional operations.

$$Var(N) \le \frac{e^{-d}}{(1-e^{-d})^2} + \left(\frac{1}{(1-e^{-d})} + \frac{\log c}{d}\right)^2 \quad (5)$$

6 Simulation

The robot moves in the three-dimensional space (see Fig. 1). However, it only moves in the two-dimensional plane of the cube due to obstacles. The cube provides four surfaces to the configuration space (the rest do not belong to the space), where there are two rectangular barriers to form a narrow channel.

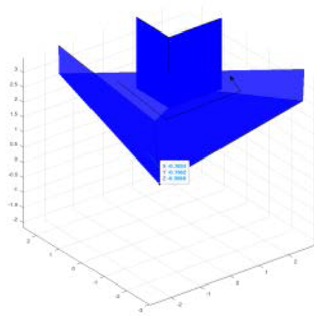

Fig. 1. 3D motion space of the robot.

Fig. 2 shows the runtime of PRM and MM-PRM at different modalities. It is found that MM-PRM has smaller runtime than PRM in different modes.

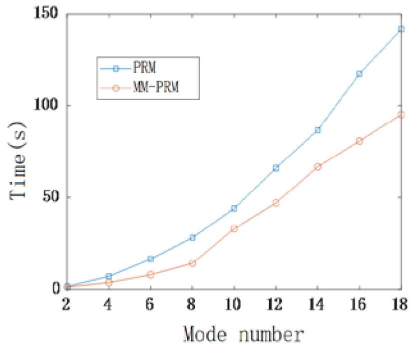

Fig. 2. Runtime of PRM and MM-PRM in different modes.

7 Conclusion

The work introduced the traditional PRM planning algorithm. After that, the planning problem of reconfigurable robot leg motions was considered as a multimodal structure composed of intersecting submanifolds in different dimensions to propose an MM-PRM planning algorithm. Simulation results showed that MM-PRM had a shorter runtime than PRM in different modes.

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References

1. S.X. Zou, P. Wang, X. Han, 2019, Improved Path Planning Algorithm Based on PRM, *Modular Machine Tool & Automatic Manufacturing Technique*, 539 (01): 6-8.
2. K. Hauser, J.C. Latombe, 2010, Multi-modal Motion Planning in Non-expansive Spaces, *International Journal of Robotics Research*, 29 (7): 897-915.
3. F. Xu, 2016, Robot Obstacle Avoidance and Path Planning Based on Improved Artificial Potential Field Method, *Computer Science*, 43 (12): 293-296.
4. T.F. Xu, Q. Luo, H. Wang, 2015, Dynamic Path Planning of Mobile Robots Based on Vector Fields, *Computer Science*, 42 (5): 237-244.
5. L.X. Wei, S.K. Wu, H. Sun, 2019, Path Planning of Mobile Robots Based on Multi-behavior, *Control and Decision*, v. 34 (12): 219-224.
6. M. Abidin, S. Azimi, S. Buyamin, et al., 2021, Multi-objective Route Planning for Underwater Cleaning Robot in Water Reservoir Tank, *Journal of Intelligent & Robotic Systems*, 101(9): 1-16.
7. H.M. Wu, M.J. Su, Y.S. Guan, 2016, Multi-robot Collision-free Motion Planning with Dynamic Adjustment of Operation Sequence, *Robot*, (6).
8. T.H. Luo, Q.Y. Li, X.Y. Ma, 2018, Multimodal Perception of Pipeline Dredging Robot in Unstructured Environment, *Mechanical Science and Technology*, v. 37; No.279 (05): 33-39.
9. Z.K. Zhang, 2018, *Multimodal Object Recognition and Localization Technology for Intelligent Service Robots*, University of Science and Technology of China.
10. R. Kala, 2019, On Sampling inside Obstacles for Boosted Sampling of Narrow Corridors, *Computational Intelligence*, 35 (2): 430-447.